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# NCERT SOLUTIONS

## TEXTBOOK+EXEMPLAR

### CHAPTERWISE & TOPICWISE

**New NCERT Textbook (Edition 2018-19) Exercises with Solutions**
**New NCERT Exemplar (Edition 2018-19) Problems with Solutions**

# CLASS 11

## PHYSICS

OSWAAL EXAM TOOLS

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# CLASS 11

## PHYSICS


**Includes :**

- Previous Years' Questions
- Mind Maps
- QR Codes for Informative Videos

WITH MIND MAPS

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# CONTENTS



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# PREFACE

In order to maintain a **Uniform Education System** across the country, many **State Boards** are implementing a **Single Curriculum System** via National Council of Educational Research & Training **(NCERT).** In the best interest of Secondary and Senior Secondary students, the Department of Education in Science & Mathematics (DESM) and NCERT have together developed Exemplar Problems in Science & Mathematics. These include practice questions of various typologies and difficulty levels which aid in in depth learning of concepts. They also contain conceptual problems which are a part of the CBSE Board Syllabus as well as the Syllabus of various Competitive Exams like IIT JEE, NEET, AIIMS, etc.

Considering the diversity and the varied difficulty level of questions in these two subjects, we, at **Oswaal Books** have launched **OSWAAL NCERT SOLUTIONS.** They are a compilation of all the Questions of the **Latest Editions of NCERT Textbook & NCERT Exemplar** in a Chapter-wise & Topic-wise format along with their complete solutions. These also include **Previous Years' Examination Questions** fully solved with their respective sources.

We believe that **OSWAAL NCERT SOLUTIONS** will help the students in school and after school in practicing and preparing extensively for both, Final Examinations as well as Competitive Examinations with utmost confidence!

**Some Special Features of Oswaal NCERT Solutions are:**

- ***Chapter-wise & Topic-wise*** presentation
- ***Chapter Objectives :*** A sneak peek into the chapter
- ***Mind Map :*** A single page snapshot of the entire chapter
- ***Quick Review :*** Concept-based study material
- ***Tips & Tricks :*** Useful guidelines for attempting each question perfectly
- ***Some Commonly Made Errors :*** Most common and unidentified errors made by students discussed
- ***Expert Advice :*** Oswaal Expert Advice on how to score more!
- ***Oswaal QR Codes :*** For Quick Revision on your Mobile Phones & Tablets

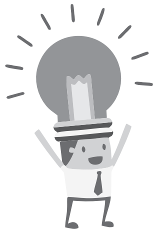
We hope that **OSWAAL NCERT SOLUTIONS** will help you at every step as you move closer to your educational goals. We wish you all great success ahead!!

**All the Best !**

**TEAM OSWAAL**

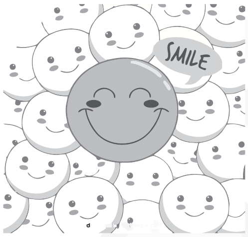
# Be Happy - It's all that matters !!

Students, remember that most of your stress comes from the way that you respond to different situations in life. This includes preparing yourself for your school/board examinations. We at Oswaal Books bring you our Success Mantra that will train you to adjust your attitude and get rid of all that unnecessary stress.



### Begin with a positive attitude

You must feel blessed that your life has a meaning. You should be privileged to have access to unlimited knowledge, good books, good teachers and loving parents.

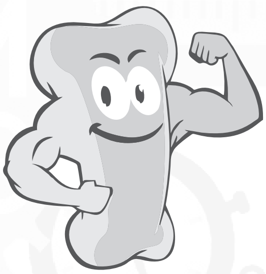



### Be happy & believe in yourself

A happy child learns better and performs better. If you believe in yourself you can be more helpful towards your fellow students/classmates and most importantly, you will realize that you have the courage to step over your insecurities and succeed.

### Being physically active

Active students are happier, healthier and more satisfied with life. Children must play outdoor games in order to get stronger muscles & bone, increased stamina, better sleep and a sharp mind.



### Perseverance & Passion

These are the two secrets of academic success. Gritty students are more likely to achieve their goals before others.

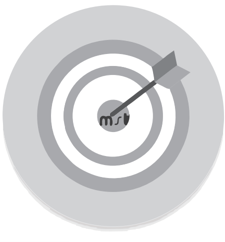
How to be Grittier

(a) Pick your Goal.

(b) Make mistakes - Every mistake made increases your chance to make progress. No matter how many mistakes you make or how slow you progress, you are still ahead of everyone who is not trying

(c) Accept that failures are a part of hardwork. Don't let them discourage you.

(d) Never give up.



### Focus on studying, rather than getting good grades

**Why?**

Students who study to understand do better than those who aim at getting good scores. Students who study to understand hit the books more often, approach various information sources, and feel less anxious about examinations. If, as a student, you keep in mind the above aspects it would definitely keep the negative emotions at bay and add to your happiness level.

# Writing a Perfect Answer Simplified !

OSWAAL EXAM TOOLS

Imagine you are seated in the exam hall, waiting to get your hands on that much anticipated piece of paper- THE EXAMINATION PAPER!! You are loaded with a ton of information in your brain. Your finger tips are numb and your face blank! It is the time to get down to serious business!

As another Exam Tool, Oswaal Books teaches you how to give every Question your best shot and write the PERFECT ANSWER everytime!!

**STEP 1**

- Student should write the data / information as given in the question carefully

**STEP 2**

- Student should write the concept / formula which is applicable in the given question
- If the question asked is related to a particular law / phenomena, then the student should write the name of that law / phenomena carefully

**STEP 3**

- Check the nature of the question –

  If the question is based on a **Theoretical Concept** :

  - After writing all the details given in the question, the student should be able to identify the concept applicable in the question
  - The student should then explain all the required points as asked in given question

  If the question is based on a **Numerical** :

  - Using the required Formula, the student should insert the particular data / values which are given / assumed
  - Then the numerical can be solved to get the final answer

# POSITIVE AFFIRMATIONS

*Our mind starts believing what we repeatedly think or say. We, at Oswaal Books resonate with this belief. So, we want all our readers to create their own affirmations ! A positive affirmation is something spoken aloud that you want to believe or want to be true. Repeating positive affirmations daily can help shift your internal dialogue from negative to positive.*

***So lets get started !***

1. I enjoy the subject I am studying.
2. During exams, I recall information quickly and easily.
3. Getting good grades is natural for me.
4. ........................................................................................................................
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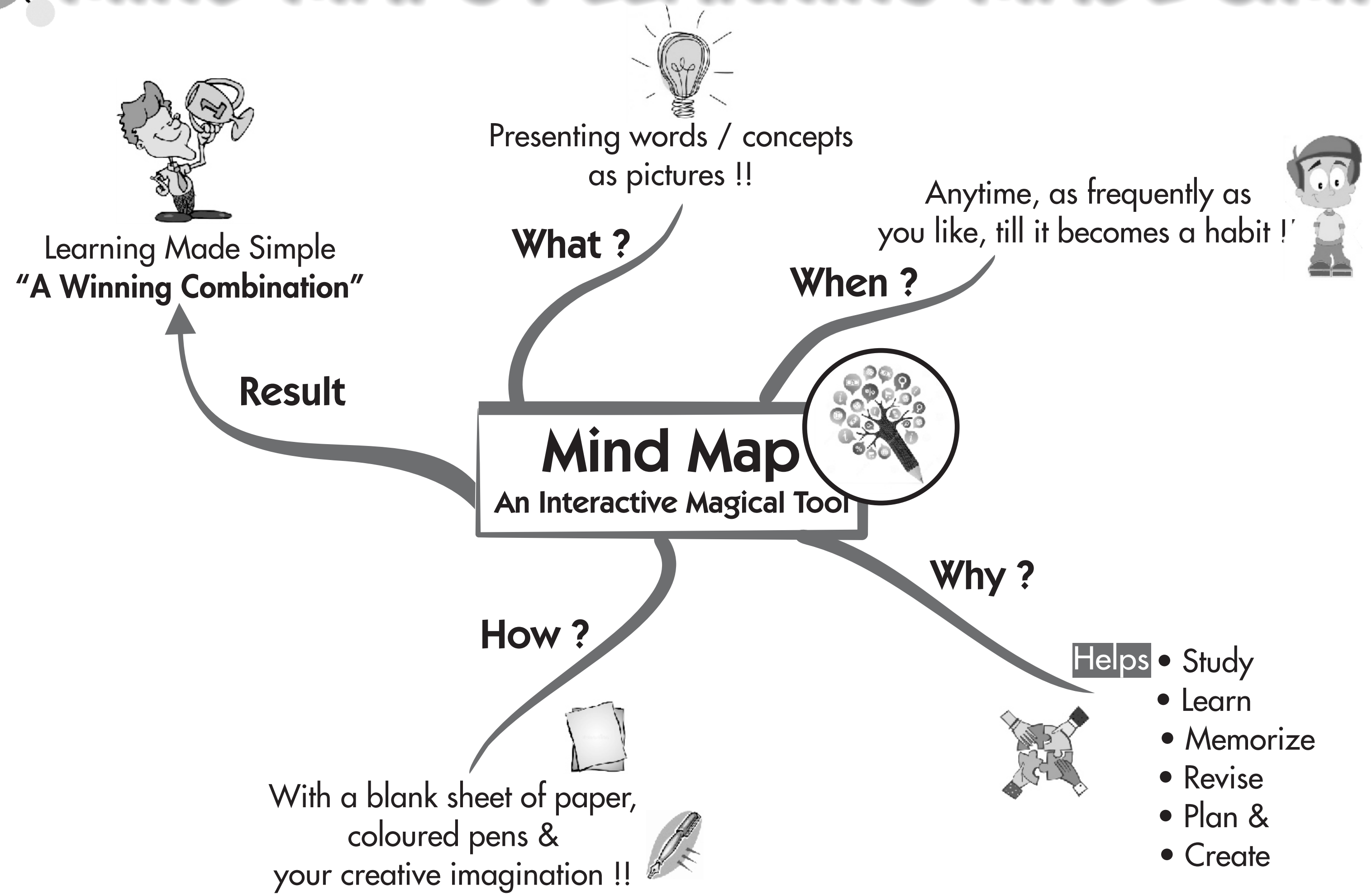
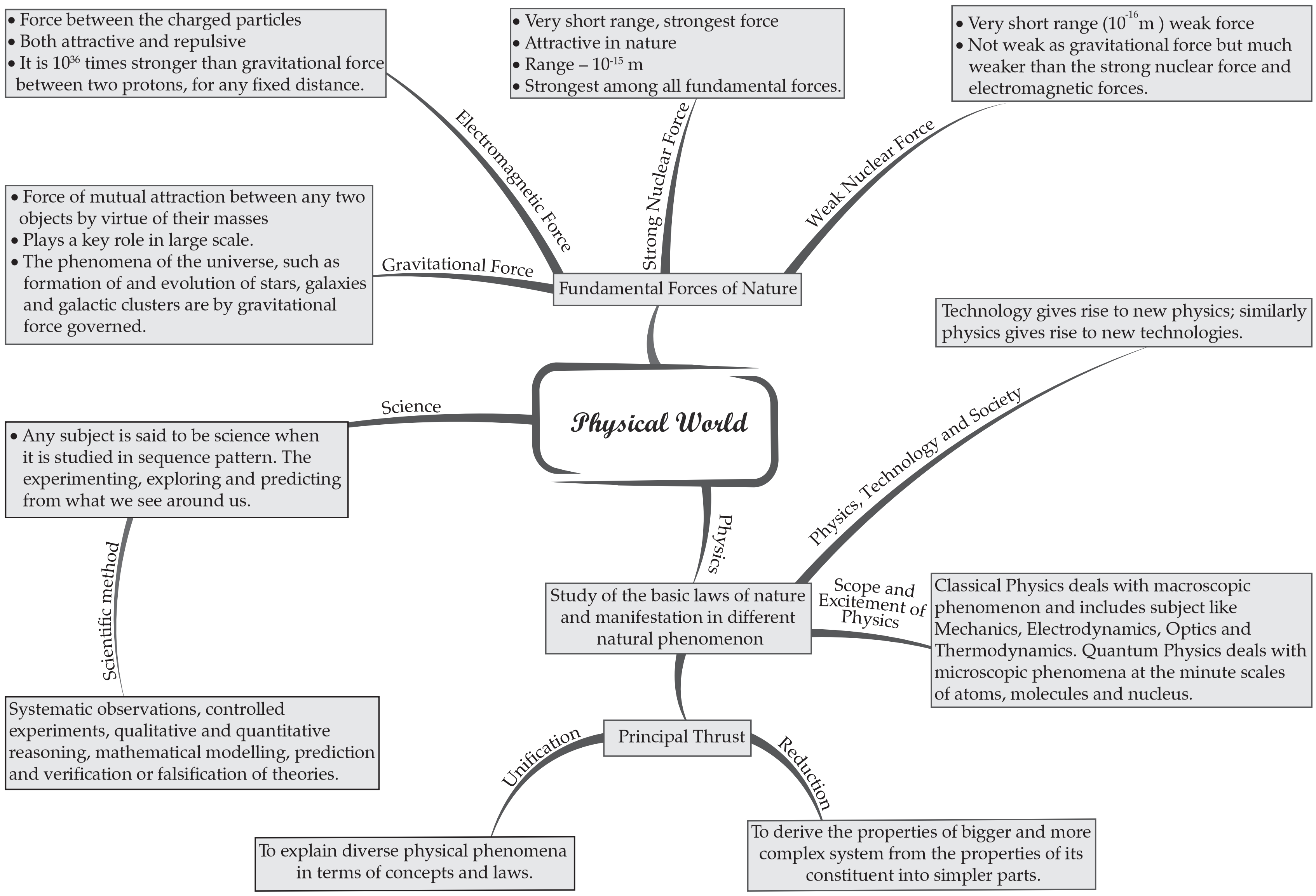
# MIND MAPS : LEARNING MADE SIMPLE



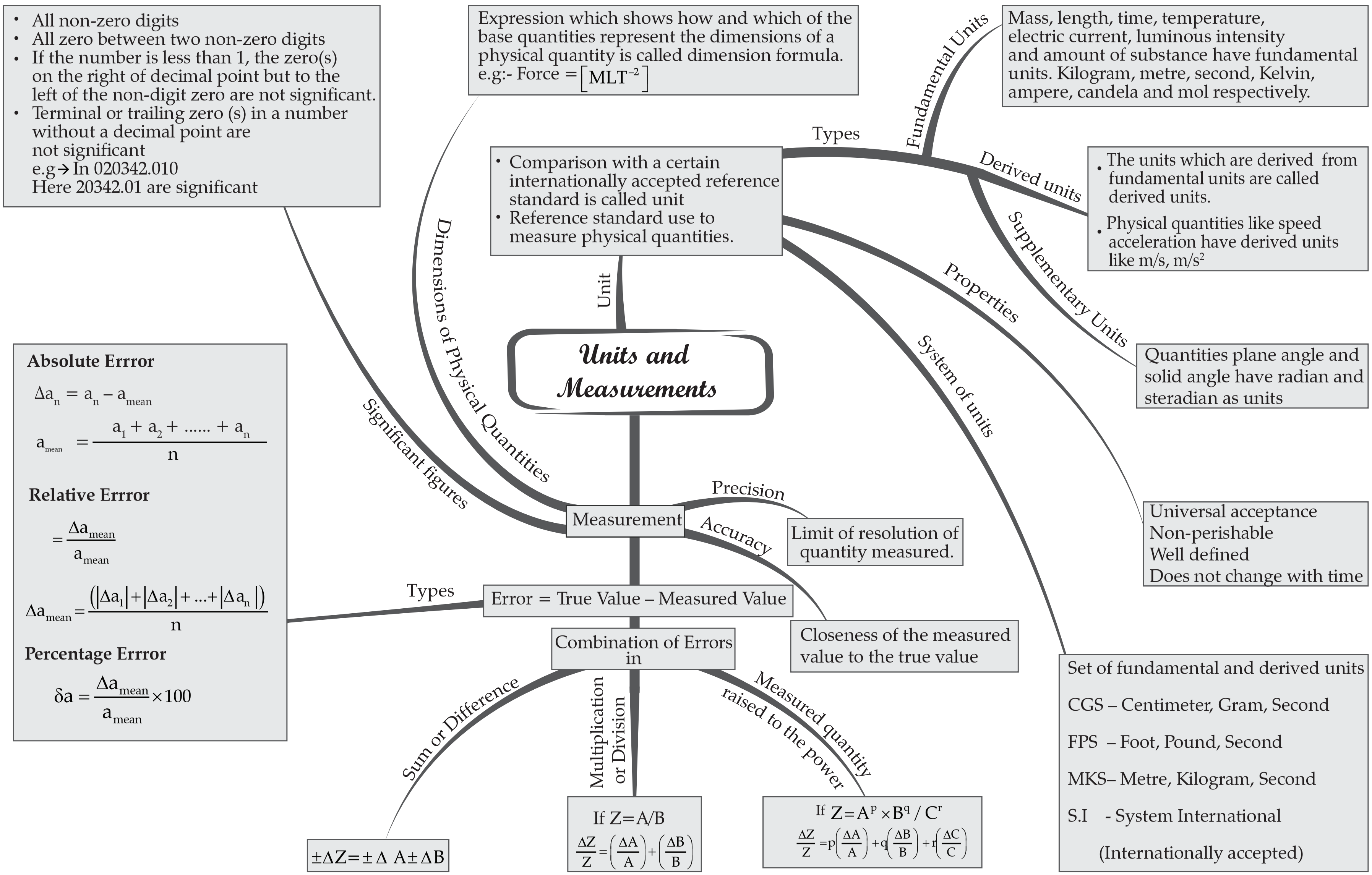

**"Mind Maps" are a novel feature of these Question Banks. Mind maps have been given for each chapter in the beginning, to help students understand all the topics & concepts better.**

# MIND MAP : LEARNING MADE SIMPLE
CHAPTER - 1

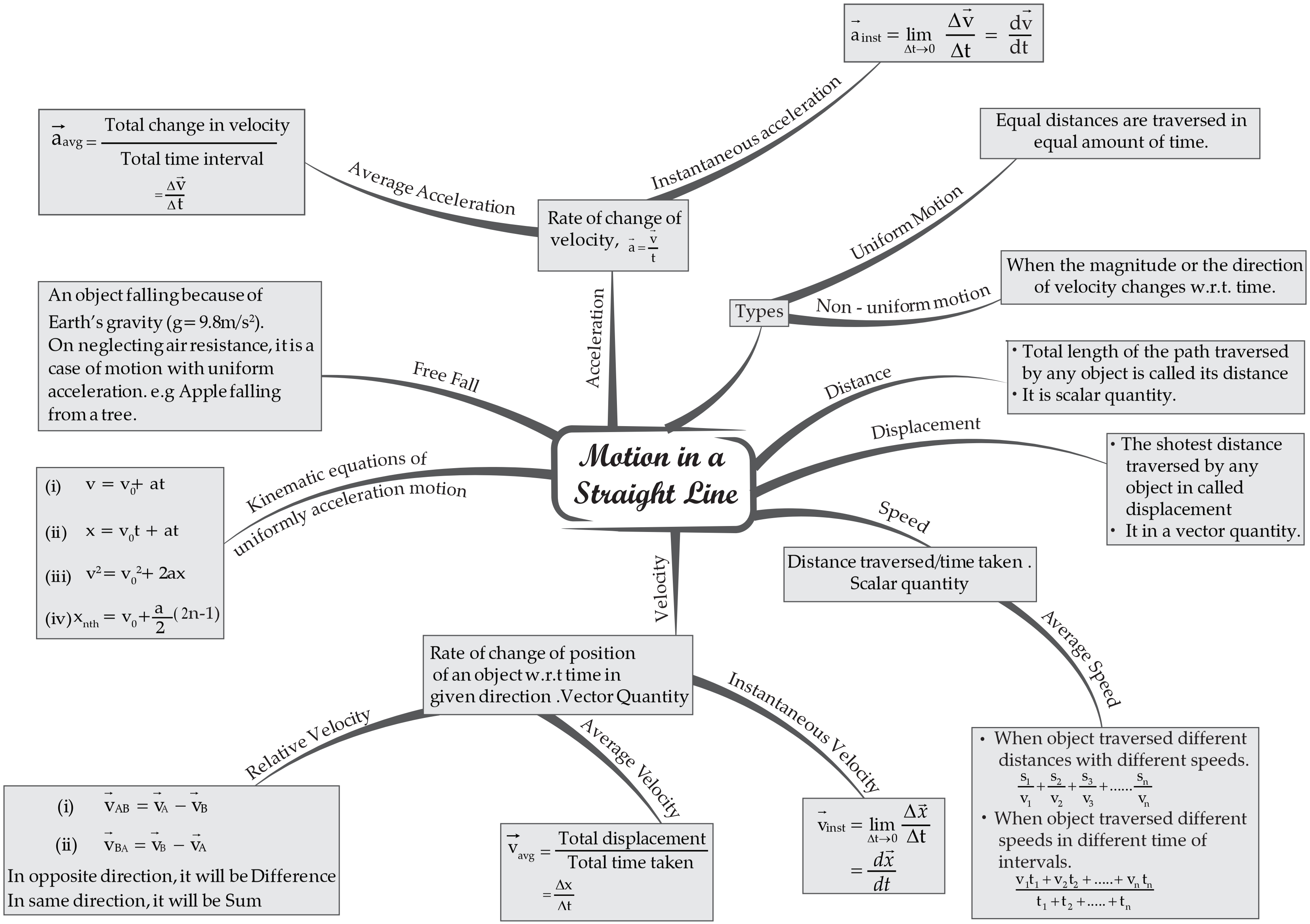


# MIND MAP : LEARNING MADE SIMPLE

CHAPTER - 2

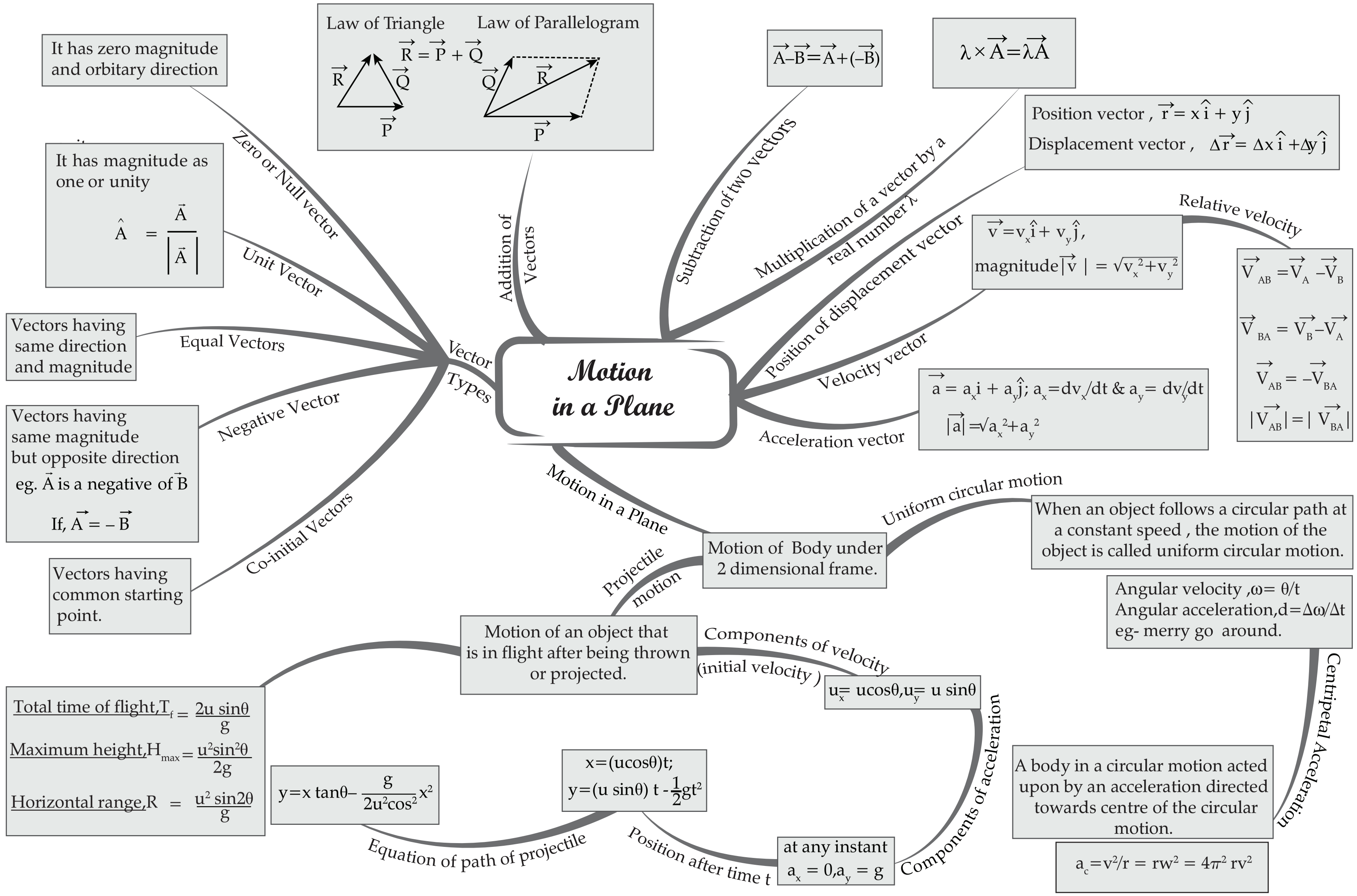


# MIND MAP : LEARNING MADE SIMPLE CHAPTER - 3

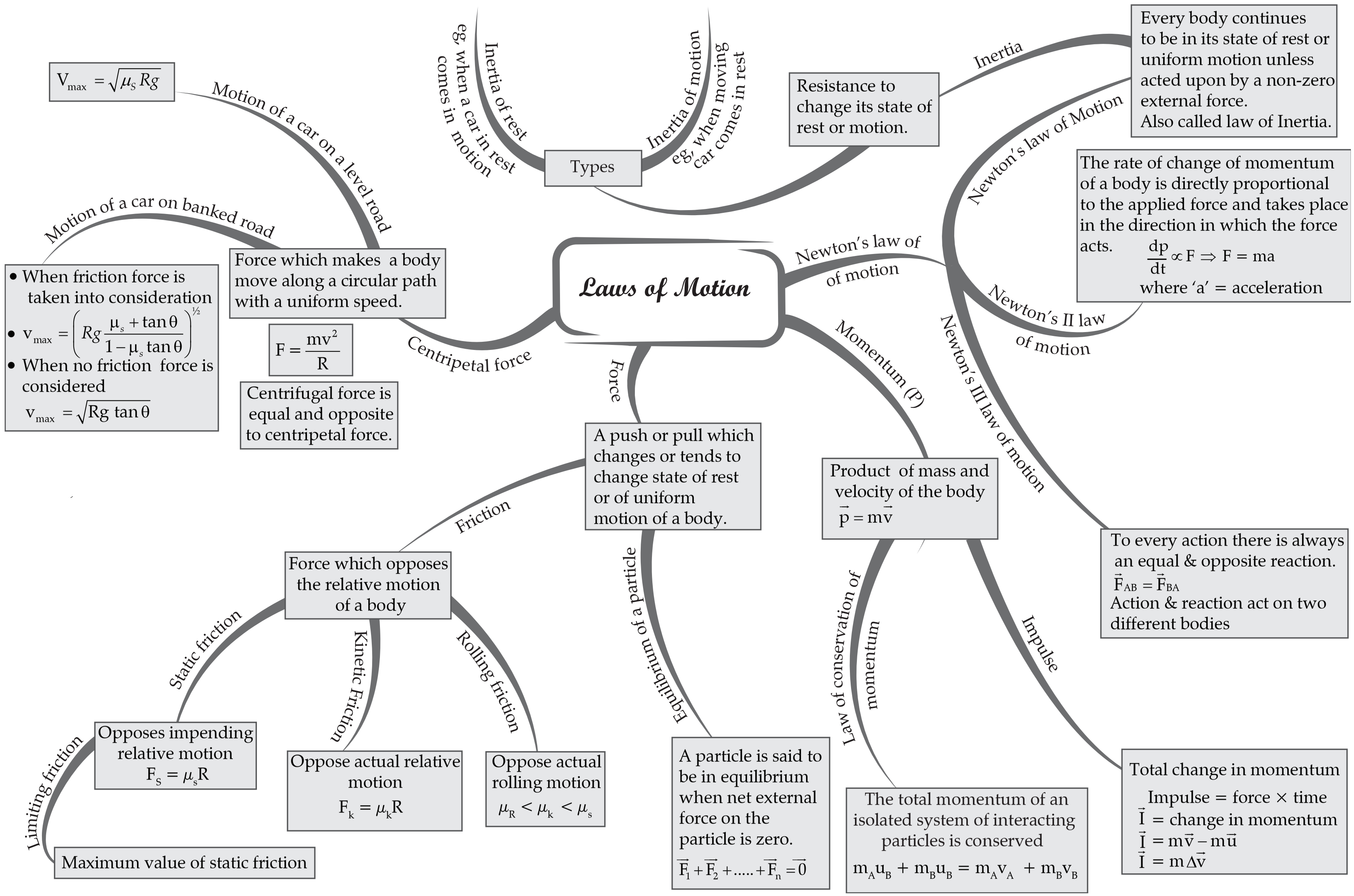


# MIND MAP : LEARNING MADE SIMPLE

## CHAPTER - 4



# MIND MAP : LEARNING MADE SIMPLE CHAPTER - 5



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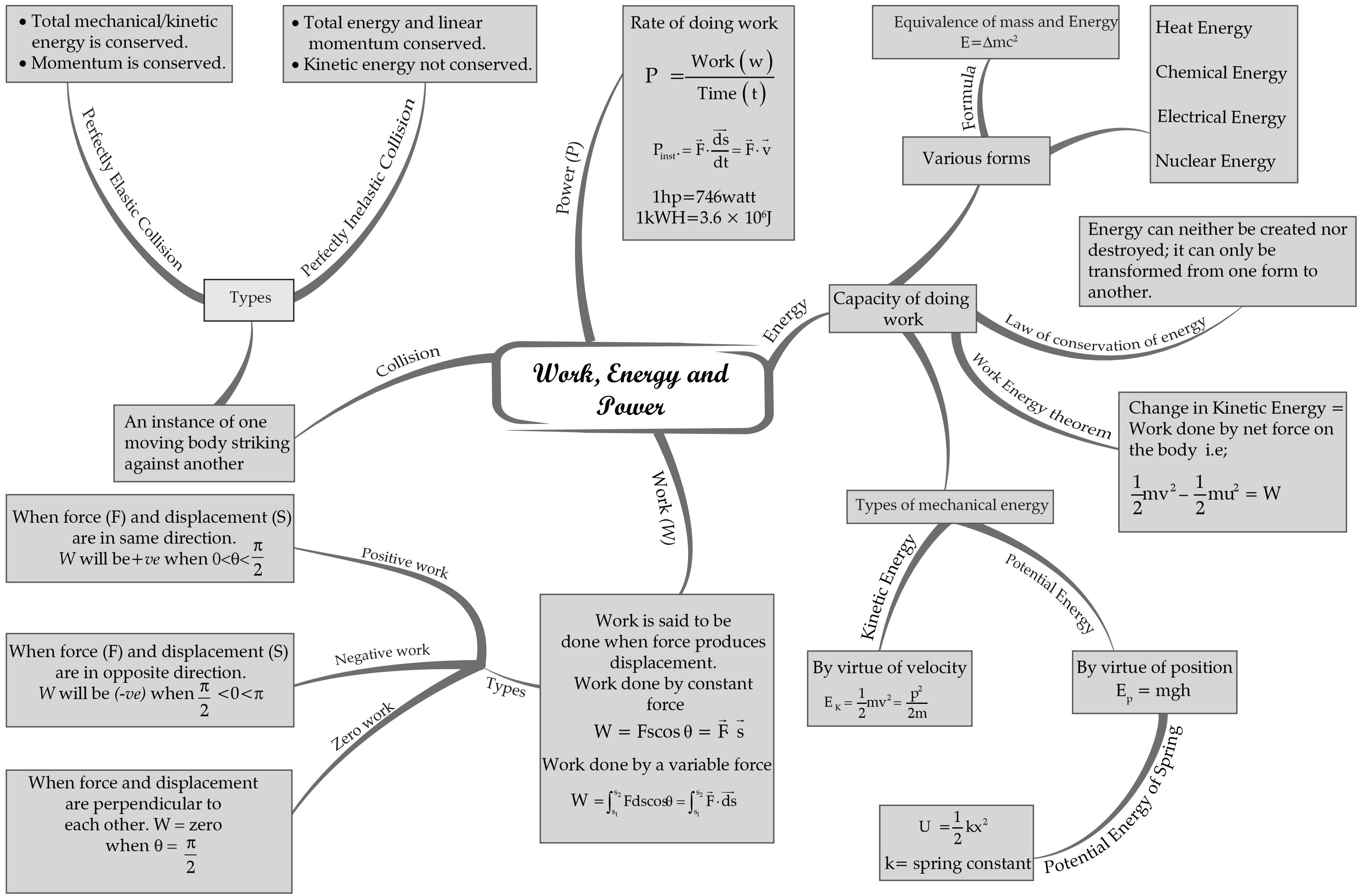

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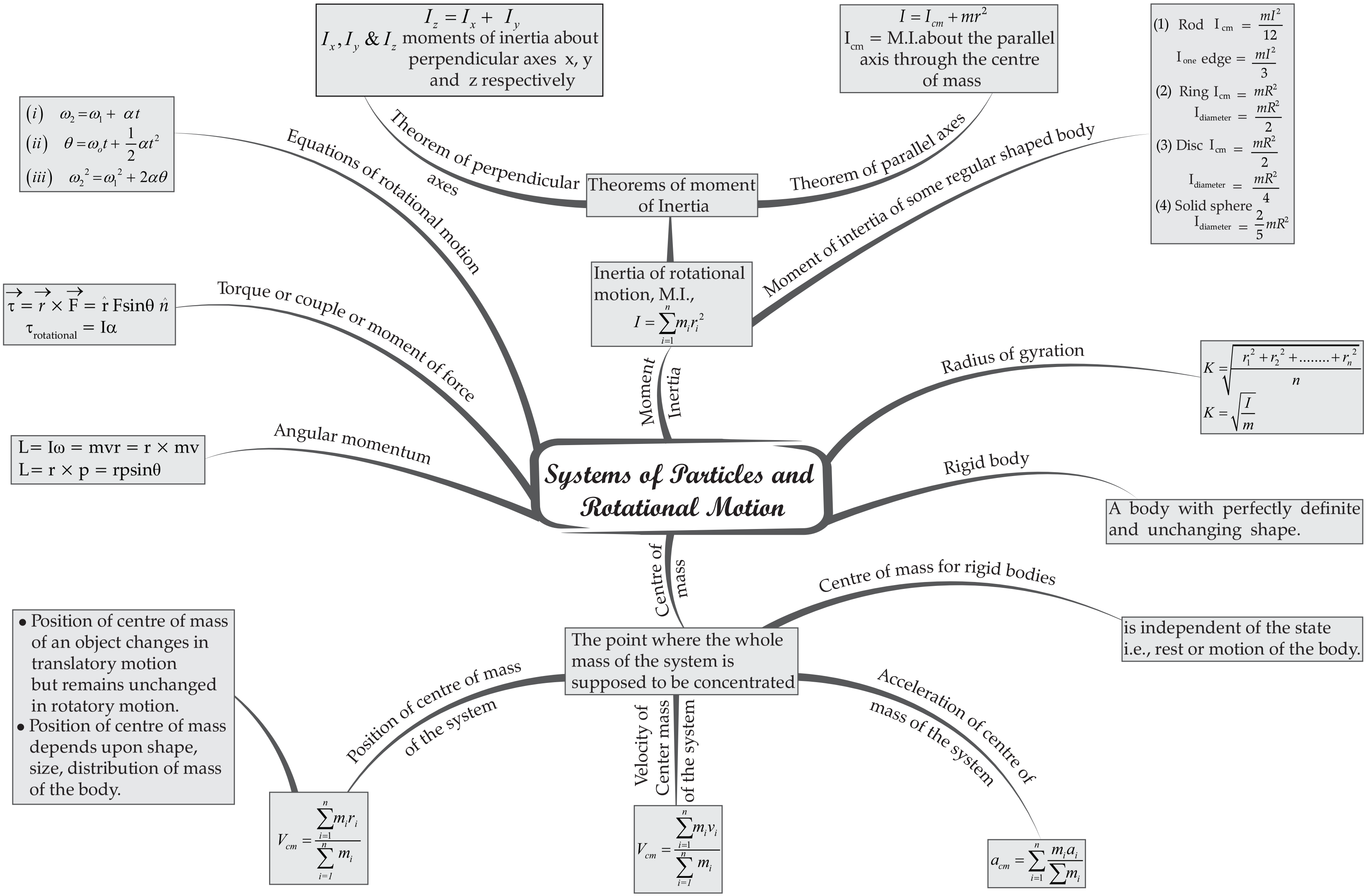


# MIND MAP : LEARNING MADE SIMPLE — CHAPTER - 8

## Gravitation

### Satellite
Heavenly object that revolves around a planet

**Geostationary satellite**
- Height from surface of earth =36000km
- Radius of orbit = 42,400km
- Time periods = 24 hours
- Orbital velocity = 3.1km/s

**Polar satellite**
- Height from earth's surface = 880km
- Time period = 84min
- Orbital velocity = 7.92km/s

**Escape and orbital speed**
- Escape speed $v_{esc} = \sqrt{2GM/R} = \sqrt{2gR}$
- Earth 's $v_{esc}$ = 11.2km/s
- Orbitals speed $v_{orb} = \sqrt{gR} = \frac{v_e}{\sqrt{2}}$
- Total Energy of a satellite
- E = K.E + P.E
- $= \left(\frac{GMm}{2r}\right) + \left(\frac{-GMm}{r}\right) = -\frac{1}{2} G\frac{Mm}{r}$

**Weightlessness**
During free fall under gravity inside a spacecraft or satellite a body is weightless i.e., effective weight becomes zero.

### Acceleration due to gravity (g)
Acceleration possessed by an object during free fall due to earth's gravitational pull,

$$g = \frac{G M_e}{R_e^2}$$

### Variation in acceleration due to gravity (g)
(i) with height $g' = g\left(1 - \frac{2h}{R_E}\right)$,
i.e. g decreases with height

(ii) with depth $g' = g\left(1 - \frac{d}{R_E}\right)$,
i.e. g decreases with depth

(iii) with rotation of earth about its own axis
$g' = g - R\omega^2 \cos^2 \lambda$,
At poles, $\lambda = 90^0$, g maximum
At equator, $\lambda = 0^0$, g minimum

### Characteristics of gravitational force
(i) Weakest force in Nature.
(ii) Central as well as conservative.
(iii) Always attractive in nature.
(iv) Applicable for all bodies irrespective of their shape, size and position.

### Newton's universal law of Gravitation
When two mass bodies are separted by a distance, they experience an attractive force which is directly proportional to the product of their masses and inversaly proportional to the square of their separation.

$$F = \frac{Gm_1m_2}{r^2}, \; G = 6.67 \times 10^{-11} Nm^2kg^{-2}$$

### Gravitational Field and potential
- Gravitational field Intensity
  $I = \frac{Gm}{r^2}$
- Gravitational potential
  $V_g = \frac{work\ done}{mass} = \frac{-Gm}{r}$
- Gravitational Potential Energy
  $U = \frac{-GMm}{r}$

### Kepler's Law-of planetary motion
(i) Law of Orbit: Every planet revolves around the Sun in an elliptical orbit and Sun is at its one focus.

(ii) Law of Area: The radius vector drawn from the Sun to a planet sweeps out equal areas in equal intervals of time.

$$\frac{dA}{dt} = \frac{L}{2m} = \text{Constant}$$

(iii) Law of Periods: $T^2 \propto a^3$
where, a = length of semi-major axis

# MIND MAP : LEARNING MADE SIMPLE — CHAPTER - 9

## Mechanical Properties of Solids

### Elasticity
Property of material by virtue of which it regains it original shape & size after the removal of deforming force

Relation between Y, B, η & σ
- (i) Y=3B (1 − 2 σ)
- (ii) Y=2η (1 + σ)
- (iii) $\sigma = \frac{3B-2\eta}{2\eta+6B}$
- (iv) $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$ or $Y = \frac{9B\eta}{\eta+3B}$

Poision's ratio(σ) = $\frac{\text{Laternal strain } (\beta)}{\text{Longitudinal strain } (\alpha)} = \frac{\Delta d / d}{\Delta l / l}$

Value of σ lies between 0 and 0.5

### Elastic Potential energy
Elastic potential energy in a stretched wire=$\frac{1}{2}$×stress × strain × volume of the wire

### Hooke's law
Within the elastic limit, stress is directly proportional to strain. i.e., stress ∝ strain

#### Stress
Restoring force per unit area i.e., stress= $\frac{F}{A}$

Types
- (i) Longitudinal stress $= \frac{\text{Deforming force applied normal}}{\text{Area}} = \frac{F_n}{A}$
- (ii) Volumetric stress $= \frac{\text{Force Changing Volume}}{\text{Area}} = \frac{F_V}{A}$
- (iii) Shearing area or tangential stress $= \frac{\text{Tangential Force}}{\text{Area}} = \frac{F_t}{A}$

#### Strain
Ratio of change in configuration to original configuration

Strain = $\frac{\text{change in configuration}}{\text{original configuration}}$

Types
- (i) Longitudinal strain $= \frac{\textit{change in length}}{\textit{original length}} = \frac{\Delta l}{l_o}$
- (ii) Volumetric strain $\varepsilon_V = \frac{\textit{change in volume}}{\textit{original volume}} = \frac{\Delta V}{V_o}$
- (iii) Shearing strain=angular displacement of the plane perpendicular to the fixed surface = θ

### Types of Modulus of Elasticity

#### Young's Modulus
Young's modulus of elasticity

$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/\Delta l}$

$Y = \frac{F\Delta l}{Al} = \frac{Mg\Delta l}{\pi r^2 l}$

#### Bulk Modulus
Bulk modulus or volume modulus of elasticity

$B = \frac{\text{hydraulic stress}}{\text{volume strain}}$

$B = -\frac{\Delta P}{\frac{\Delta V}{V}}; \frac{1}{B}$ = compressibility

#### Modulus of Rigidity
Rigidity or shear modulus of elasticity

$\eta = \frac{\textit{tangential stress}}{\textit{shearing strain}}$

$\eta = \frac{\sigma_s}{\varepsilon_s} = \frac{F/A}{\theta} = \frac{F}{A\theta}$

# MIND MAP : LEARNING MADE SIMPLE

## CHAPTER - 10

### Mechanical Properties of fluids

That can flow like liquids and gases

**Flow of fluids**

**Streamline :** In liquid flow when the Velocity is less than critical velocity, each particle of the liquid passing through a point travels along the same path and same velocity as the preceding particles.

**Turbulent :** When velocity of liquid flow is greater than critical velocity and particles follow zig-zag path.

**Bernoulli's principle**

For an incompressible non-viscous streamline,irrotational flow of fluid,

$$P+\frac{1}{2}\rho v^2+\rho gh=\text{constant}$$

**Applications**

- **Venturimeter**: Device used to measure the rate of flow of liquid. Volume of liquid flowing per second

$$Q=a_1a_2\sqrt{\frac{2h\rho_m g}{\rho(a_1^2-a_2^2)}}$$

- **Toricell's Law**: Velocity of efflux of liquid through an orifice $V=\sqrt{2gh}$
- o Lift of an aircraft wing.
- o Sprayer or atomizer
- o Blowing off the roofs during windstorm.

**Viscosity**

Opposing force between different layers of fluid in relative motion

Viscous drag $F=-\eta A\frac{dv}{dx}$

$\eta$=coefficient of viscosity

Stroke's law $F=6\,\eta vr$

**Formulae**

- Surface tension $S=\frac{F}{l}$
- Surface Energy $=\frac{\text{work done in increasing area}}{\text{increase in surface area}}=\frac{W}{\Delta A}$
- Capillary rise or fall, $h=\frac{2S\cos\theta}{r\rho g}$
- Excess Pressure inside a drop (liquid) $P_{excess}=\frac{2S}{R}$
- Excess Pressure inside a bubble (soap) $P_{excess}=\frac{4S}{R}$

**Laws of Fluids**

- **Equation of continuity**
  $m=a_1v_1\rho_1=a_2v_2\rho_2$
  for an incompressible liquid,
  $\rho_1=\rho_2$ then $a_1v_1=a_2v_2$
  or av=constant
- **Pascal's law :** The pressure exerted at any point on an enclosed liquid is transmitted equally in all direction. Hydraulic brakes and hydraulic lifts are based on Pacal's law.

**Pressure**

$$\text{Pressure}(P)=\frac{\text{thrust}(F)}{\text{area}(A)}=\lim_{\Delta A\to 0}\frac{\Delta E}{\Delta A}=\frac{dF}{dA}$$

Pressure exerted by a liquid column of height h, $(p)=h\rho g$

Density$(\rho)=\frac{\text{Mass(m)}}{\text{Volume(v)}}$

Density of water at 4°C i.e., maximum density of water$=1.0\times10^3$ kg/m$^3$

Relative Density or specific gravity $=\frac{\text{density of substance}}{\text{density of water at 4°C}}$

- **Atmospheric Pressure(Pa)**: Pressure (atm) exerted by the atmosphere. At sea level, 1 atm=pressure exerted by 0.76m of $Hg=h\rho g=0.76\times13.6\times10^3\times9.8=1.013\times10^5\,Nm^{-2}$ =101.3kPa
- **Absolute Pressure(P)**: Total or actual pressure at a point. Absolute pressure= atmospheric pressure + gauge pressure
- **Gauge Pressure (Pg)**: Difference between the absolute pressure at a point and the atmospheric pressure. Pg=absolute pressure(P) – atmospheric pressure(Pa)

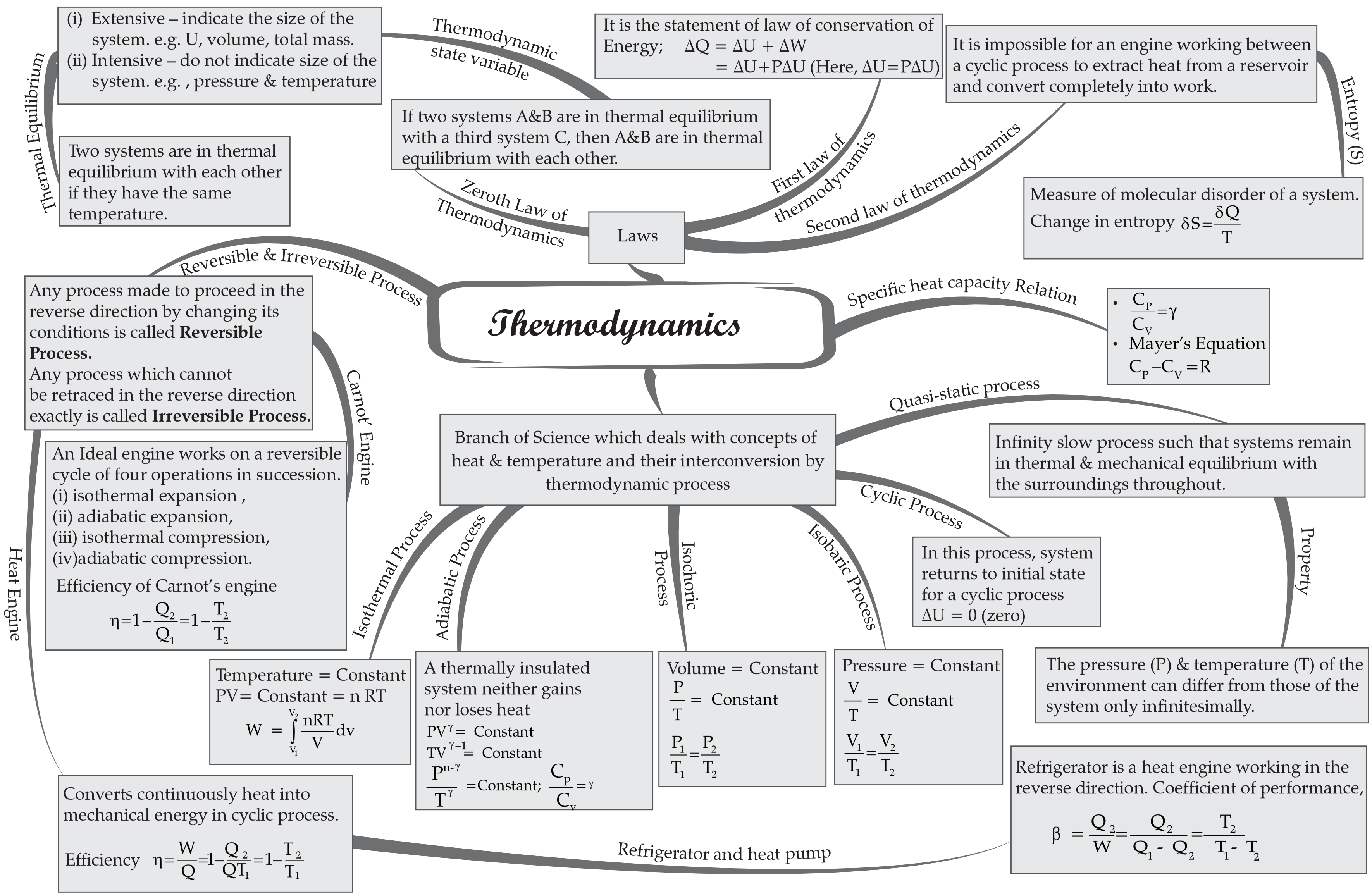
# MIND MAP : LEARNING MADE SIMPLE

## CHAPTER - 11

### Thermal Properties of Matter

- **Thermal Laws**
  - Krichoff's law: At any given temperature $\frac{e_\lambda}{a_\lambda} = E_\lambda$ = constant
  - Wien displacement law: $\lambda_m \propto \frac{1}{T}$ or $\lambda_m = \frac{b}{T}$; b(wine's constant) = $2.9\times10^{-3}$ mK
  - Stefan's Boltzmann law: $E \propto T^4$; $E = \sigma T^4$; $\sigma = 5.672 \ \text{Js}^{-1}\text{m}^{-2}\text{k}^{-4}$. For a perfectly black body $E = \sigma\left(T^4 - T_0^4\right)$
  - Newton's law of cooling: For small temperature difference between a body and its surroundings, the loss of heat is given by $-\frac{dQ}{dt} = k\,(T_2 - T_1)$
- **Thermal conductivity**: $(K) = \frac{Q\,x}{A(T_2 - T_1)\,t}$
- **Heat**: A form of energy, transferred between two systems by virtue of temperature difference.
  - Latent Heat: Heat required to change the state of unit mass substance without changing its temperature, $L = \frac{Q}{m}$. For water, latent heat of fusion, $L_f = 3.33\times10^5$ J/ kg. Latent heat of vapourisation, $L_v = 22.6\times10^5$ J/ kg
  - Specific Heat:
    - Specific heat capacity $C = \frac{s}{m} = \frac{Q}{m\Delta T}$
    - Heat capacity $s = \frac{\Delta Q}{\Delta T}$
    - Molar specific heat capacity, $c = \frac{s}{n} = \frac{\Delta Q}{n\Delta T}$
- **Principle of calorimetry**: Heat lost = Heat Gain
- **Mode of heat transfer**
  - (i) **Conduction :** heat transfer through molecular collisions without any actual motion of matter.
  - (ii) **Convection :** heat transfer by actual motion of matter within the medium. Land breeze, sea breeze, trade winds based on natural convection are some examples.
  - (iii) **Radiation :** method of heat transfer requiring no material medium.
- **Temperature**: Degree of hotness or coldness of a body measuring device = Thermometer
  - Relation among different temperature scales: $\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_K - 273.15}{373.15 - 273.15} = \frac{R - 0}{80 - 0}$
- **Thermal Expansion**: Increase in dimensions due to increase in temperature
  - Thermal Stress: $\frac{\Delta F}{A} = Y\left(\frac{\Delta l}{l}\right)$, Y = Young's modulus
  - Types (In solids)
    - Linear expansion: Increase of length of a solid on heating. Coefficient of linear expansion $\alpha = \frac{\Delta l}{l_0\Delta T} = \frac{L - l_0}{l_0(T - T_0)}$
    - Superficial or area Expansion: Increase in area of a solid on heating. Coefficient of superficial expansion $\beta = \frac{\Delta A}{A\Delta T} = \frac{dA}{A\,dT} = \frac{A - A_0}{A_0(T - T_0)}$
    - Relation: $\gamma = 2\beta = 3a$ or $a:b:\gamma = 1:2:3$
    - Cubical or volume expansion: Increase in volume of a solid on heating. Coefficient of cubical expansion. $\gamma = \frac{\Delta V}{V_0\Delta T} = \frac{dV}{V_0\,dT} = \frac{V - V_0}{V_0(T - T_0)}$

# MIND MAP : LEARNING MADE SIMPLE CHAPTER - 12



# MIND MAP : LEARNING MADE SIMPLE

## CHAPTER - 13

## Kinetic Theory

- **Behaviour of Gases**: An ideal gas satisfies equation $PV = nRT$ at all pressure and temperature $n$ = no. of moles, $R = N_A k_B$ universal gas constant
  - **Guy Lussac's Law**: $P \propto T$ (If V = constant); $\frac{P}{T}$ = constant or $\frac{P_1}{T_1} = \frac{P_2}{T_2}$
  - **Charles's Law**: $V \propto T$ (If P = constant); $\frac{V}{T}$ = constant or $\frac{V_1}{T_1} = \frac{V_2}{T_2}$
  - **Boyle's Law**: $P \propto \frac{1}{V}$ (If T = constant); PV = constant or $P_1V_1 = P_2V_2$
  - **Avogadro's law**: Under the same condition of temperature and pressure equal volumes of all gases contain equal no. of molecules. i.e. $N_1 = N_2$
  - **Dalton's Law of Partial Pressure**: Total pressure of a mixture of non - reacting gases, $P = P_1 + P_2 + ...... + P_n$
- **Pressure exerted by gas**: $P = \frac{1}{3}\rho v_{rms}^2$
- **Relation between Pressure (P) and Kinetic Energy (E)**: $E = \frac{3}{2}PV$
- **Degree of freedom (f)**:
  - For monoatomic gas: $f = 3$
  - For diatomic gas :
    - (i) at room temperature, $f = 5$
    - (ii) at high temperature, $f = 7$
  - For polyatomic gas:
    - (i) at room temperature, $f = 6$
    - (ii) at high temperature, $f = 8$
- **Mean Free Path**: $\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{k_B T}{\sqrt{2}\pi d^2 P}$
- **Law of Equipartition Energy**: Energy associated with each degree of freedom per molecule $= \frac{1}{2}kT$
- **Assumption of Kinetic Theory of Gases**:
  - All the molecules of a gas are identical.
  - The molecules of different gases are different.
  - The molecules of gases are in a state of random motion.
  - The collisions of gases molecules are perfectly elastic.
- **Specific Heat Capacity**:
  - Specific Heat Capacity for an ideal gas, $C_p - C_v = R$
  - for monoatomic $\frac{C_p}{C_v} = \gamma = \frac{5}{3}$
  - for diatomic $\frac{C_p}{C_v} = \gamma = \frac{7}{5}$
  - for polyatomic gases, $\frac{C_p}{C_v} = \gamma = \frac{(4+f)}{(3+f)}$
  - Hence, $f$ is the degree of freedom
- **Most probable Speed**: $v_{mp} = \sqrt{\frac{2RT}{M}} = \left(\sqrt{\frac{2}{3}}\right) v_{rms} = 0.816\ v_{rms}$
- **Average Speed**:
  - $v_{av} = \frac{v_1 + v_2 + v_3 + ... + v_N}{N}$
  - $v_{av} = \sqrt{\frac{8RT}{\pi M}} = \left(\sqrt{\frac{8}{3\pi}}\right) v_{rms} = 0.92\ v_{rms}$
- **Root mean square Speed**: $v_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$

# MIND MAP : LEARNING MADE SIMPLE — CHAPTER - 14(A)

## *Oscillations*

- Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.
- To and fro motion repeatedly about a fixed point in a definite interval of time.

### SHM

Simple Harmonic Motion is the simplest form of oscillatory motion

- **Displacement in SHM**
  - $x(t) = A(\cos\omega t + \phi)$
  - A = Amplitude i.e., maximum displacement of particles.
  - **Phase**
    - Time varying quantity $(\omega t + \phi)$
      - Phase constant or Phase angle $(\phi)$
      - It depends upon velocity $(v)$ and displacement of particle at t = 0
- **Time period (T)**
  - The smallest interval of time after which the motion is repeated
  - **Frequency**
    - The number of oscillations per second i.e., $f = \frac{1}{T} = \frac{\omega}{2\pi}$
- **Velocity in SHM**
  - $v = \frac{dx}{d(t)} = -\omega A \sin(\omega t + \varphi)$
  - $v_{max} = \omega A$
- **Acceleration in SHM**
  - $a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \varphi)$
  - $a_{max} = \omega^2 A$
- **Energy in SHM**
  - Kinetic Energy
    - $K.E = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \varphi)$
  - Potential Energy
    - $P.E = \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \varphi)$
  - Total energy, T.E. = K.E. + P.E.
    - $= \frac{1}{2} kA^2 [\cos^2(\omega t + \varphi) + \sin^2(\omega t + \varphi)]$
  - Force law equation for SHM
    - $F = -k(x),\ k = m\omega^2$
    - $\therefore \omega = \sqrt{\frac{k}{m}}$

### Simple pendulum

Time period $T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{l}{g}}$

### Oscillations due to a spring

Time period $T = 2\pi\sqrt{\frac{m}{k}}$

Here, $k = \frac{F}{x}$ = *spring constant*

### Damped Oscillations

Oscillation of a body whose amplitude goes on decreasing with time.

Damping force $F_d = -bv$, b = damping constant

Displacement of damped oscillator

$$x = x_m e^{-bt/2m} \sin(\omega t + \varphi)$$

Amplitude decreases with time as

$$x = x_m e^{-(b/2m)t}$$

### Forced Oscillations

Driving force, $F(t) = F_o \cos\omega_d t$

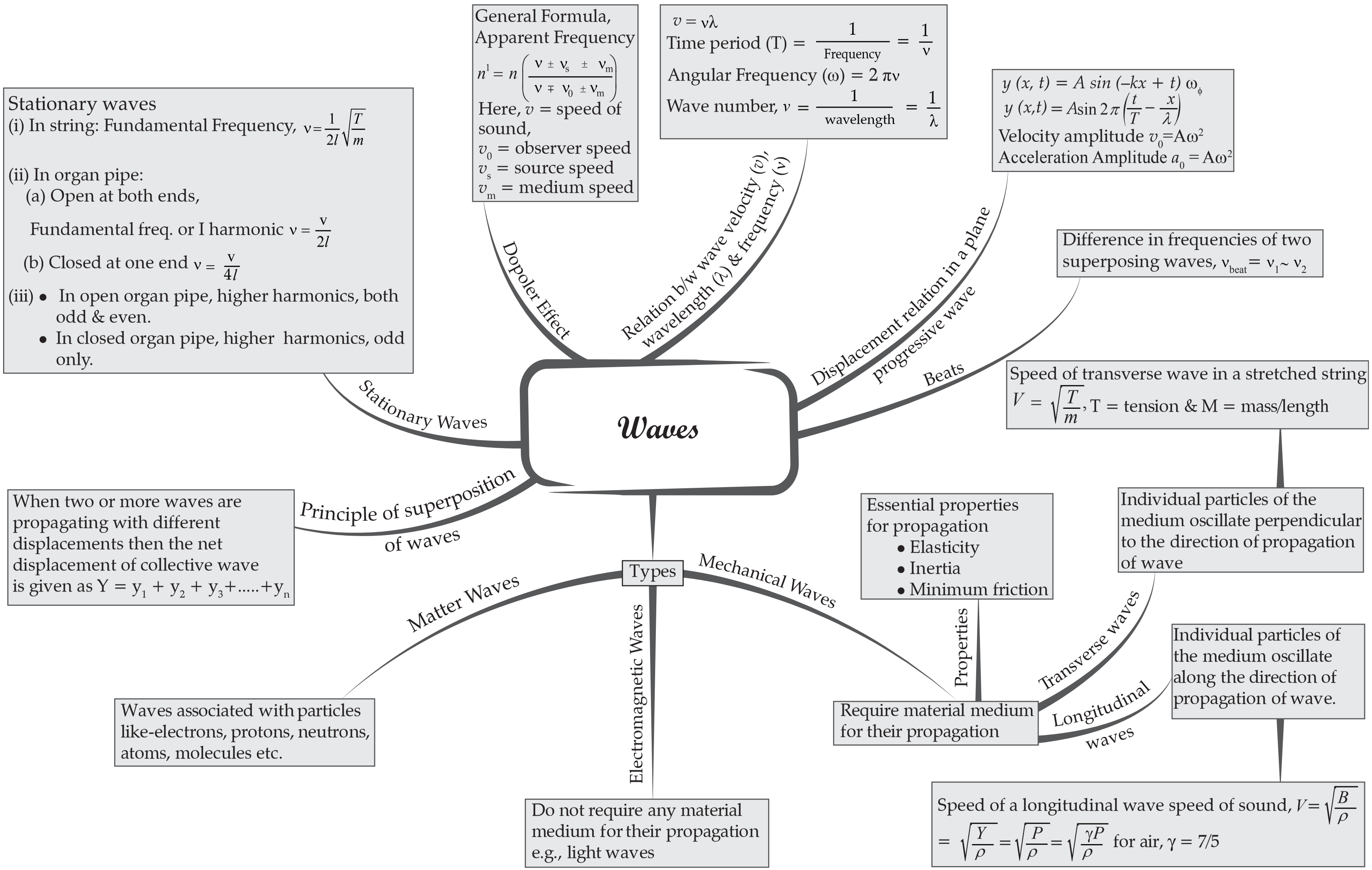
Displacement, $x(t) = A\cos(\omega_d + \varphi)$

Amplitude, $A = \frac{F_o}{\left\{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2\right\}^{\frac{1}{2}}}$

$\tan\varphi = \frac{-v_o}{\omega_d x_o}$

- **Resonance**
  - When frequency of external force is equal to natural frequency of oscillator.

# MIND MAP : LEARNING MADE SIMPLE CHAPTER - 14(B)



# MIND MAP : LEARNING MADE SIMPLE

## CHAPTER - 15

### Ray Optics

**Reflection & Laws**

When beam of incident light bounces in the same medium.

**Laws :**

(i) Incident ray, reflected ray and normal lie in the same plane.

(ii) Angle of incidence (∠i) = Angle of reflection (∠r)

**Refraction & Laws**

When beam of light goes from one medium to another medium, it deviates from its path, such deviation of light in called refraction of light.

**Laws :**

(i) Incident ray, refracted ray & normal lie in the same plane.

(ii) Snell's Law, refractive index (η) $= \frac{\sin i}{\sin r}$

**Dispersion of light**

The separation of visible light into its different colours.

$\delta_{red} < \delta_{violet}$ because $\eta_{red} < \eta_{violet}$

(ii) Angular dispersion : $\theta = \delta_v - \delta_r = (\eta_v - \eta_r)A$

(iii) Dispersive power : $\omega = \frac{\delta_v - \delta_r}{\delta_r}$

**Spherical Mirrors**

(1) Focal length $f = \frac{R}{2}$

Mirror formula $\Rightarrow \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

(2) Linear magnification, $m = \frac{I}{O} = \frac{-v}{u} = \frac{f}{f-u} - \frac{f-v}{f}$

**Len's Maker Formula**

$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

**Astronomical Telescope**

Magnifying Power (MP)

- When image is at ∞

$MP = -\frac{f_o}{f_e}$ & $L = f_o + f_e$

- When image is at $D$

$MP = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

& $L = f_o + \left(\frac{f_e}{f_e + D}\right)$

**Compound Microscope**

Magnifying Power (MP)

- When image is at ∞

$MP = -\frac{v}{u}\left(\frac{D}{f_e}\right)$ & L = u + $f_e$

- When image is at $D$

$MP = -\frac{v}{u}\left(1 + \frac{D}{f_e}\right) = \frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$

**Combination of thin lenses and power**

- $P = \frac{1}{f}$
- $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$
- $P = P_1 + P_2$

**Dispersion of light through Prism**

- $\delta = i + e - A$
- $A = r_1 + r_2$
- $\mu = \frac{\sin(\delta_{min} + A)/2}{\sin A/2}$
- $\delta = A(\mu - 1)$

**Optical Instrument**

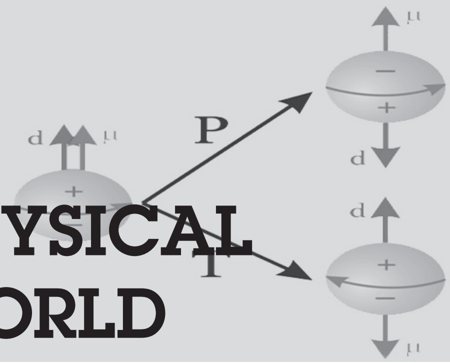
- Compound microscope
- Astronomical microscope
- Newtonian Telescope

**Total Internal Reflection**

The complete reflection of a light ray reaching an interface with a less dense medium when the angle of incidence exceeds the critical angle.

CHAPTER

# 1 PHYSICAL WORLD



## Chapter Objective

**This chapter will help you understand :**

- ***Physics*** *– Scope and excitement, Nature of Physical laws.*
- ***Physics****–Technology and society.*

# TOPIC-1
## Physical Science

## Quick Review



- **Scope of Physics :**

  The scope of Physics is wide, covering a tremendous range of magnitude of physical quantities.

  Physics covers larger phenomenon as well as smaller quantities. For example,

**(i)** Life span of most unstable particle to life of an average star i.e., from $10^{-22}$ s to $10^{18}$ s on time scales.

**(ii)** Celestial Body's mass to electron's mass i.e., from $10^{55}$ kg to $10^{-30}$ kg on mass scales.

- **Excitement in Physics :**

  Few reasons that makes physics exciting—

**(i)** It is very challenging and exciting to unfold and detect secrets of nature with imaginative new experiments.

**(ii)** There is an innovative approach for applying physical laws to many new inventions *i.e.*, projects, machines, etc.

**(iii)** Few basic concepts, fundamental principles and laws helps in explaining the wide range of physical quantities, complicated phenomena of nature, etc.

## Know the Terms

- **Science** is a systematic attempt to understand a natural phenomenon in as much detail and depth as possible and use the knowledge so gained to predict, modify and control phenomenon.
- **Physics** is the study of the basic laws of nature and their manifestation in different phenomenon.
- **Mechanics :** It is an area of science concerned with behaviour of physical bodies when subjected to forces or displacements and subsequent effect of bodies on environment. It can also be defined as branch of science which deals with motion of forces on objects.
- **Electro-dynamics :** It deals with rapidly changing electric and magnetic fields.
- **Optics :** It involves the behaviour and properties of light including its interactions with matter and construction of instruments that use or detect it.
- **Thermodynamics :** It is concerned with heat and temperature and their relation to energy and work.

- **Mesoscopic Physics :** It is the sub-discipline of condensed matter physics which deals with materials of an intermediate length scale between size of quantity of atoms and of materials measuring micrometres.

## Know the Links

- *www.learncbse.in*
- *www.examfear.com*
- *https://www.atpeducation.com*

# TOPIC-2
## Physical laws : Nature and Forces

## Quick Review

- **Nature of Physical Laws :**

  Nature of physical laws can be explained on the basis of certain laws. Certain quantities remain same but several quantities may change with time. Those quantities which remain constant known as conserved quantities and this concept is called law of conservation.

- **Laws of Conservation of Physical Quantities :**

  **There are four laws of conservation in classical physics.**

**(a)** **Law of Conservation of Energy :** It states that energy can neither be created nor destroyed but it can be changed from one form to another *i.e.* total sum of all types of energy in the whole universe remains constant.

**(b)** **Law of Conservation of Mass :** It states that matter can neither be created nor can be destroyed. Einstein's theory of relativity modified this statement to $E = mc^2$ where $m$ is mass and $c$ is the speed of light in vacuum. According to this theory mass and energy are inter-convertible.

**(c)** **Law of Conservation of Momentum :** It is classified into two laws—

**(i)** **Law of Conservation of Linear Momentum :** It states that if no external force acts on a system, then its linear momentum remains constant.

**(ii)** **Law of Conservation of Angular Momentum :** It states that if no external torque acts on a system, then its angular momentum remains constant.

**(d)** **Law of Conservation of Charge :** It states that charge can neither be created nor destroyed. It can be transferred from one body to another. It is the basic law of conservation in nature.

## Know the Terms

- **Law** is precise and summarised statement related to duly verified and authentic observations of natural phenomena eg. Newton's laws of motion etc.
- **Conserved Quantities** are those physical quantities which remain constant in a process *i.e.* total energy, total momentum, etc.
- **Technology** is defined as the study of new techniques of producing machines, gadgets, etc. by using scientific discoveries and advancements. It is application of physics.

## Know the Links

- *https://www.vedantu.com*
- *www.learncbse.in*
- *www.exemfear.com*

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. Some of the most profound statements on the nature of science have come from Albert Einstein, one of the greatest scientist of all time. What do you think did Einstein mean when he said "The most incomprehensible thing about the world is that it is comprehensible."**

**[NCERT Ex. Q. 1.1, Page 13]**

**Ans.** The word comprehensible means understandable. When we look at nature and natural phenomenon, we fail to understand their complexities with ordinary knowledge. But with scientific methods, we come to know that only a few basic laws and principles can very well explain most of them. 'So many phenomenon of world, governed and understood by only a few scientific principles, are otherwise not easily understandable'.

**Q. 2. "Every great physical theory starts as a hearsay and ends as a dogma." Give some examples from the history of science of the validity of this incisive remark. [NCERT Ex. Q. 1.2, Page 13]**

**Ans.** Whenever a good theory is stated it is treated like a rumor (hearsay) but after passage of time and reverification it becomes a principle and authority (dogma). Galileo was punished for his theory that earth revolves around the sun because at that time it was felt by people that he was spreading wrong information. After passage of some time, Kepler as well as Newton supported his theory. Now this theory is an authentic principle.

**Q. 3. "Politics is the art of possible." Similarly, "Science is the art of soluble." Explain this beautiful aphorism on the nature and practice of science.**

**[NCERT Ex. Q. 1.3, Page 13]**

**Ans.** It is said that word 'impossible' is missing in the dictionary of a politician. Politics is, therefore, treated as an art of making every impossible thing possible for a politician. Similarly, there is hardly any scientific problem which cannot be solved by a scientific approach. Science is therefore, treated as an art of getting solutions for those problems also which appear to have no solution because even most complex phenomena of nature have their explanations in terms of a few basic laws of Physics.

**Q. 4. Though India now has a large base in science and technology which is fast expanding, it is still a long way from realizing its potential for becoming a world leader in science. Name some important factors, which in your view have hindered the advancement of science in India.**

**[NCERT Ex. Q. 1.4, Page 13]**

**Ans.** **(i)** Indian society is full of superstitions. So, they slowly adopt the technology and innovation.

**(ii)** Bureaucracy in science education.

**(iii)** Highly poor condition and infrastructure for quality education and research in schools and colleges of India.

**(iv)** No collaboration between industries and scholars in India.

**(v)** Lack of scientific planning.

**Q. 5. No physicist has ever 'seen' an 'electron'. Yet all physicists believe in the existence of electrons. An intelligent but superstitious man advances this analogy to argue that 'ghosts' exist even though no one has seen one. How will you refute his argument ? [NCERT Ex. Q. 1.5, Page 13]**

**Ans.** It is simply a superstition that ghosts exist. There is not even a single authentic evidence that proves that ghosts exist. On the other hand it is a fact that atoms exist. There are many examples to prove this fact, for eg. atomic power plant, atomic bombs, atomic clocks etc.,

**Q. 6. The shells of crabs found around a particular coastal location in Japan seem mostly to resemble the legendary face of the Samurai. Given below are the two explanations of this observed fact. Which of these strikes you as a scientific explanation ?**

**(a) A tragic sea accident several centuries ago drowned a young Samurai. As a tribute to his bravery, nature through its inscrutable ways immortalized his face by imprinting it on the crab shells in that area.**

**(b) After the sea tragedy, fisherman in that area in a gesture of honour to their dead hero, let free any crab shell caught by them which accidentally had a shape resembling the face of the Samurai. Consequently, the particular shape of the crab shell survived longer and in course of time, the shape was genetically propagated. This is an example of evolution by artificial selection.**

**[NCERT Ex. Q. 1.6, Page 13]**

**Ans.** **(b)** It is a correct scientific explanation of the observed fact .

**Q. 7. The Industrial Revolution in England and Western Europe more than two centuries ago was triggered by some key scientific and technological advances. What were these advances ?**

**[NCERT Ex. Q. 1.7, Page 14]**

**Ans.** Some of the key scientific and industrial advances which led to industrial revolution prior to 1750 A.D. were **(i)** steam engine, **(ii)** blast furnace, **(iii)** power loom etc.

**Q. 8. It is often said that the world is witnessing now a second Industrial Revolution which will transform the society as radically as did the first. List some key contemporary areas of science and technology, which are responsible for this revolution. [NCERT Ex. Q. 1.8, Page 14]**

**Ans.** The key areas which are transforming radically the present society are **(i)** super computers, **(ii)** biotechnology, **(iii)** development of super conductors at room temperature, **(iv)** Nanotechnology, **(v)** robots.

**Q. 9. Write in about 1,000 words a fiction piece based on your speculation on the science and technology of the twenty second century.**

**[NCERT Ex. Q. 1.9, Page 14]**

**Ans.** Our Scientists launch a spaceship towards a distant star, about 500 light year away. This spaceship contains matter and antimatter and uses the water as fuel. The spaceship also has some nanorobots as passengers who consume the electricity generated from an electric motor with nanotubes and superconducting wires at very high temperature.

**OR**

Imagine you along with your friends are in a spaceship which is moving towards Mars. The body of the spaceship is made of specially designed matter which becomes more harder as its temperature increases. The spaceship is using nuclear fuel and there are three nuclear power plants in spaceship. Two of them work alternatively and the third is for emergency. The speed of the spaceship is very high and all of you are very happy. The energy produced in power plants is converted into electric energy which runs the motors of the spaceship. You along with your friends reach safely on Mars, collects data, takes photographs and then returns to Earth. On the return journey, the spaceship collides with an object in the space due to which two power plants stop to work. Now, only one power plant is working and due to overheating its efficiency is decreasing continuously. You and your friends try to reduce the temperature of the power plant by flowing air in the plant and try to repair the fuse of the other power plants. Finally, fuse of one other plant is required and start to work before the first plant crosses the danger limit of an excess of temperature. Finally, you and your friends return safely to earth.

**Q. 10. Attempt to formulate your 'moral' views on the practice of science. Imagine yourself stumbling upon a discovery, which has a great academic interest but is certain to have nothing but dangerous consequences for the human society. How, if at all, will you resolve your dilemma ?**

**[NCERT Ex. Q. 1.10, Page 14]**

**Ans.** Yes, any discovery good or bad must be made public. Something which appears dangerous today, may be put to use in some form later. So, a discovery, which reveals a truth of nature, should not be concealed.

**Q. 11. Science, like any knowledge, can be put to good or bad use, depending on the user. Given below are some of the applications of science. Formulate your views on whether the particular application is good, bad or something that cannot be so clearly categorized :**

**(i) Mass vaccination against small pox to curb and finally eradicate this disease from the population. (This has already been successfully done in India.)**

**(ii) Television for eradication of illiteracy and for mass communication of news and ideas.**

**(iii) Parental sex determination.**

**(iv) Computers for increase in work efficiency.**

**(v) Putting artificial satellites around the Earth.**

**(vi) Development of nuclear weapons.**

**(vii) Development of new and powerful techniques of chemical and biological warfare.**

**(viii) Purification of water for drinking.**

**(ix) Plastic surgery.**

**(x) Cloning. [NCERT Ex. Q. 1.11, Page 14]**

**Ans.** **(i)** Good, **(ii)** Good, **(iii)** Bad, **(iv)** Good, **(v)** Good, **(vi)** Bad, **(vii)** Bad, **(viii)** Good, **(ix)** Good, **(x)** Good.

**Q. 12. India has had a long and unbroken tradition of great scholarship in mathematics, astronomy, linguistics, logic and ethics yet, in parallel with this, several superstitious and obscurantist attitudes and practices flourished in our society and unfortunately continue even today among many educated people too. How will you use your knowledge of science to develop strategies to counter these attitudes?**

**[NCERT Ex. Q. 1.12, Page 14]**

**Ans.** We can remove these illogical practices and superstitious and obscurantist attitudes only by educating the society. Mass media *i.e.* radio, television, newspapers, magazines, internet programmers, social sites, etc. can play a vital role in it, so programme should be framed and spread by use of media to target these practices.

**Q. 13. Though the law gives women equal status in India, many people hold unscientific views on a woman's innate nature, capacity and intelligence and in practice given them a secondary status and role. Demolish this view using scientific**

**arguments and by quoting examples of great women in science and other spheres; and persuade yourself and others that, given equal opportunity, women are on par with men.**

**[NCERT Ex. Q. 1.13, Page 14]**

**Ans.** There is no difference in the capacity of the women in taking good and quick decisions, in doing hard work and intelligence. The development of human brain depends on the nutrition contents of prenatal and postnatal diet and it does not depend on the gender. Anything which can be achieved by a man can also be achieved by a woman. In every field of life, women have proved herself. Madam Curie, a Physicist, won Nobel prize. Mother Teresa was a great saint, Kalpana Chawla an astronaut, Mrs. Indira Gandhi, Margret Thatcher, Lata Mangeshkar etc., are well known personalities in different fields. Therefore, women should be given equal opportunity on par with men.

**Q. 14. "It is more important to have beauty in the equation of physics than to have them agree with experiments." The great British physicist P.A.M. Dirac held this view. Criticize this statement. Look out for some equations and results in this book which strike you as beautiful.**

**[NCERT Ex. Q. 1.14, Page 14]**

**Ans.** It is a general feeling that physics is a dry subject and its main aim is to give quantitative and qualitative treatment, *i.e.*, any derived relations or equations must be verified through experimentation. It is felt that truth of an equation is more important than the simplicity, wonderfulness, symmetry or beauty of the equation.

We have some simple and beautiful equation in physics like

$E = mc^2$ (Energy of light)

$KE=\frac{1}{2}mv^2$ (kinetic energy of a moving particle)

$W = F.d$ (work done)

$V = IR$ (Ohm's law)

**Q. 15. Though the statement quoted above be disputed, most physicists do have a feeling that the great laws of physics are at once simple and beautiful. Some of the notable Physicists, besides Dirac, who have articulated this feeling are Einstein, Bohr, Heisenberg, Chandrasekhar and Feynman. You are urged to make special efforts to get access to the general books and writings by these and other great masters of physics. Their writings are truly inspiring ! Find out its value.**

**[NCERT Ex. Q. 1.15, Page 14]**

**Ans.** It is absolutely true that great laws of physics are simple and beautiful. Few examples are given below :

**(i)** Einstein's mass-energy equivalance relation $E = mc^2$ is simple and beautiful.

**(ii)** According to Max Planck's quantum theory, the energy of a photon is $E = h\nu$, is also a simple and useful equation.

**(iii)** de-Broglie wavelength associated with a particle of mass $m$ is given by $\lambda = \frac{h}{mv}$. It is also a simple and useful equation.

**Q. 16. Textbooks on science may give you a wrong impression that studying science is dry and too serious and that scientists are absent minded introverts who never laugh or grin. This image of science and scientists is patently false. Scientists like any other group of humans have their share of humorists and many have led their lives with a great sense of fun and adventure even as they seriously pursued their scientific work. Two great physicists of this generation are Gamow and Feynman. Find out its value.**

**[NCERT Ex. Q. 1.16, Page 15]**

**Ans.** It is true that scientists like any other group of humans have their share of humorists. Two great physicists of this genre are Gamow and Feynman. Few other scientists whose name can be added in this list are CV Raman, Einstein, Bohr, former Indian president. APJ Abdul Kalam etc.

**TIPS... & TRICKS...**

*Study scope of Physics*

*Understand various part of physics*

## Some Commonly Made Errors

- Students ignore to learn the initial concept of solid state chemistry.
- Generally, students skip the diagrams or tables in crystal lattices while answering in examination.

## EXPERT ADVICE

☞ *Always study chronologically rather than priority order.*

☞ *Try to learn qualitative than quantitative.*

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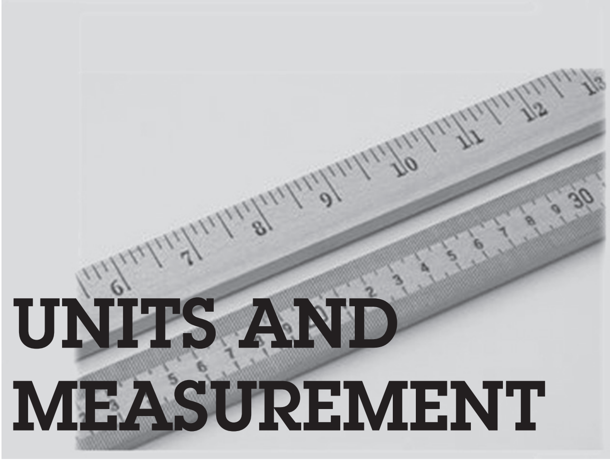
CHAPTER

# 2 UNITS AND MEASUREMENT



## Chapter Objective

**This chapter will help you understand :**

- **Unit system and Measurement :** Need for measurement, Units of measurement, the international system of units fundamental and derived units, length, mass and time measurements. Significant figures.
- **Dimensional Analysis and Error :** Dimensions of Physical quantities, Dimensional analysis and its applications. Accuracy and Precision of measuring instruments. Errors in measurement.

## TOPIC-1
### Units System and Measurement



### Quick Review

- **Units :** It is the chosen standard of measurement of a quantity which has essentially the same nature as that of the quantity.

**(i) Fundamental Units :** The physical quantity which are treated as independent of other, it means that they cannot be defined in terms of other units, are known as fundamental physical quantity. These are the units of measurement of length, mass, time etc.

**Seven Fundamental physical quantities in SI system of units are :**

**(a)** Mass - kg
**(b)** Length - m
**(c)** Time - s
**(d)** Temperature - K
**(e)** Electric current - A
**(f)** Luminous Intensity - cd
**(g)** Quantity of Matter - mol

**(ii) Derived Units :** These are the units of measurement of all other physical quantities which are derived from fundamental units. eg. Velocity - (m/s), Acceleration - $m/s^2$, Pressure - Pa, Force - N.

- **System of Units :**

**(a) F. P. S. system —** Foot, Pound, Second.
**(b) C. G. S. system —** Centimetre, Gram, Second.
**(c) M. K. S. system —** Meter, Kilogram, Second.

- **Length Measurement :**

**(a) Direct Methods :**
(i) a metre scale for distances from $10^{-3}$ m to $10^2$ m.
(ii) a vernier callipers for distances upto $10^{-4}$ m.
(iii) a screw gauge and a spherometer for distances upto $10^{-5}$ m.

**(b) Indirect Methods :**
(i) Parallax method.
(ii) Astronomical Telescope for size of an astronomical object.
(iii) Tunneling microscope for size of molecule.

- **Mass Measurement :**

**(a)** For large masses - gravitational methods.
**(b)** For small masses - mass spectrograph.

- **Time-intervals Measurement**

**(a)** Electric oscillators
**(b)** Electronic oscillators
**(c)** Solar clock
**(d)** Quartz crystal clock
**(e)** Atomic clock
**(f)** Decay of elementary particles using photographic emulsion techniques.
**(g)** Radio-active Dating Technique.

## Know the Terms

- **Mass of a body** is defined as the quantity of matter in a body which can never be zero.
- **Length of an object** may be defined as the distance of separation between any two points at the extreme ends of the object.

## Know the Formulae

- 1 AU = $1.496 \times 10^{11}$ m.
- 1 ly = $9.46 \times 10^{15}$ m.
- 1 parsec = $3.1 \times 10^{16}$ m.
- 1 Å = $10^{-10}$ m; 1 nm = $10^{-9}$ m
  1 μm = $10^{-6}$ m, 1 mm = $10^{-3}$ m
- 60 seconds (of arc) = 1 min (arc)
- 60 min. (of arc) = 1 degree (of arc)
- 180 degree (of arc) = $\pi$ radian
- Indirect methods for long and small distances–

  Angular diameter $(\theta) = \frac{l}{r}$

  where, length = $l$
  radius = $r$
- **Time—**

  $$\text{Fractional Error} = \frac{\text{Difference in time}}{\text{Time interval}}$$
- **Magnification—**

**(a)** $\text{Linear Magnification} = \frac{\text{Size of image}}{\text{Size of object}}$

**(b)** $\text{Linear Magnification} = \sqrt{\text{Areal Magnification}}$

## Know the Links

☞ *https://www.vedantu.com*
☞ *www.learncbse.com*
☞ *www.examfear.com*

## MCQ/Fillups/True or False **(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. The number of significant figures in 0.06900 is**

**(a)** 5 **(b)** 4 **(c)** 2 **(d)** 3

**[NCERT Exemp. Q. 2.1, Page 5]**

**Ans. Correct option:** (b) 4

*Explanation:* If the number is less than 1, the zeroes on the left of decimal point are not significant, but all the zeroes between two non-zero digits are significant. So in

not Significant    Significant

0.0 6900

**Q. 2. The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is**

**(a) 663.821** **(b) 664**

**(c) 663.8** **(d) 663.82**

**[NCERT Exemp. Q. 2.2, Page 5]**

**Ans. Correct option:** (c) 663.8

*Explanation:*

$$\begin{array}{r} 436.32 \\ 227.2 \\ +\ 0.301 \\ \hline 663.821 \end{array}$$

Answer in significant figures = 663.8

In addition, the final result should retain as many decimal places as there in the number with least decimal places.

**Q. 3. The mass and volume of a body are 4.237 g and 2.5 $cm^3$, respectively. The density of the material of the body in correct significant figures is**

**(a) 1.6048 g $cm^{-3}$** **(b) 1.69 g $cm^{-3}$**

**(c) 1.7 g $cm^{-3}$** **(d) 1.695 g $cm^{-3}$**

**[NCERT Exemp. Q. 2.3, Page 5]**

**Ans. Correct option:** (c) 1.7 g $cm^{-3}$

*Explanation:*

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{4.237\text{g}}{2.5\text{ cm}^3} = 1.6948\text{ gcm}^{-3}$$

$= 1.7$ g $cm^{-3}$ (In significant figure)

**Q. 4. The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give**

**(a) 2.75 and 2.74** **(b) 2.74 and 2.73**

**(c) 2.75 and 2.73** **(d) 2.74 and 2.74**

**[NCERT Exemp. Q. 2.4, Page 6]**

**Ans. Correct option:** (d) 2.74 and 2.74

*Explanation:* 2.745 = 2.74 (In 3 significant figures)

2.735 = 2.74

If the digit to be round off is 5, then :

**(a)** preceding digit is left unchanged, if it is even.

**(b)** preceding digit is increased by one, if it is odd.

## (B) Fill in the blanks

**Q. 5. (a) The volume of a cube of side 1 cm is equal to ..... $m^3$.**

**(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ..... $mm^2$.**

**(c) A vehicle moving with a speed of 18 $kmh^{-1}$ covers .... m in 1 s.**

**(d) The relative density of lead is 11.3. Its density is ....... g $cm^{-3}$ or ...... kg $m^{-3}$.**

**[NCERT Ex. Q. 2.1, Page 35]**

**Ans. (a)** $10^{-6}$ **(b)** 15072

**(c)** 5 **(d)** 11.3, 11300

*Explanation:*

**(a)** Volume of cube is

$V = (a)^3 = (1\text{ cm})^3 = (1 \times 10^{-2}\text{ m})^3 = 10^{-6}\text{ m}^3$

**(b)** Surface Area of solid cylinder

$$\begin{aligned} S &= 2\pi r(r + h) \\ &= 2 \times 3.14 \times 2\,(2 + 10)\text{ cm}^2 \\ &= 12.56 \times 12\text{ cm}^2 \\ S &= 150.72\text{ cm}^2 \\ &= 15072\text{ mm}^2 \end{aligned}$$

[as 1 $cm^2$ = 100 $mm^2$]

**(c)** $v = 18\text{ kmh}^{-1} = 18 \times \frac{5}{18}\text{ m sec}^{-1}$

$= 5\text{ m sec}^{-1} = 5$ m in 1 sec

**(d)** Relative density = 11.3

$$\begin{aligned} \text{density} &= 11.3 \times 1\text{ g cm}^{-3} = 11.3\text{ g cm}^{-3} \\ &= 11.3 \times \frac{1}{1000}\text{ kg} \times \left(\frac{1}{100}\text{ m}\right)^{-3} \\ &= 11.3 \times \frac{1}{10^3} \times 10^{-6} = 11300\text{ kg m}^{-3} \end{aligned}$$

**Q. 6. Fill in the blanks by suitable conversion of units :**

**(a) 1 kg $m^2s^{-2}$ = ....g $cm^2s^{-2}$**

**(b) 1 m = ........ ly (light year)**

**(c) 3.0 $ms^{-2}$ = ..... $kmh^{-2}$**

**(d) G = 6.67 × $10^{-11}$ $Nm^2$ $kg^{-2}$ = .... $cm^3$ $s^{-2}$ $g^{-1}$**

**Ans. (a)** $10^7$ **(b)** $1.053 \times 10^{-16}$

**(c)** $3.888 \times 10^4$ **(d)** $6.67 \times 10^{-8}$

**[NCERT Ex. Q. 2.2, Page 35]**

*Explanation:*

**(a)** $1\text{ kg m}^2\text{ s}^{-2} = 1000\text{ g} \times (100\text{ cm})^2 \times 1\text{ s}^{-2}$

$= 10^7\text{ g cm}^2\text{ s}^{-2}$

**(b)** $1\text{ m} = \frac{1}{9.46 \times 10^{15}} = 1.057 \times 10^{-16}$ light year

**(c)** $3.0\text{ ms}^{-2} = 3.0 \times \left(\frac{1}{1000}\text{ km}\right) \times [(3600)^2\text{h}^{-2}]$

$$= \frac{3 \times 3600 \times 3600}{1000}\text{ km h}^{-2}$$

$= 3.888 \times 10^4\text{ km h}^{-2}$

**(d)**

$$\begin{aligned} G &= 6.67 \times 10^{-11}\text{ N m}^2\text{ kg}^{-2} \\ &= 6.67 \times 10^{-11}\text{ m}^3\text{ s}^{-2}\text{ kg}^{-1} \\ &= 6.67 \times 10^{-11} \times (100\text{ cm})^3 \times \text{s}^{-2} \times (1000\text{ g})^{-1} \\ &= \frac{6.67 \times 10^{-11} \times 10^6}{10^3}\text{ cm}^3\text{ s}^{-2}\text{ g}^{-1} \\ &= 6.67 \times 10^{-8}\text{ cm}^3\text{ s}^{-2}\text{g}^{-1} \end{aligned}$$

## Very Short Answer Type Questions (1 mark each)

**Q. 1. Why do we have different units for same physical quantity?**

**[NCERT Exemp. Q. 2.19, Page 8]**

**Ans.** The value of any given physical quantity may vary over a wide range

Because, bodies differ in order of magnitude significantly in respect to the same physical quantity. For example, interatomic distances are of the order of angstroms, Inter-city distances are of the order of km and interstellar distances are of the order of light year.

**Q. 2. The radius of atom is of the order of 1Å and radius of nucleus is of the order of Fermi. How many magnitudes higher is the volume of atom as compared to the volume of nucleus?**

**[NCERT Exemp. Q. 2.20, Page 8]**

**Ans.** Given : Radius of atom, $r_a = 1\text{Å} = 10^{-10}$ m

Radius of nucleus, $r_n = 1$ fermi $= 10^{-15}$m

Volume of atom; $V_1 = \frac{4}{3}\pi r_a^3$

Volume of nucleus, $V_2 = \frac{4}{3}\pi r_n^3$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_a^3}{\frac{4}{3}\pi r_n^3} = \left(\frac{r_a}{r_n}\right)^3 = \left(\frac{10^{-10}}{10^{-15}}\right)^3$$

$$\frac{V_1}{V_2} = 10^{15}$$

**Q. 3. Name the device used for measuring the mass of atom and molecules.**

**[NCERT Exemp. Q. 2.21, Page 8]**

**Ans.** Mass spectrograph.

**Q. 4. Express unified atomic mass unit in kg.**

**[NCERT Exemp. Q. 2.22, Page 9]**

**Ans.** 1 atomic Mass unit $= \frac{1}{12}$ of mass of a $_6C^{12}$ atom

Mass of 1 mole of $_6C^{12}$ atom = 12 gm

No. of atom in 1 mole = Avogadro number

Mass of $_6C^{12}$ atom $= \frac{12}{6.023 \times 10^{23}}$ gm

1 atomic mass unit $= \frac{1}{12} \times \frac{12}{6.023 \times 10^{23}}$ g

$= 1.67 \times 10^{-27}$ gm

**Q. 5. Why length, mass and time are chosen as base quantities in mechanics?**

**[NCERT Exemp. Q. 2.24, Page 9]**

**Ans.** Because all other quantities of mechanics can be expressed in terms of length, mass and time through simple relations.

And length, mass & time cannot be derived from one another, *i.e.* they are independent quantities.

**Q. 6. Explain the statement clearly :**

**"To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison." In view of this, reframe the following statements wherever necessary :**

**(a) Atoms are very small objects.**

**(b) A jet plane moves with great speed.**

**(c) The mass of Jupiter is very large.**

**(d) The air inside this room contains a large number of molecules.**

**(e) A proton is much more massive than an electron.**

**(f) The speed of sound is much smaller than the speed of light.** **[NCERT Ex. Q. 2.4, Page 35]**

**Ans.** The physical quantities are called large or small in comparison to some standard units of measurement. So, statement is correct :

**(a)** As size of an atom is smaller than the sharp tip of pin.

**(b)** As a jet plane moves faster than a superfast train.

**(c)** As mass of Jupiter is very large as compared to the mass of earth.

**(d)** As air inside the room contains more number of molecules than in one mole of air.

**(e)** Statement is true.

**(f)** Statement is true.

**Q. 7. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance ?** **[NCERT Ex. Q. 2.5, Page 35]**

**Ans.** Distance between Sun and Earth is

= Speed of light × Time taken by light to cover the distance

$= 3 \times 10^8$ m/s $\times 500$ s

$= 500 \times 3 \times 10^8$ m

As the speed of light in vacuum = 1 $ms^{-1}$ (in new system). So, distance between Sun and Earth.

= 500 new units.

**Q. 8. Which of the following is the most precise device for measuring length :**

**(a) A vernier calipers with 20 division on the sliding scale ?**

**(b) A screw guage of pitch 1 mm and 100 divisions on the circular scale ?**

**(c) An optical instrument that can measure length to within a wavelength of light ?**

**[NCERT Ex. Q. 2.6, Page 35]**

**Ans. (a)** Least count of vernier calipers

$$= \frac{1}{20} = 0.05 \text{ mm} = 5 \times 10^{-5} \text{ m}$$

**(b)** Least count of screw gauge

$$= \frac{1\times10^{-3}}{100} = 1 \times 10^{-5} \text{ m}$$

**(c)** Least count of optical instrument

$$= 5 \times 10^{-7} \text{ m.}$$

∴ Optical instrument is most precise instrument.

**Q. 9. A student measures the thickness of a human hair with the help of a microscope of magnification 100. He takes 20 readings and finds that the average width of the hair in the field of view of microscope is 3.5 mm. What is the estimate on the thickness of the hair ?**

**[NCERT Ex. Q. 2.7, Page 35]**

**Ans.** Magnification,

$$\text{Real width} = \frac{\text{Observed width } (y)}{\text{Magnification}}$$

$$= \frac{3.5}{100} = 0.035 \text{ mm}$$

**Q. 10. Answer the following :**

**(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread ?**

**(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ?**

**(c) The mean diameter of a thin brass rod is to be measured by vernier calipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?**

**[NCERT Ex. Q. 2.8, Page 35]**

**Ans. (a)** By winding the thread in close turns on a pencil and using the formula *i.e.*

$$\text{Diameter} = \frac{\text{Average length}}{\text{No. of turns}}$$

**(b)** Yes, by increasing the number of divisions on circular scale as

$$\text{Least count} = \frac{\text{Pitch}}{\text{No. of divisions on circular scale}}$$

**(c)** Random error is reduced on taking large number of observations.

**Q. 11. The photograph of a house occupies an area of 1.75 cm$^2$ on a 35 mm slide. The slide is projected on to a screen and the area of the house on the screen is 1.55 m$^2$. What is the linear magnification of the projector–screen arrangement ?**

**[NCERT Ex. Q. 2.9, Page 35]**

**Ans.** Here, size of an object = Area of object

$$= 1.75 \text{ cm}^2$$

$$= 1.75 \times 10^{-4} \text{ m}^2$$

Size of the image = Area of the image

$$= 1.55 \text{ m}^2$$

$$\therefore \text{ A real magnification} = \frac{\text{Area of image}}{\text{Area of object}}$$

$$= \frac{1.55}{1.75\times10^{-4}} = 8.857\times 10^3$$

Linear magnification = $\sqrt{8857}$ = 94.1

**Q. 12. State the number of significant figure in:**

**(a) 0.007 m$^2$** **(b) 2.64 × 10$^{24}$**

**(c) 0.2370 g cm$^{-3}$** **(d) 6.320 J**

**(e) 6.032 Nm$^{-2}$** **(f) 0.0006032 m$^2$**

**[NCERT Ex. Q. 2.10, Page 35]**

**Ans.(a)** 1 **(b)** 3

**(c)** 4 **(d)** 4

**(e)** 4 **(f)** 4

**Q. 13. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. Find its area and volume of the sheet to correct significant figures.**

**[NCERT Ex. Q. 2.11, Page 36]**

**Ans.**

$$\text{Length} = 4.234 \text{ m}$$

$$\text{Breadth} = 1.005 \text{ m}$$

$$\text{Thickness} = 2.01 \text{ cm} = 2.01 \times 10^{-2} \text{ m}$$

$$\text{Area} = l \times b$$

$$= 4.234 \times 1.005$$

$$= 4.25517$$

$$= 4.26 \text{ m}^2$$

$$\text{Volume} = 4.234 \times 1.005 \times (2.01 \times 10^{-2})$$

$$= 8.55289 \times 10^{-2}$$

$$= 0.0855 \text{ m}^3.$$

**Q. 14. The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the mass of the pieces to corrects significant figures ?**

**[NCERT Ex. Q. 2.12, Page 36]**

**Ans.** Here mass of the box,

$$m = 2.300 \text{ kg}$$

Mass of one gold piece,

$$m_1 = 20.15 \text{ g} = 0.02015 \text{ kg}$$

Mass of second gold piece,

$$m_2 = 20.17 \text{ g} = 0.02017 \text{ kg}$$

**(a)** Total mass $= m + m_1 + m_2$

$$= (2.300 + 0.02015 + 0.02017) \text{ kg}$$

$$= 2.34032 \text{ kg.}$$

As the least number of significant figures in the mass of box is 2, so, maximum number of significant figures in the result can be 2.

Total mass = 2.3 kg.

**(b)** Difference in masses,

$$m = m_2 - m_1$$
$$= 20.17 - 20.15 = 0.02 \text{ g}$$

Since, there are two significant figures, so, the difference in masses to the correct significant figures is 0.02 g.

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. (a) The earth-moon distance is about 60 earth radius. What will be the diameter of the earth (approximately in degrees) as seen from the moon?**

**(b) Moon is seen to be of $\left(\frac{1}{2}\right)$ diameter from the earth. What must be the relative size compared to the earth?**

**(c) From parallax measurement, the sun is found to be at a distance of about 400 times the earth-moon distance. Estimate the ratio of sun-earth diameters. [NCERT Exemp. Q. 2.25, Page 9]**

**Ans.** **(a)** Given : $\theta = \frac{l}{r}$, $l = R_E$

$$r = 60 R_E$$

$$\theta = \frac{R_E}{60R_E} = \frac{1}{60} \text{ rad} = 1°$$

Hence, angle subtended by diameter of earth = 2θ = 2°

∴ Diameter of the earth as seen from the moon is about 2.

**(b)** At earth-moon distance, moon is seen as (1/2)° diameter and earth is seen as 2° diameter. Hence, diameter of earth is 4 times the diameter of moon.

$$\frac{D_{\text{earth}}}{D_{\text{moon}}} = \frac{\left(\frac{2}{\pi}\right)\text{rad}}{\left(\frac{1}{2\pi}\right)\text{rad}} = 4 \quad \text{(i)}$$

**(c)** From parallax measurement, sun is at distance of about 400 times the earth-moon distance,

$$\frac{r_{\text{sun}}}{r_{\text{moon}}} = 400 = \frac{D_{\text{sun}}}{D_{\text{moon}}} \quad \text{(ii)}$$

(Here $r$ stands for distance and $D$ for diameter.)

dividing eq$^n$ (ii) by (i)

we get,

$$\frac{D_{\text{sun}}/D_{\text{moon}}}{D_{\text{earth}}/D_{\text{moon}}} = \frac{400}{4}$$

$$\frac{D_{\text{sun}}}{D_{\text{earth}}} = 100$$

**Q. 2. The distance of a galaxy is of the order of $10^{25}$ m. Calculate the order of magnitude of time taken by light to reach us from the galaxy. [NCERT Exemp. Q. 2.27, Page 9]**

**Ans.** Given: Distance of galaxy = $10^{25}$m

Speed of light = $3 \times 10^8$ m/s.

Time taken to reach us from galaxy by light,

$$T = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{10^{25}}{3\times10^8}\frac{\text{m}}{\text{m/s}}$$

$$= 3.33 \times 10^{16} \text{ s}$$

$$= 3 \times 10^{16} \text{ sec.}$$

**Q. 3. During a total solar eclipse the moon almost entirely covers the sphere of the sun. Write the relation between the distances and sizes of the sun and moon. [NCERT Exemp. Q. 2.29, Page 9]**

**Ans.** $R_{me}$ = distance of moon from earth

$R_{se}$ = distance of sun from earth

Let θ be angle made by sun and moon and $A_s$= area of sun, $A_m$= area of moon

$$\Rightarrow \quad \theta = \frac{A_s}{R_{se}^2} = \frac{A_m}{R_{mc}^2}$$

$$\Rightarrow \quad \theta = \frac{\pi R_s^2}{R_{se}^2} = \frac{\pi R_m^2}{R_{mc}^2}$$

$$\Rightarrow \quad \left(\frac{R_s}{R_{se}}\right)^2 = \left(\frac{R_m}{R_{mc}}\right)^2$$

$$\Rightarrow \quad \frac{R_s}{R_m} = \frac{R_{se}}{R_{mc}}$$

**Q. 4. Calculate the length of the arc of a circle of radius 31.0 cm which subtends an angle of $\frac{\pi}{6}$ at the centre. [NCERT Exemp. Q. 2.32, Page 10]**

**Ans.** Angle, $\theta = \frac{l}{r}$ radian

From question, $\theta = \frac{\pi}{6} = \frac{l}{31}$

$$\Rightarrow \quad \text{length, } l = 31 \times \frac{\pi}{6} \text{ cm}$$

$$= \frac{31\times3.14}{6}$$

$$= 16.22 \text{ cm}$$

$$= 16.2 \text{ cm}$$

**Q. 5. Calculate the solid angle subtended by the periphery of an area of 1 cm$^2$ at a point situated symmetrically at a distance of 5 cm from the area. [NCERT Exemp. Q. 2.33, Page 10]**

**Ans.** As solid angle $\Omega = \frac{\text{Area}}{(\text{distance})^2}$

[Area = 1 cm$^2$, distance = 5 cm]

$$= \frac{1\ \text{cm}^2}{(5\ \text{cm})^2}$$

$$= \frac{1}{25} = 4 \times 10^{-2}\ \text{steradian.}$$

Solid angle, $\Omega = 4 \times 10^{-2}$ steradian

**Q. 6. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å. 1 Å = $10^{-10}$ m. The size of the hydrogen atom is about 0.5 Å. What is the total atomic volume in m$^3$ of a mole of hydrogen atoms ?**

**[NCERT Ex. Q. 2.16, Page 36]**

**Ans.** Here, $r = 0.5$ Å

$= 0.5 \times 10^{-10}$ m

$V_1$ = Volume of each hydrogen atom

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3$$

$$= 5.233 \times 10^{-31}\ \text{m}^3$$

According to Avogadro's hypothesis, one mole of hydrogen contains

$N = 6.023 \times 10^{23}$ atoms

∴ Atomic volume of 1 mole of hydrogen atoms,

$V = NV_1$

$V = 6.023 \times 10^{23} \times 5.233 \times 10^{-31}$

$= 3.152 \times 10^{-7}\ \text{m}^3$

$= 3.152 \times 10^{-7}\ \text{m}^3$.

**Q. 7. One mole of an ideal gas at NTP and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen ? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large ?**

**[NCERT Ex. Q. 2.17, Page 36]**

**Ans.** Atomic volume $= \frac{4}{3}\pi R^3 \times N$

$$= \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3 \times 6.023 \times 10^{23}$$

$= 3.154 \times 10^{-7}\ \text{m}^3$.

Molar volume $= 22.4$ L

$= 22.4 \times 10^{-3}\ \text{m}^3$

$$\frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{3.154 \times 10^{-7}}$$

$= 7.1 \times 10^4$

The large value of the ratio shows that the inter molecular separation in a gas is much larger than the size of a molecule.

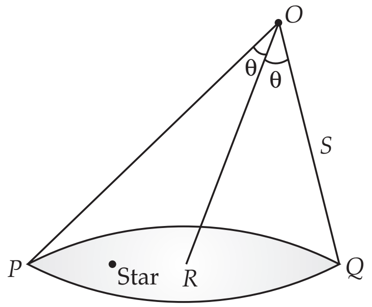
**Q. 8. Explain this common observation clearly. If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant, objects (hill tops, the moon, the stars etc.) seem to be stationary (In fact, since you are aware that you are moving, these distant objects seem to move with you). [NCERT Ex. Q. 2.18, Page 36]**

**Ans.** The line joining the object to the eye is line of sight. If we move rapidly, the line of sight nearby tree changes its direction of motion rapidly. It means near objects make greater angle than distant objects. Thus, the trees appear to run in opposite direction. But in case of moon, stars etc., this change is negligible, so they appear stationary.

**Q. 9. The principle of 'parallax' is used in the determination of distance of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit = $3 \times 10^{11}$ m. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1″ (second) or arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1″ (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metre ?**

**[NCERT Ex. Q. 2.19, Page 36]**

**Ans.**




From parallax method,

$$S = \frac{b}{\theta}$$

$\theta' = \angle AOB = 2\angle ROB = 2\theta$

O = Position of star,

$b$ = PB = base line

= Diameter of earth's orbit

$= 3 \times 10^{11}$ m

$\theta = 1\ \text{sec} = 4.85 \times 10^{-6}$ rad

So, $$S = \frac{b}{2\theta} = \frac{3 \times 10^{11}}{2 \times 4.85 \times 10^{-6}}$$

$= 3.09 \times 10^{16}$ m

$\approx 3.1 \times 10^{16}$ m

**Q. 10. The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs ? How much parallax would this star (named Alpha Centauri) show, when viewed from two locations of the Earth six months apart in its orbit around the sun ?**

**[NCERT Ex. Q. 2.20, Page 37]**

**Ans.** **(b)** $4.29\ ly = 4.29\ ly \times \left(\frac{1\ \text{parsec}}{3.262\ ly}\right) = 1.31$ parsec.

Let $\partial = x$, $\theta = \dfrac{l}{\partial}$

$$t = \frac{2AU}{x}$$

$$= \frac{2 \times 1.494 \times 10^{11}}{4.29 \times 9.46 \times 10^{15}}$$

$$= 7.36 \times 10^{-6} \text{ sec.}$$

**Q. 11. A LASER is a source of very intense, monochromatic and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the moon from the earth has been already determined very precisely using a laser as a source of light. A laser light beamed moon takes 2.56 s to return after reflection at moon surface. What is the radius of the orbit of moon (lunar orbit) around the earth ? [NCERT Ad. Ex. Q. 2.29, Page 37]**

**Ans.** Here, $t = 2.56$ s

Velocity of laser light in vacuum

$$c = 3 \times 10^8 \text{ m/s}$$

The radius of lunar orbit is the distance of moon from earth. Let it be $x$.

$$\therefore \quad x = \frac{c \times t}{2}$$

$$\therefore \quad x = \frac{3 \times 10^8 \times 2.56}{2}$$

$$= 3.84 \times 10^8 \text{ m.}$$

**Q. 12. A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR, the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m/s.) [NCERT Exemp. Q. 2.30, Page 38]**

**Ans.** Given: Time, t = 77s, v = 1450 m/s.

$$\text{Distance of enemy submarine} = \frac{v \times t}{2}$$

$$= \frac{1450 \times 77}{2}$$

$$= 55825 \text{ m.}$$

**Q. 13. The farthest object in our universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the earth. These objects (known as quasars) have many puzzling features which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us ? [NCERT Ad. Ex. Q. 2.31, Page 38]**

**Ans.** Time taken,

$$t = 3 \times 10^9 \text{ years}$$

$$= 3 \times 10^9 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

Velocity of light,

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$\therefore$ Distance of quasar from earth

$$= ct$$

$$= 3 \times 10^8 \times 3 \times 10^9 \times 365 \times 24 \times 3600 \text{ m}$$

$$= 2.84 \times 10^{25} \text{ m}$$

$$= 2.84 \times 10^{22} \text{ km.}$$

**Q. 14. It is a well known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the sun. Determine the approximate diameter of the moon. [NCERT Ad. Ex. Q. 2.32, Page 38]**

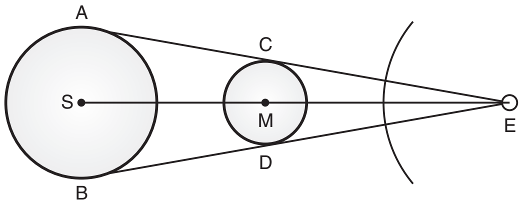
**Ans.** Distance of moon from earth = $3.84 \times 10^8$ m

Distance of sun from earth = $1.496 \times 10^{11}$ m

Diameter of sun AB = $1.39 \times 10^9$ m

The situation is shown in figure, $\Delta$ABE and $\Delta$CDE are similar

$$\therefore \quad \frac{AB}{CD} = \frac{SE}{ME}$$




$$CD = \frac{AB \times ME}{SE}$$

$$= \frac{1.39 \times 10^9 \times 3.89 \times 10^8}{1.496 \times 10^{11}}$$

$$= 3.5679 \times 10^6 \text{ m}$$

$$= 3567.9 \text{ km}$$

The diameter of the moon is 3567.9 km.

## Long Answer Type Questions (5 marks each)

**Q. 1. In an experiment to estimate the size of a molecule of oleic acid, 1 mL of oleic acid is dissolved in 19 mL of alcohol. Then 1 mL of this solution is diluted to 20 mL by adding alcohol. Now 1 drop of this diluted solution is placed on water in a shallow trough. The solution spreads over the surface of water forming one molecule thick layer. Now, lycopodium power is sprinkled evenly over the film and its diameter is measured. Knowing the volume of the drop and are of the film we can calculate the thickness of the film which will give us the size of oleic acid molecule.**

**Read the passage carefully and answer the following questions :**

**(a) Why do we dissolve oleic acid in alcohol?**

**(b) What is the role of lycopodium powder?**

**(c) What would be the volume of oleic acid in each mL of solution prepared?**

**(d) How will you calculate the volume of *n* drops of this solution of oleic acid?**

**(e) What will be the volume of oleic acid in one drop of this solution?**

**[NCERT Exemp. Q. 2.42, Page 11]**

**Ans.** (a) Because oleic acid dissolves in alcohol but does not dissolve in water.

(b) When lycopodium powder is spread on water, it spreads on the entire surface. When a drop of the prepared solution is dropped on water, oleic acid does not dissolve in water. On the other hand, it spreads on the water surface pushing the lycopodium powder away to clear a circular area where the drop falls. This allows measuring the area where oleic acid spreads.

(c) In each mL of solution prepared, volume =

$$\left[\frac{1}{20}\text{mL}\times\frac{1}{20}=\frac{1}{400}\text{mL}\right]$$

(d) By means of a burette and measuring cylinder and measuring the number of drops.

(e) If *n* drops of the solution make 1 mL, the volume of oleic acid in one drop will be (1/400)*n* mL.

**Q. 2. (a) How many astronomical units (A. U.) make 1 parsec?**

**(b) Consider a sun like star at a distance of 2 parsecs. When it is seen through a telescope with 100 magnifications, what should be the angular size of the star? Sun appears to be (1/2)° from the earth. Due to atmospheric fluctuations, eye can't resolve objects smaller than 1 arc minute.**

**(c) Mars has approximately half of the earth's diameter. When it is closest to the earth it is at about 1/2 A. U. from the earth. Calculate what size it will appear when seen through the same telescope.**

**(Comment : This is to illustrate why a telescope can magnify planets but not stars.)**

**[NCERT Exemp. Q. 2.43, Page 11]**

**Ans.** By definition of parsec

$$\therefore \quad 1\text{ parsec} = \left(\frac{1\text{ A}U}{1\text{ arc sec}}\right)$$

$$1\text{ deg} = 3600\text{ arcsec}$$

$$1\text{ parsec} = \frac{\pi}{3600\times180}\text{ radians}$$

$$1\text{ parsec} = \frac{3600\times180}{\pi}\text{ A}U$$

$$= 206265\text{ A}U = 2\times10^5\text{ A}U$$

**(b)** At 1 AU distance, sun is (1/2) in diameter.

Therefore, at 1 parsec, star is $\frac{1/2}{2\times10^5}$ degree in diameter $= 0.25\times10^{-5}$ arc min

With 100 magnification, it should look $0.25\times10^{-3}$ arc min. However, due to atmospheric fluctuations, it will still look of about 1 arc min.

∴ It can't be magnified using telescope.

**(c)** $\frac{D_{moon}}{D_{earth}}=\frac{1}{2},\frac{D_{earth}}{D_{sun}}=\frac{1}{100}$ [Here, D = diameter]

$$\frac{D_{sun}}{D_{moon}}=400,\frac{D_{earth}}{D_{moon}}=4\Rightarrow\frac{D_{moon}}{D_{sun}}=\frac{1}{2}\times\frac{1}{100}$$

At 1 AU sun is seen as 1/2 degree in diameter, and mars will be seen as 1/400 degree in diameter. i.e.

mars diameter $=\frac{1}{2}\times\frac{1}{200}=\frac{1}{400}$ at 1/2 AU, mars diameter $=\frac{1}{400}\times2°=\left(\frac{1}{200}\right)^{\circ}$

At ½ AU mars will be seen as 1/400 degree in diameter. With 100 magnification mars will be seen

$$\text{mars diameter} = \frac{1°}{200}\times100=\left(\frac{1}{2}\right)^{\circ}=30.$$

This is larger than resolution limit due to atmospheric fluctuations. Hence, it looks magnified.

**Q. 3. Einstein's mass energy relation emerging out of his famous theory of relativity relates mass (*m*) to energy (*E*) as E = *mc*$^2$, where *c* is speed of light in vacuum. At the nuclear level, the magnitudes of energy are very small. The energy at nuclear level is usually measured in MeV, where 1 MeV = 1.6 × 10$^{-13}$ J; the masses are measured in unified atomic mass unit (u) where 1u = 1.67 × 10$^{-27}$ kg.**

**(a) Show that the energy equivalent of 1 u is 931.5 Mev.**

**(b) A student writes the relation as 1 u =931.5 MeV. The teacher points out that the relation is dimensionally incorrect. Write the correct relation.**

**[NCERT Exemp. Q. 2.44, Page 12]**

**Ans.** **(a)** We can apply Einstein's mass energy relation in this problem, E = *mc*$^2$, to calculate the energy equivalent of the given mass.

Here,

1 amu = 1u = $1.67\times10^{-27}$ kg

Applying E = *mc*$^2$,

$$E = (1.67\times10^{-27})(3\times10^{-8})^2\text{ J}$$

$$= 1.67\times9\times10^{-11}\text{ J}$$

$$\text{or } E = \frac{1.67\times9\times10^{-11}}{1.6\times10^{-13}}\text{ MeV}$$

$$= 939.3\text{ MeV}$$

$$= 931.5\text{MeV}$$

(b) As $E = mc^2 \Rightarrow m = \frac{E}{c^2}$

According to this 1 u = $\frac{931.5 \text{ meV}}{c^2}$

Hence, the dimensionally correct relation is

1 amu × $c^2$ = 1u × $c^2$

= 931.5 MeV

**Q. 4. The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding $10^7$ K and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the sun to be, in the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data :**

**Mass of sun = $2.0 \times 10^{30}$ kg,**

**Radius of the sun = $7.0 \times 10^8$ m**

**[NCERT Ex. Q. 2.23, Page 37]**

**Ans.** Here, $M = 2 \times 10^{30}$ kg

$R = 7.0 \times 10^8$ m

∴ Mass density of sun

$$= \frac{\text{Mass of sun}}{\text{Volume of sun}} = \frac{M}{V}$$

$$= \frac{M}{\frac{4}{3}\pi R^3}$$

$$= \frac{2\times 10^{30}}{\frac{4}{3}\times 3.14\times\left(7\times 10^8\right)^3}$$

$$= \frac{3\times 2\times 10^{30}}{4\times 3.14\times 343\times 10^{24}} \text{ kgm}^{-3}$$

$$= 1.39 \times 10^3 \text{ kg m}^{-3}$$

This density is the range of density of solids and not gases.

**Explanation—**The temperature of the inner core of the sun exceeds $10^7$ K while the temperature of the outer surface of the sun is nearly 6000 K. At such extremely high temperature, no substance can exist either in a solid or in a liquid phase. So, the sun is made of ionized matter, *i.e.*, hot plasma. The high density of the plasma is due to the inward gravitational attraction on outer layers due to the inner layers of the sun.

**Q. 5. When the planet Jupiter is at a distance of 824.7 million kilometers from the earth, its angular diameter is measured to be 35.72″ of arc. Calculate the diameter of Jupiter.**

**[NCERT Ex. Q. 2.24, Page 37]**

**Ans.** Here, $d$ = Distance of Jupiter from earth

= 824.7 million kms

$d = 824.7 \times 10^6$ km

$\theta = 35.72''$

$= 35.72 \times 4.85 \times 10^{-6}$ rad

($\because$ 1″ = $4.85 \times 10^{-6}$ rad)

D = Diameter of jupiter = ?

∴ Using the relation,

$\theta = \frac{D}{d}$, we get

$D = \theta \times d$

$= 35.72 \times 4.85 \times 10^{-6} \times 824.7 \times 10^6$ km

= 142873 km

$= 142.873 \times 10^3$ km

$= 1.42873 \times 10^5$ km

$= 1.429 \times 10^5$ km

**Q. 6. A man walking briskly in rain with speed $v$ must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and $v$ : tan θ = $v$ and checks that the relation has a correct limit as $v \to 0$, $\theta \to 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically on a stationary man). Do you think this relation can be correct ? If not, guess the correct relation.**

**[NCERT Ad. Ex. Q. 2.25, Page 37]**

**Ans.** Here, given relation is tan θ = $v$

No, the relation is not correct.

Since, the left hand side of this relation is a trigonometrical function which is dimensionless, so R.H.S. must also be dimensionless. So, ν must be ν/u, where $u$ = speed of rainfall.

Hence, the correct relation becomes

$$\tan\theta = \frac{v}{u}$$

**Q. 7. It is claimed that the two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring time interval of 1 s ?**

**[NCERT Ad. Ex. Q. 2.26, Page 37]**

**Ans.** Here,

time interval = 100 years

= 100 × 365 × 24 × 60 × 60s

$= 3.155 \times 10^9$ s

Difference in time = 0.02 s

$$\text{Fractional error} = \frac{\text{Difference in time (s)}}{\text{Time interval (s)}}$$

$$= \frac{0.02}{3.155\times 10^9}$$

$= 6.34 \times 10^{-12}$

$\approx 10 \times 10^{-12}$

$\approx 10^{-11}$

∴ In 1 s, the difference is $10^{-11}$ to $6.34 \times 10^{-12}$.

Hence, degree of accuracy shown by the atomic clock in 1 s is 1 part in $\frac{1}{10^{-11}}$ to $\frac{1}{6.34\times 10^{-12}}$

$= 10^{11}$ to $1.587 \times 10^{11}$.

**Q. 8. Estimate the average mass density of sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro's number, and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase: 970 kg $m^{-3}$. Are the two densities of the same order of magnitude ? If so, why ?**

**[NCERT Ad. Ex. Q. 2.27, Page 37]**

**Ans.** Average radius of sodium atom

$= 1.25 \times 10^{-10}$ m

$\therefore$ Volume of sodium atom

$$= \frac{4}{3}\pi r^3$$

$$= 8.18 \times 10^{-30} \text{ m}^3$$

$\therefore$ Mass of sodium atom,

$$M = \frac{23\times10^{-3}}{6.023\times10^{23}} \text{ kg}$$

$$= 3.82 \times 10^{-26} \text{ kg}$$

$\therefore$ Density of sodium atom

$$= \frac{3.82\times10^{-26}}{8.18\times10^{-30}}$$

$$= 4.67 \times 10^3 \text{ kg/m}^3$$

Density of sodium in crystalline phase

$= 970$ kg/m$^3$

$\therefore$ Both densities are of the same order of magnitude.

**Q. 9. The unit of length convenient on the nuclear scale is a fermi :**

$$\mathbf{1\ f = 10^{-15}\ m.}$$

**Nuclear sizes obey roughly the following empirical relation :**

$$r = r_0 \mathbf{A}^{1/3}$$

**where, $r$ is the radius of the nucleus, A its mass number and $r_0$ is a constant equal to about 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in above question.**

**[NCERT Ad. Ex. Q. 2.28, Page 37]**

**Ans.** Here, radius of nucleus,

$$r = r_0 A^{1/3}$$

$\therefore$ Volume of the nucleus (assuming it to be spherical),

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi [r_0 A^{1/3}]^3$$

$$= \frac{4}{3}\pi r_0^3 A.$$

Mass of nucleus having mass number A is

$= A$ amu

$= A \times 1.66 \times 10^{-27}$ kg

and, $r_0 = 1.2 \text{ f} = 1.2 \times 10^{-15}$ m

$\therefore$ Density of nucleus,

$$\rho = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{A\times1.66\times10^{-27}\text{kg}}{\frac{4}{3}\pi r_0^3 A\text{m}^3}$$

$$= \frac{1.66\times10^{-27}}{\frac{4}{3}\pi r_0^3} \text{ kg m}^{-3} \quad ...(i)$$

$$= \frac{3\times1.66\times10^{-27}\text{ kg m}^{-3}}{4\times3.14\times(1.2\times10^{-15})^3}$$

$$= 2.29 \times 10^{17} \text{ kg m}^{-3}$$

$$= 2.3 \times 10^{17} \text{ kg m}^{-3}$$

From equation (i), it is clear that ρ is independent of A, so nuclear mass density is constant for different nuclei and this must be the density of sodium nucleus also. Thus, density of sodium nucleus $= 2.3 \times 10^{17}$ kg m$^{-3}$. Average mass density of sodium atom is $\rho = 4.67 \times 10^3$ kg m$^{-3}$

$$\frac{\rho}{\rho'} = \frac{2.3\times10^{17}\text{kgm}^{-3}}{4.67\times10^3\text{kgm}^{-3}}$$

$$= 0.49 \times 10^{14} = 4.9 \times 10^{13}$$

**Q. 10. A great physicist of this century (P.A.M. Dirac) loved playing with the numerical values of fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics $m_e$, $m_p$ and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (~ 15 billion yrs). From the table of fundamental constants in this book, try to see if you too can reach that number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants ? What is its value ?**

**[NCERT Ad. Ex. Q. 2.33, Page 38]**

**Ans.** Few basic constants of atomic physics are given below.

Charge of an electron ($e$) $= 1.6 \times 10^{-19}$ C

Speed of light in vacuum ($c$) $= 3 \times 10^8$ m/s

Gravitational constant (G) $= 6.67 \times 10^{-11}$ N-m$^2$/kg$^2$

Mass of electron ($m_e$) $= 9.1 \times 10^{-31}$ kg

Mass of proton ($m_p$) $= 1.67 \times 10^{-27}$ kg

Permittivity of free space ($\varepsilon_o$) $= 8.85 \times 10^{-12}$ N-m$^2$/C$^2$

On trying with these basic constants, we can get the quantity whose dimension is equal to the dimension of time. One such quantity is on writing dimensions of a quantity on RHS,

$$x = \left(\frac{e^2}{4\pi\varepsilon_o}\right)^2 \times \frac{1}{m_e^2 m_p c^3 G}$$

$$[x] = \frac{[AT]^4}{[M^{-1}L^{-3}T^4A^2]^2 \times [M]\times[M]^2} \times [LT^{-1}]^3 \times [M^{-1}L^3T^{-2}]$$

$$= [M^{2-1-2+1} L^{6-3-3} T^{4-8+2+3} A^{4-4}]$$

$$= [M^{3-3} L^{6-6} T^{9-8} A^{4-4}]$$

$$= [M^0L^0TA^0]$$

$$= [T]$$

Now, substituting values of all constants in the given relation,

$$x = \frac{(1.6\times10^{-19})^4}{16\times(3.14)^2\times(8.854\times10^{-12})^2\times(1.67\times10^{-27})} \times (9.1 \times 10^{-31})^2 \times (3 \times 10^8)^3$$

$= 2.18 \times 10^{16}$ sec

$= 6.9 \times 10^8$ yr

$= 10^9$ yr = 1 billion yr

The estimate value of the quantity $x$ is close to the age of the universe.

# TOPIC-2
## Dimensional Analysis and Error

## Quick Review

- **Use of Dimensional Equations :**
  - **(i)** Conversion of one system of units into another.
  - **(ii)** Checking the accuracy of various formulae.
  - **(iii)** Derivation of formulae.
- **Error :**
  The difference in the true value and the measured value of a quantity is called error of measurement.
- **Types of Errors :**
  - **(a)** Systematic Error
  - **(b)** Random Error
  - **(c)** Gross Error.

## Know the Terms

- **Dimensions of physical quantity** are the powers to which the units of base quantities are raised to represent a derived unit of that quantity.
- **Dimensional formula of the given physical quantity** is the expression which shows how and which of the base quantities represent the dimensions of a physical quantity.
- **Dimensional constants** are the quantities whose values are constant and they possess dimensions *e.g.* velocity of light, etc.
- **Dimensional variables** are the quantities whose values are variable and they possess dimensions *e.g.* area, volume, etc.
- **Dimensional less constants** are the quantities whose value are constant but they do not possess dimensions *e.g.* mathematical constants– $\pi$.
- **Dimensional less variables** are the quantities whose values are variable and they do not have dimensions *e.g.* angle, strain, etc.
- **Accuracy** is a measure of how close the measured value is to true value of quantity.
- **Precision** describes the limitation of a measuring instrument.

## Know the Formulae

- Conversion of one system of units into another

$$n_2 = \frac{n_1 u_1}{u_2} = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

- Mean absolute error

$$\Delta a_{mean} = \frac{1}{n}\times\sum_{i=1}^{i=n}|\Delta a_i|$$

- Fractional or Relative error

$$\delta a = \frac{\text{Mean absolute error}}{\text{True value}}$$

$$= \pm \frac{\Delta a_{mean}}{a_m}$$

$a_m$ = arithmetic mean

➢ Percentage error

$$\delta a = \pm \frac{\Delta a_{mean}}{a_m} \times 100\%$$

➢ If $x = a + b$, $\Delta x = \pm (\Delta a + \Delta b)$

➢ If $x = a - b$ $\Delta x = \pm (\Delta a + \Delta b)$

➢ If $x = a \times b$, $\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

➢ If $x = \frac{a}{b}$, $\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

➢ If $x = a^n$, $\frac{\Delta x}{x} = \pm n \left(\frac{\Delta a}{a}\right)$

## Know the Links

☞ *https://www.vedantu.com*

☞ *www.learncbse.in*

☞ *www.examfear.com*

## MCQ/Fillups/True or False (1 mark each)

### (A) Multiple Choice Questions

**Q. 1. The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is**

**(a) 164 ± 3 cm$^2$** **(b) 163.62 ± 2.6 cm$^2$**

**(c) 163.6 ± 2.6 cm$^2$** **(d) 163.62 ± 3 cm$^2$**

**[NCERT Exemp. Q. 2.5, Page 6]**

**Ans. Correct option:** (a) 164 ± 3 cm$^2$

***Explanation:***

$l = (16.2 \pm 0.1)$ cm

$b = (10.1 \pm 0.1)$ cm

Area, A = $l \times b$ = (16.2 cm) × (10.1 cm)

= 163.62 cm$^2$

By rounding off we get A = 164 cm$^2$

If ΔA is error in the area,

then relative error is $\frac{\Delta A}{A}$

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$= \left(\frac{0.1}{16.2} + \frac{0.1}{10.1}\right)$$

$$= \frac{2.63}{163.62}$$

$$\Delta = A \times \frac{2.63}{163.62} = 163.62 \times \frac{2.63}{163.62}$$

$$= 2.63 \text{ cm}^2$$

By rounding off ΔA = 3 cm$^2$

So, area = (164 ± 3) cm$^2$

**Q. 2. Which of the following pairs of physical quantities does not have same dimensional formula?**

**(a) Work and torque.**

**(b) Angular momentum and Planck's constant.**

**(c) Tension and surface tension.**

**(d) Impulse and linear momentum.**

**[NCERT Exemp. Q. 2.6, Page 6]**

**Ans. Correct option:** (c) Tension and surface tension

***Explanation:***

$$\text{Tension} = \text{Force} = [MLT^{-2}]$$

$$\text{Surface Tension} = \frac{\text{Force}}{\text{length}} = \frac{[MLT^{-2}]}{[L]}$$

$$= [ML^0T^{-2}]$$

**Q. 3. Measure of two quantities along with the precision of respective measuring instrument is**

**A = 2.5 m s$^{-1}$ ± 0.5 m s$^{-1}$**

**B = 0.10 s ± 0.01 s**

**The value of AB will be**

**(a) (0.25 ± 0.08) m** **(b) (0.25 ± 0.5) m**

**(c) (0.25 ± 0.05) m** **(d) (0.25 ± 0.135) m**

**[NCERT Exemp. Q. 2.7, Page 6]**

**Ans. Correct option:** (a) (0.25 ± 0.08) m

***Explanation:***

$$AB = (2.5 \times 0.10)\text{m}$$

$$= 0.25 \text{ m}$$

$$\frac{\Delta AB}{AB} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta A}{A}\right) = \left(\frac{0.5}{2.5} + \frac{0.01}{0.10}\right)$$

$$= 0.3$$

$$\Delta AB = 0.3 \times 0.25 \text{ m} = 0.075 \text{ m}$$

$= \pm 0.08$ m

$\therefore \quad AB = 0.25 \pm 0.08$ m

**Q. 4. You measure two quantities as A=1.0 m± 0.2 m, B=2.0 m±0.2 m. We should report correct value for $\sqrt{AB}$ as :**

**(a) 1.4 m ± 0.4 m** **(b) .41 m ± 0.15 m**
**(c) 1.4 m ± 0.3 m** **(d) 1.4 m ± 0.2 m**

**[NCERT Exemp. Q. 2.8, Page 6]**

**Ans. Correct option:** (d) 1.4 m ± 0.2 m

*Explanation:*

$$AB = 1 \times 2 = 2\text{ cm}^2$$

$$\sqrt{AB} = \sqrt{2}\text{ m} = 1.414\text{ m} = 1.4\text{ m}$$

$$\frac{\Delta\sqrt{AB}}{\sqrt{AB}} = \frac{1}{2}\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)=\frac{1}{2}\left(\frac{0.2}{1.0}+\frac{0.2}{2.0}\right)=\frac{0.3}{2}$$

$$\Delta\sqrt{AB} = \frac{0.3}{2}\times\sqrt{AB}=\frac{0.3}{2}\times 1.414 = 0.212\text{ m}$$

$$\Delta\sqrt{AB} = 0.2$$

$$\therefore \quad \sqrt{AB} = 1.4\text{ m} \pm 0.2\text{ m}$$

**Q. 5. Which of the following measurements is most precise?**

**(a) 5.00 mm** **(b) 5.00 cm**
**(c) 5.00 m** **(d) 5.00 km.**

**[NCERT Exemp. Q. 2.9, Page 7]**

**Ans. Correct option:** (a) 5.00 mm

*Explanation:* 5.00 mm is most precise, because least count of this measurement is 0.01 mm.

**Q. 6. The mean length of an object is 5cm. Which of the following measurements is most accurate?**

**(a) 4.9 cm** **(b) 4.805 cm**
**(c) 5.25 cm** **(d) 5.4 cm**

**[NCERT Exemp. Q. 2.10, Page 7]**

**Ans. Correct option:** (a) 4.9 cm

*Explanation:* 4.9 cm near to 5 cm.

**Q. 7. Young's modulus of steel is 1.9 × 10$^{11}$ N/m$^2$. When expresses in CGS units of dyne/cm$^2$, it will be equal to (1 N = 10$^5$ dyne, 1 m$^2$ = 10$^4$ cm$^2$)**

**(a) 1.9 × 10$^{10}$** **(b) 1.9 × 10$^{11}$**
**(c) 1.9 × 10$^{12}$** **(d) 1.9 × 10$^{13}$**

**[NCERT Exemp. Q. 2.11, Page 7]**

**Ans. Correct option:** (c) 1.9 × 10$^{12}$

*Explanation:*

$$y = 1.9 \times 10^{11}\left(\frac{10^5\text{ dyne}}{10^4\text{ cm}^2}\right) = 1.9 \times 10^{12}\text{ dyne/cm}^2$$

**Q. 8. If momentum (p), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula**

**(a) ($p^1A^{-1}T^1$)** **(b) ($p^2A^1T^1$)**
**(c) ($p^1A^{-1/2}T^1$)** **(d) ($p^1A^{1/2}T^{-1}$)**

**[NCERT Exemp. Q. 2.12, Page 7]**

**Ans. Correct option:** (d) ($p^1A^{1/2}T^{-1}$)

*Explanation:* $E = Kp^aA^bT^c$ {Here K is constant} ...(i)

$E = [ML^2T^{-2}]$ ...(ii)

$[ML^2T^{-2}] = p^aA^bT^c$ {by (i) & (ii)}

$= [MLT^{-1}]^a\,[M^0L^2T^0]^b\,[M^0L^0T]^c$

$\therefore a = 1, a + 2b = 2, -a + c = -2$

On solving this we get

$a = 1, b = \frac{1}{2}, c = -1$

$\therefore \quad E = [p^1A^{1/2}T^{-1}]$

**Q. 9. On the basis of dimensions, decide which of the following relations for the displacement of a particle undergoing simple harmonic motion is not correct:**

**(a) $y = a\sin 2\pi t/T$**
**(b) $y = a\sin vt$**
**(c) $y = \frac{a}{t}\sin\left(\frac{t}{a}\right)$**
**(d) $y = a\sqrt{2}\left(\sin\frac{2\pi t}{T} - \cos\frac{2\pi t}{T}\right)$**

**[NCERT Exemp. Q. 2.13, Page 7]**

**Ans. Correct option:** (b) and (c)

*Explanation:* Here, trigonometric functions are dimensionless and option (b) and (c) are dimensionally incorrect.

**Q. 10. If P, Q, R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?**

**(a) (P – Q)/R** **(b) PQ – R**
**(c) PQ/R** **(d) (PR – Q$^2$)/R**
**(e) (R + Q)/P**

**[NCERT Exemp. Q. 2.14, Page 7]**

**Ans. Correct option:** (a) and (e)

*Explanation:* Addition and Subtraction of physical quantities are possible if both have some dimensions.

**Q. 11. Photon is quantum of radiation with energy E = hν where ν is frequency and *h* is Planck's constant. The dimensions of *h* are the same as that of**

**(a) Linear impulse** **(b) Angular impulse**
**(c) Linear momentum** **(d) Angular momentum**

**[NCERT Exemp. Q. 2.15, Page 8]**

**Ans. Correct option:** (b) and (d)

*Explanation:* Dimension of $h = \frac{[E]}{[\nu]} = \frac{[ML^2T^{-2}]}{[T^{-1}]}$

$= [ML^2T^{-1}]$

**(b)** Dimension of Angular Impulse = [Iω]
$= (ML^2T^{-2}) \times [T] = (ML^2T^{-1})$

**(d)** Dimension of angular momentum = $[mvr]$
$= [M]\,[LT^{-1}]\,[L] = [ML^2T^{-1}]$

Hence (b) and (d) has same dimension as *h*.

**Q. 12. If Planck's constant (*h*) and speed of light in vacuum (c) are taken as two fundamental quantities, which one of the following can, in addition, be taken to express length, mass and time in terms of the three chosen fundamental quantities?**

**(a) Mass of electron ($m_e$)**
**(b) Universal gravitational constant (G)**
**(c) Charge of electron (*e*)**
**(d) Mass of proton ($m_p$)**

**[NCERT Exemp. Q. 2.16, Page 8]**

**Ans. Correct option:** (a), (b) and (d)

*Explanation:* (a), (b), (d) has dimensions in term of length, mass and time.

**Q. 13. Which of the following ratios express pressure?**

**(a) Force/Area** **(b) Energy/Volume**
**(c) Energy/Area** **(d) Force / Volume**

**[NCERT Exemp. Q. 2.17, Page 8]**

**Ans. Correct option:** (a) and (b)

*Explanation:* **(a)** $\frac{\text{Force}}{\text{Area}} = \frac{[\text{MLT}^{-2}]}{[\text{L}^2]} = [\text{ML}^{-1}\text{ T}^{-2}]$

**(b)** $\frac{\text{Energy}}{\text{Volume}} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{L}^3]} = [\text{ML}^{-1}\text{ T}^{-2}]$

Option (a) and (b) has dimensions of Pressure.

**Q. 14. Which of the following are not a unit of time?**

**(a) Second** **(b) Parsec**
**(c) Year** **(d) Light year**

**[NCERT Exemp. Q. 2.18, Page 8]**

**Ans. Correct option:** (b) and (d)

*Explanation:* Option (b) and (d) are unit of length.

## Very Short Answer Type Questions **(1 mark each)**

**Q. 1. A function $f(\theta)$ is defined as :**

$$f(\theta) = 1 - 0 + \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} ...$$

**Why is it necessary for $f(\theta)$ to be a dimensionless quantity ?**

**[NCERT Exemp. Q. 2.23, Page 9]**

**Ans.** Since $f(\theta)$ is a sum of different powers of $\theta$, it has to be dimensionless.

According to homogeneity principle, if RHS is dimensionless, then LHS should also be dimensionless.

**Q. 2. A book with many printing errors contains four different formulae for the displacement $y$ of a particle undergoing a certain periodic motion :**

**(a)** $y = a \sin \frac{2\pi t}{T}$

**(b)** $y = a \sin vt$

**(c)** $y = \frac{a}{T} \sin \left(\frac{t}{a}\right)$

**(d)** $y = \left(\frac{a}{\sqrt{2}}\right)\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)$

**($a$ = maximum displacement of the particle,**

**$v$ = speed of the particle,**

**$T$ = time period of motion). Rule out the wrong formulae on dimension grounds.**

**[NCERT Exemp. Q. 2.14, Page 36]**

**Ans.** According to the principle of homogeneity of dimensions, the dimensions of L.H.S. = dimensions of R.H.S. of an, equation. Also argument of a trigonometric function should be dimensionless.

Thus relation **(a)** and **(d)** are true to the above facts whereas **(b)** and **(c)** are false.

**Q. 3. A famous relation in physics relates 'moving mass' $m$ to the 'rest mass' $m_0$ of a particle in terms of its speed $v$ and speed of light $c$. (This relation first arose as a consequence of special theory of relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant $c$. He writes $m = \frac{m_0}{(1-v^2)^{1/2}}$. Guess where to put the missing $c$.**

**[NCERT Ex. Q. 2.15, Page 36]**

**Ans.** From principle of homogeneity of dimensions, power of M, L and T on either side of the formula must be equal. In the given formula, on R.H.S., the denominator $(1 - v^2)^{1/2}$ must be dimensionless.

Hence, the correct formula is $\frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$.

**Q. 4. Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precision measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.**

**[NCERT Ex. Q. 2.21, Page 37]**

**Ans. (i)** Precise measurement of physical quantities, time, mass and length, are needed for atomic reactions.

**(ii)** The vast applications of laser depend on the interval of time and distance ($\approx 10^{-9}$ second).

**Q. 5. Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity) :**

**(i) The total mass of rain-bearing clouds over India during the Monsoon.**

**(ii) The mass of an elephant.**

**(iii) The wind speed during storm.**

**(iv) The number of strands of hair on our head.**

**(v) The number of air molecules in your classroom.**

**[NCERT Ex. Q. 2.22, Page 37]**

**Ans. (i)** Mass of rain water over India

= Average rain fall × Area of India × Density of water

**(ii)** By lever (mass about 3000 kg for adult one)

**(iii)** By couple of devices including balloon ($\approx$ 80 $kmh^{-1}$ – 300 $kmh^{-1}$)

**(iv)** No. of human air

$$= \frac{\text{Area of head}}{\text{Area of cross-section of hair}}$$

**(v)** No. of air molecules in a given classroom

$$= \frac{\text{Volume of classroom}}{22.4\times10^{-3}m^3} \times \text{Avogadro's number.}$$

**Q. 6. Which of the following time measuring devices is most precise?**

**(a) A wall clock.**

**(b) A stop watch.**

**(c) A digital watch.**

**(d) An atomic clock.**

**Give reason for your answer.**

**[NCERT Ex. Q. 2.26, Page 9]**

**Ans.** An atomic clock is the most precise time measuring device because atomic oscillations are repeated with a precision of 1s in $10^{13}$ s.

**Q. 7. The displacement of a progressive wave is represented by $y$ = A sin ($\omega t$ – $kx$), where $x$ is distance and $t$ is time. Write the dimensional formula of (i) $\omega$ and (ii) $k$**

**[NCERT Ex. Q. 2.34, Page 10]**

**Ans. (i)** Dimensional formula of $\omega = T^{-1}$.

**(ii)** Dimensional formula of $k = L^{-1}$.

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. The vernier scale of a travelling microscope has 50 divisions which coincide with 49 main scale divisions. If each main scale division is 0.5 mm, calculate the minimum inaccuracy in the measurement of distance.**

**[NCERT Exemp. Q. 2.28, Page 9]**

**Ans.** Given: 50 VSD = 49 MSD (VSD-Vernier Scale divisions, MSD-Main scale divisions)

$$\Rightarrow \quad 1\text{ MSD} = \frac{50}{49}\text{ VSD}$$

$$1\text{ VSD} = \frac{49}{50}\text{ MSD}$$

Inaccuracy(minimum)= 1MSD-1VSD

$$= 1\text{MSD} - \frac{49}{50}\text{MSD} = \frac{1}{50}\text{MSD}$$

= 0.5 mm

$$\text{Inaccuracy (minimum)} = \frac{1}{50}\times 0.5\text{ mm}$$

= 0.01 mm

**Q. 2. If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s, what is the unit of mass in this system of units?**

**[NCERT Exemp. Q. 2.30, Page 9]**

**Ans.** Given:

Dimensions of Force, F = $[MLT^{-2}]$

= 100N (1)

Time, T = 100 s (2)

Length, L = 10 m (3)

Substituting eq$^n$ (2) & eq$^n$ (3) in eq$^n$ (1)

$M \times 10 \times (100)^{-2} = 100$

$M = 100 \times 1000$

$M = 10^5$ kg

**Q. 3. Give an example of**

**(a) a physical quantity which has a unit but no dimensions.**

**(b) a physical quantity which has neither unit nor dimensions.**

**(c) a constant which has a unit.**

**(d) a constant which has no unit.**

**[NCERT Exemp. Q. 2.31, Page 10]**

**Ans. (a)** Angle or solid angle.

**(b)** Relative density, etc.

**(c)** Planck's constant, universal gravitational constant, etc.

**(d)** Reynold's number.

**Q. 4. Time for 20 oscillations of a pendulum is measured as $t_1$ = 39.6 s; $t_2$ = 39.9 s; $t_3$ = 39.5 s. What is the precision in the measurements? What is the accuracy of the measurement?**

**[NCERT Exemp. Q. 2.35, Page 103]**

**Ans. (a)** Precision is given by the least count of the instrument.

For 20 oscillations, precision = 0.1 s

For 1 oscillation, precision = 0.005 s.

Average time for 20 oscillations

$$t = \frac{39.6+39.9+39.5}{3} s = 39.7 \text{ sec}$$

Absolute errors, $\Delta t_1 = t - t_1$

$= 39.7 - 39.6$

$= 0.1$s.

$\Delta t_2 = t - t_2$

$= 39.7 - 39.9 = -0.2$s

$\Delta t_3 = t - t_3$

$= 39.7 - 39.5 = 0.2$s

$$\text{Mean absolute error,} = \frac{|\Delta t_1|+|\Delta t_2|+|\Delta t_3|}{3}$$

$$= \frac{0.1+0.2+0.3}{3} = \frac{0.5}{3}$$

= 0.17

= ± 0.2 s.

# Long Answer Type Questions (5 marks each)

**Q. 1. A new system of units is proposed in which unit of mass is α kg, unit of length β m and unit of time γ s. How much will 5 J measure in this new system? [NCERT Exemp. Q. 2.36, Page 10]**

**Ans.** Dimension of energy = $[ML^2T^{-2}]$,

$$n_2u_2 = n_1u_1$$

$$\Rightarrow \quad n_2 = n_1\frac{u_1}{u_2}$$

$$= n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-3}$$

Given:
$n_1 = 5$ J
$M_2 = \alpha$ kg,
$M_1 = 1$ kg,
$L_2 = \beta$ m, $T_1 = 1$ sec,
$T_2 = \gamma$ s, $L_1 = 1$m,

$$\therefore \quad n_2 = 5J\left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1\left[\frac{1\text{ m}}{\beta\text{ m}}\right]^2\left[\frac{1\text{ sec}}{\gamma\text{ sec}}\right]^{-2}$$

$$n_2 = 5J[\alpha^{-1}\beta^{-2}\gamma^2]$$

New system = $\frac{5\gamma^2}{\alpha\beta^2}J$

**Q. 2. The volume of a liquid flowing out per second of a pipe of length $l$ and radius $r$ is written by a student as**

$$V = \frac{\pi}{8}\frac{Pr^4}{\eta l}$$

**where P is the pressure difference between the two ends of the pipe and η is coefficient of viscosity of the liquid having dimensional formula $[ML^{-1}T^{-1}]$. Check whether the equation is dimensionally correct. [NCERT Exemp. Q. 2.37, Page 10]**

**Ans.** The dimensional part in the expression is $\frac{Pr^4}{\eta l}$.

Therefore, the dimensions of the right hand side comes out to be $\frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L]} = \frac{[L^3]}{[T]}$, which is volume upon time. As $[V] = \frac{L^3}{T}$, the formula is dimensional correct, because LHS = RHS.

**Q. 3. A physical quantity $x$ is related to four measurable quantities $a$, $b$, $c$ and $d$ as follows:**

$$X = a^2 b^3 c^{5/2} d^{-2}$$

**The percentage error in the measurement of $a$, $b$, $c$ and $d$ are 1%, 2%, 3% and 4%, respectively. What is the percentage error in quantity $x$? If the value of $x$ calculated on the basis of the above relation is 2.763, to what value should you round off the result.**

**[NCERT Exemp. Q. 2.38, Page 10]**

**Ans.** Given: $X = a^2 b^3 c^{5/2} d^{-2}$

The fractional error in X is

$$\frac{\Delta X}{X} = \frac{2\Delta a}{a} + \frac{3\Delta b}{b} + \frac{\frac{5}{2}\Delta c}{c} + \frac{2\Delta(d)}{d}$$

$$= \pm\left[2 + 6 + \frac{15}{2} + 8\right] = \pm\ 23.5\%$$

$$= 24\%$$

Mean absolute error in X = ± 0.235 = ± 0.24
(rounding off upto two significant digits)
Since the error is in first decimal, hence the result should be rounded off as 2.8.
i.e. X = 2.8

**Q. 4. In the expression $P = El^2 m^{-5} G^{-2}$, Here, E, $m$, $l$ and G denote energy, mass, angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity.**

**[NCERT Exemp. Q. 2.39, Page 11]**

**Ans.** Since E, I and G have dimensional formulas:
$E = [ML^2T^{-2}]$
$I = [ML^2T^{-1}]$
$G = [L^3M^{-1}T^{-2}]$
Hence, $P = E\,l^2\,m^{-5}\,G^{-2}$ will have dimensions:

$$[P] = \frac{[ML^2T^{-2}][M^2L^4T^{-2}]}{[M^5][L^6M^{-2}T^{-4}]}$$

$$= M^0L^0T^0$$

Thus, P is dimensionless quantity.

**Q. 5. If velocity of light c, Planck's constant h and gravitational constant G are taken as fundamental quantities then express mass, length and time in terms of dimensions of these quantities.**

**[NCERT Exemp. Q. 2.40, Page 11]**

**Ans.** Let $m \propto c^x h^y G^z$

$m = Kc^x h^y G^z$ (A)

$h = [ML^2T^{-1}]$, $c = [LT^{-1}]$, $G = [M^{-1}L^3T^{-2}]$
($k$ = dimensionless)

or $[ML^0T^0] = [LT^{-1}]^x[ML^2T^{-1}]^y[M^{-1}L^3T^{-2}]^z$
$[M^{y-z}L^{x+2y+3z}T^{-x-y-2z}]$

Comparing powers–

$y - z = 1$ ...(1)
$x + 2y + 3z = 0$ ...(2)
$-x - y - 2z = 0$ ...(3)

Adding above all three equations-

$2y = 1 \Rightarrow y = \frac{1}{2}$

So, $z = \frac{1}{2}$, $x = \frac{1}{2}$

Putting in eqn. (A)–

$$m = kc^{\frac{1}{2}}h^{\frac{1}{2}}G^{\frac{1}{2}}$$

$$m = k\sqrt{\frac{ch}{G}}$$

**(ii)** Let $L \propto c^x h^y G^z$

$L = kc^x h^y G^z$ (B)

Substituting in B

$[M^0LT^0] = [LT^{-1}]^x \times [ML^2T^{-1}]^y \times [M^{-1}L^3T^{-2}]^z$

$= [M^{y-z}L^{x+2y+3z}T^{-x-y-2z}]$

Comparing powers-

$y - z = 0$ (a)

$x + 2y + 3z = 1$ (b)

$-x - y - 2z = 0$ (c)

Adding (a), (b), (c), we get-

$$y = \frac{1}{2}, z = \frac{1}{2}, x = -\frac{3}{2}$$

Putting in (B)-

$$L = kc^{-\frac{3}{2}}h^{\frac{1}{2}}G^{\frac{1}{2}}$$

$$L = h\sqrt{\frac{hG}{c^3}}$$

**(iii)** Let $T \propto c^x h^y G^z$

$T = Kc^x h^y G^z$ (C)

Substituting dimensions-

$[M^0L^0T] = [LT^{-1}]^x[ML^2T^{-1}]^y[M^{-1}L^3T^{-2}]^z$

$= [M^{y-z}L^{x+2y+3z}T^{-x-y-2z}]$

Comparing powers-

$y - z = 0$ (1)

$x + 2y + 3z = 0$ (2)

$-x - y - 2z = 1$ (3)

Adding (1), (2), (3), we get-

$$y = \frac{1}{2}, z = \frac{1}{2}, x = -\frac{5}{2}$$

Substituting in (C)

$$T = kc^{-\frac{5}{2}}h^{\frac{1}{2}}G^{\frac{1}{2}}$$

$$T = k\sqrt{\frac{hG}{c^5}}$$

**Q. 6. An artificial satellite is revolving around a planet of mass M and radius R, in a circular orbit of radius r. From Kepler's Third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit *r*. Show using dimensional analysis, that**

$$T = \frac{K}{R}\sqrt{\frac{r^3}{g}}$$

**Where K is dimensionless constant and g is acceleration due to gravity,**

**[NCERT Exemp. Q. 2.41, Page 11]**

**Ans.** Given From Kepler's III law, $T \propto r^{3/2}$

$T$ is also function of $g$ and $R$.

$\Rightarrow T \propto r^{3/2} R^y g^x = kr^{3/2} R^y g^x$

[$k$ = dimensionless constant of proportionality]

Substituting dimension in each term-

$\therefore \quad [L^0M^0T^1] = k[L^{3/2}M^0T^0][L^1M^0T^{-2}][L]^y$

$= k[L^{x+y+3/2}T^{-2x}]$

For $L$, $0 = \frac{3}{2} + x + y$

For $T$, $1 = -2x \Rightarrow x = -\frac{1}{2}$

Therefore, $0 = \frac{3}{2} - \frac{1}{2} + y \Rightarrow y = -1$

Thus, $T = kr^{3/2} g^{-1/2} R^{-1} = \frac{k}{R}\sqrt{\frac{r^3}{g}}$

**Q. 7. A calorie is a unit of heat or energy and it equals about 4.2 J, where 1 J = 1 kgm²s⁻². Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length β m, the unit of time is γ s. Show that a calorie has a magnitude 4.2 α⁻¹ β⁻² γ² in terms of new units.**

**[NCERT Ex. Q. 2.3, Page 35]**

**Ans.** Given : 1 cal = 4.2 J = 4.2 kg $m^2s^{-2}$

So, $a = 1$, $b = 2$, $c = -2$

In S.I. system, $n_1 = 4.2$, $M_1 = 1$ kg, $L_1 = 1$ m,

$T_1 = 1$, $n_2 = ?$, $M_2 = \alpha$ kg, $T_2 = \beta$ m, $L_2 = \gamma$ s

From equations,

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$n_2 = 4.2\left[\frac{1\text{kg}}{\alpha\text{kg}}\right]^1\left[\frac{1\text{m}}{\beta\text{m}}\right]^2\left[\frac{1\text{s}}{\gamma\text{s}}\right]^{-2}$$

$\therefore n_2 = 4.2\,\alpha^{-1}\beta^{-2}\gamma^2$ in new system.

Hence, proved.

**Q. 8. A physical quantity P is related to four observables *a*, *b*, *c* and *d* as follows :**

$$P = a^3b^2/\left(\sqrt{c}\,d\right)$$

**The percentage errors of measurement in *a*, *b*, *c* and *d* are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P ? If the value of P is calculated using the above relation turns out to be 3.763, to what value should you round off the result ?**

**[NCERT Ex. Q. 2.13, Page 36]**

**Ans.** Relative error in P is given by

$$\frac{\Delta P}{P} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}$$

So, percentage error

$$\frac{\Delta P}{P}\times 100 = 3\left(\frac{\Delta a}{a}\times 100\right) + 2\times\frac{\Delta b}{b}\times 100$$

$$+\frac{1}{2}\left(\frac{\Delta c}{c}\times 100\right)+\frac{\Delta d}{d}\times 100$$

$$= (3 \times 1\%) + (2 \times 3\%) + \left(\frac{1}{2}\times 4\%\right) + (1 \times 2\%)$$

$= 13\%$

The result should be round off to 3.8.

**TIPS... & TRICKS...**

- *Study the systems of units*
- *Understand Fundamental units and Derived units*
- *Learn various conversion in units*
- *Learn Dimensional formula of Physical quantities*
- *Write Physical quantities of same Dimensional formula*
- *Evaluate formula for maximum Percentage Error.*

## Some Commonly Made Errors

- Generally, students got confused during calculations of different types of errors.
- Students do not learn thoroughly the rules for calculating the significant figures.
- Generally, students make error during calculation of dimensional formula due to insufficient revision of it.
- Students do not convert all the units to SI during calculation of dimensional

### EXPERT ADVICE

- *Always study chronologically rather than priority order.*
- *During calculation of dimensional formula of derived quantities always use the correct formula of quantity.*
- *Make a list of all SI units of base quantities.*
- *Always check the dimensional consistency of equations by principle of homogeneity.*

### OSWAAL LEARNING TOOLS

***For* Suggested Online Videos**

**Visit :** *https://youtu.be/5UjwJ9PIUvE*

**Or Scan the Code**

**Visit :** *https://youtu.be/8Tr2PZG8I5c*

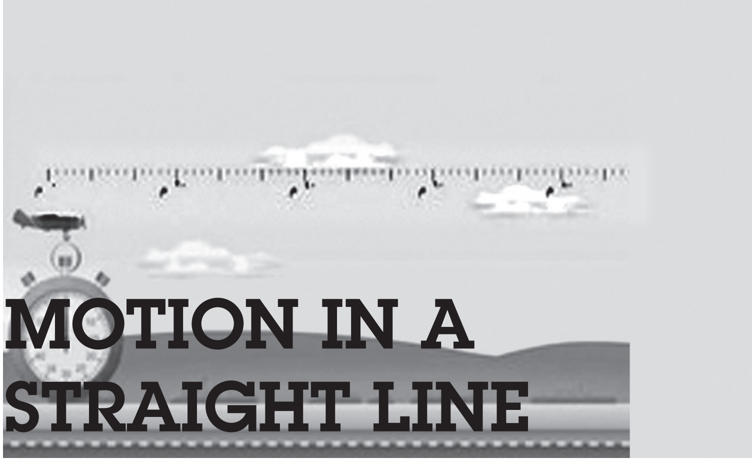
**Or Scan the Code**

**Visit :** *https://youtu.be/8Rkz4UuFS_k*

**Or Scan the Code**

**Visit :** *https://goo.gl/mDaofM*

**Or Scan the Code**



CHAPTER

# 3 MOTION IN A STRAIGHT LINE

## Chapter Objective

**This chapter will help you understand :**

- ***Motion and Velocity :*** *Frame of reference. Motion in a straight line, Position – time graph, speed and velocity. Elementary concepts of differentiation and integration for describing motion. Uniform and non-uniform motion. Average speed and instantaneous velocity.*
- ***Uniformly accelerated Motion :*** *Velocity time graph, Position-time graph relation for uniformly accelerated motion (graphical treatment).*

## TOPIC-1
## Motion and Velocity



### Quick Review

- **Rest :** An object or a particle is said to be in the state of rest when it does not change its position with time w.r.t. same reference point.

  Depending upon the position of observer, the state of rest of a particle is of two types :

**(a)** Absolute state of rest,

**(b)** Relative state of rest.

- **Motion :** An object or a particle is said to be in the state of motion when it changes its position with time w.r.t. same reference point.

  The motion of an object can be either linear or curvilinear, circular or in a plane or in a space.

**(a) Linear motion or Rectilinear or Translatory motion :**

**(i)** It is the motion in which a particle moves along a straight line with respect to a point of reference.

**(ii)** A body is said to be in linear motion if every constituent particle of the body move along parallel straight line and covers same distance in the given time.

**(b) Circular or Rotatory Motion :**

**(i)** A motion in which a particle or a point mass body is moving in a circle.

**(ii)** In rotatory motion all its constituent particles move simultaneously along concentric circles.

**(c) Oscillatory or Vibratory Motion :**

**(i)** In oscillatory motion the body moves back & forth repeatedly in definite interval of time about a fixed point.

**(ii)** If the amplitude of oscillating body is very small, the motion is called vibratory motion.

- **Dimensional Motion**

**(a) Motion in 1–D :**

**(i)** It is that motion in which a particle moves in one particular direction with respect to a point of reference.

**(ii)** In 1–D, the particle or a body moves along a straight line or a well defined path. Therefore, one dimensional motion is sometimes known as rectilinear or linear motion.

**(b) Motion in 2–D**

**(i)** If two out of three coordinates specifying the position of the object change with respect to time, the motion is called 2–D motion.

**(c) Motion in 3–D**

**(i)** If all the three coordinates specifying the position of the object change with respect to time, the motion is called 3–D motion.

➢ **Frame of Reference :** It is a system of co-ordinate axes attached to an observer having a clock with him, w.r.t. which the observer can describe position, displacement, acceleration, etc. of moving object. It is of two types :

**(a)** **Inertial frame—**It obeys Newton's first law.

**(b)** **Non-inertial frame—**It does not obey Newton's first law.

➢ **Path Length or Distance :**

**(a)** Length is defined as the actual path traversed by body during motion in a given interval of time.

**(b)** Distance is a scalar quantity.

**(c)** The S.I. unit of distance is meter and C.G.S. unit is centimeter.

**(d)** The value of distance traversed by a moving body can never be zero or negative.

➢ **Displacement :**

**(a)** Displacement of a body in a given time is defined as the change in the position of the body in a particular direction during that time. It may also defined as the shortest distance between initial and final position of the object.

**(b)** Displacement is a vector quantity as it possesses both magnitude and direction.

**(c)** Displacement of a body in a given time is represented by a vector drawn from the initial position to its final position.

**(d)** The unit of displacement is same as that of length.

**(e)** The value of displacement can be positive, zero or negative.

**(f)** The value of displacement can never be greater than the distance travelled.

**(g)** When a moving body returns to its starting point, then its effective displacement is zero.

➢ **Difference between Distance & Displacement :**

| S. No. | Distance | Displacement |
|---|---|---|
| 1. | Actual path traversed by object in given time. | Shortest distance between initial & final positions of object in given time. |
| 2. | Scalar quantity. | Vector quantity. |
| 3. | It cannot be zero or negative, it will be always positive. | It can be positive, negative or zero. |
| 4. | It is either equal or greater than displacement but never less than displacement. | It is either equal or less than distance but never greater than distance. |
| 5. | It can have many values depending upon path followed between two positions. | It has unique value. |
| 6. | Between two positions of an object, it tells type of path followed. | It does not tell type of path followed. |

➢ **Speed :** It is the ratio of total path length & corresponding time taken by an object.

**(a)** Speed is a scalar quantity.

**(b)** The speed of a body can be zero or positive but never negative.

**(c)** The speed of a body can increase or decrease with time.

**(d)** The C.G.S. unit of speed is cm/s and S.I. unit is m/s.

**(e)** If the speed of a body is zero, the body is at rest.

**(f)** The distance time graph of a body at rest, is a straight line parallel to time axis.

➢ **Types of Speed :**

**(i)** **Uniform Speed. (Dimensions are $[M^0LT^{-1}]$) :**

**(a)** A body is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, howsoever small these intervals may be.

**(b)** If a body moves with a constant speed, distance travelled by it in each second is the same.

The distance travelled at the end of 1st, 2nd, 3rd, .... second will be in the ratio 1 : 2 : 3 : .....

**(ii)** **Variable Speed :** A body is said to be moving with a variable speed if it covers equal distance in unequal intervals of time or unequal distances in equal intervals of time, howsoever small these intervals may be.

**(iii) Average or Mean Speed :** If a body is moving with a variable speed, then the average speed of the body is defined as the ratio of total distance travelled by the body to the total time taken, *i.e.*,

$$\text{Average speed} = \frac{\text{Total distance traversed}}{\text{Total time taken}}$$

**(iv) Instantaneous Speed :** When a body is moving with a variable speed, then the speed of the body at a given instant of time is known as its instantaneous speed. The body possesses different speed at different instant.

➢ **Velocity :** It is as the ratio of displacement & the corresponding time interval taken by object.

**(a)** It is vector quantity.

**(b)** It can be positive, negative or zero.

**(c)** Unit—cm/s in C.G.S. system & m/s in SI.

**(d)** Dimensions—$[LT^{-1}]$

**(i) Uniform velocity :** A body is said to be moving with a uniform velocity, if it undergoes equal displacements in equal intervals of time, howsoever small these intervals may be.

**(ii) Variable velocity :**

**(a)** The body is said to be moving with a variable velocity if it covers equal displacements in unequal intervals of time or unequal displacements in equal intervals of time or changes direction of motion while moving with a constant speed.

**(b)** The position-time graph of a body moving with variable velocity is a curve.

**(iii) Average or Mean velocity :**

**(a)** When a body is moving with a variable velocity, then the average velocity of the body for a given time is defined as the ratio of the total displacement of the body to the total time taken, *i.e.*,

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

**(b)** When a body is moving with a uniform velocity, its average velocity is equal to its uniform velocity.

**(iv) Instantaneous velocity :** If a body is moving with a variable velocity, then the velocity of the body at a given instant of time is called its instantaneous velocity.

## Know the Terms

➢ **Mechanics** deals with study of motion of material objects, with respect to the time.

➢ **Statics** deals with study of material objects at rest, with respect to the time.

➢ **Kinematics** deals with study of motion of material objects without taking into account the factors like nature of force, nature of bodies, etc. with respect to the time.

➢ **Dynamics** deals with study of motion of objects taking into account the factors which cause motion.

➢ **Uniform motion** is said to be in an object when velocity is uniform *i.e.* it undergoes equal displacements in equal intervals of time, howsoever small these intervals may be.

➢ **Non–uniform motion** is said to be in an object when it undergoes equal displacements in unequal intervals of time, howsoever small these intervals may be.

## Know the Formulae

➢ Path length or distance, D = Speed × Time

➢ Displacement = Velocity × Time

➢ $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

➢ $\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$

➢ Relative velocity —

- $\vec{v}_{BA} = \vec{v}_A - \vec{v}_B$
- $\vec{v}_{AB} = \vec{v}_B - \vec{v}_A$

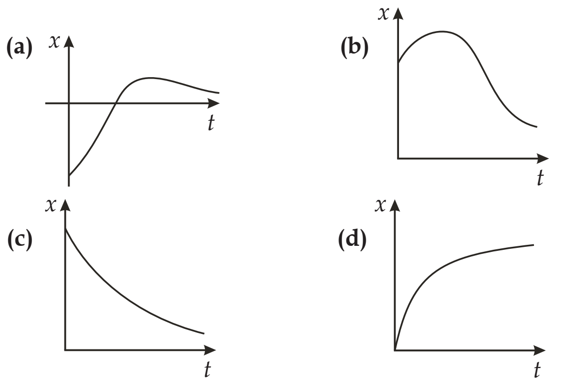
## Know the Links

- *https://schools.aglasem.com*
- *www.vedantu.com*
- *www.learncbse.in*

## MCQ/Fillups/True or False **(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. Among the four graphs, there is only one graph for which average velocity over the time interval (0, $T$ ) can vanish for a suitably chosen $T$. Which one is it?**




**[NCERT Exemp. Q. 3.1, Page 13]**

**Ans. Correct option:** (b)

*Explanation:* In (b) for the value of displacement, two timings are there. Therefore for one time, the average velocity is positive and for other time is equal but negative. Due to this average velocity can vanish.

**Q. 2. In one dimensional motion, instantaneous speed $v$ satisfies $0 \le v < v_0$**

**(a) The displacement in time T must always take non-negative values.**

**(b) The displacement $x$ in time T satisfies $-v_0T < x < v_0T$.**

**(c) The acceleration is always a non-negative number.**

**(d) The motion has no turning points.**

**[NCERT Exemp. Q. 3.3, Page 14]**

**Ans. Correct option:** (b)

*Explanation:* Since Maximum distance covered in time $T = v_0T$ . That's why, for the object having one dimensional motion, the displacement $x$ in time $T$ satisfies $-v_0T < x < v_0T$ .

**Q. 3. A vehicle travels half the distance $L$ with speed $V_1$ and the other half with speed $V_2$ then its average speed is**

**(a)** $\frac{V_1 + V_2}{2}$

**(b)** $\frac{2V_1 + V_2}{V_1 + V_2}$

**(c)** $\frac{2V_1V_2}{V_1 + V_2}$

**(d)** $\frac{L(V_1V_2)}{V_1 + V_2}$

**[NCERT Exemp. Q. 3.4, Page 14]**

**Ans. Correct option:** (c)

*Explanation:* Time taken to travel first half distance,

$$t_1 = \frac{\frac{L}{2}}{V_1} = \frac{L}{2V_1}$$

Time taken to travel second half distance,

$$t_2 = \frac{\frac{L}{2}}{V_2} = \frac{L}{2V_2}$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{L}{t_1 + t_2} = \frac{L}{\frac{L}{2V_1} + \frac{L}{2V_2}} = \frac{1}{\frac{1}{2}\left[\frac{1}{V_1} + \frac{1}{V_2}\right]}$$

$$= \frac{2V_1V_2}{V_2 + V_1}$$

**Q. 4. The displacement of a particle is given by $x = (t - 2)^2$ where $x$ is in metres and $t$ in seconds. The distance covered by the particle in first 4 seconds is**

**(a) 4 m** **(b) 8 m**

**(c) 12 m** **(d) 16 m**

**[NCERT Exemp. Q. 3.5, Page 14]**

**Ans. Correct option:** (b)

*Explanation:*

At $t = 0, x = x_0 = (0 - 2)^2 = 4$ m

$t = 1$ s, $x = x_1 = (1 - 2)^2 = 1$ m

$t = 2$ s, $x = x_2 = (2 - 2)^2 = 0$ m

$t = 3$ s, $x = x_3 = (3 - 2)^2 = 1$ m

$t = 4$ s, $x = x_4 = (4-2)^2 = 4$ m

The distance covered by the particle in 1st second is $D_1 = x_0 - x_1 = 3$ m

Similarly, $D_3 = 1$ m, $D_4 = 3$ m

The distance covered by the particle in first 4 second is

$$D = D_1 + D_2 + D_3 + D_4$$
$$= 3\text{ m} + 1\text{ m} + 1\text{ m} + 3\text{ m} = 8\text{ m}$$

**Q. 5. At a metro station, a girl walks up a stationary escalator in time $t_1$. If she remains stationary on the escalator, then the escalator take her up in time $t_2$. The time taken by her to walk up on the moving escalator will be**

**(a) $(t_1 + t_2)/2$** **(b) $t_1t_2/(t_2 - t_1)$**

**(c) $t_1t_2/(t_2 + t_1)$** **(d) $t_1 - t_2$**

**[NCERT Exemp. Q. 3.6, Page 14]**

**Ans. Correct option:** (c)

***Explanation:***

Let $L$ be the length of escalator.

Velocity of girl w.r.t escalator, $V_{ge} = \dfrac{L}{t_1}$

$\therefore$ Velocity of escalator, $V_e = \dfrac{L}{t_2}$

$\therefore$ Velocity of girl w.r.t ground would be $V_g = V_{ge} + V_e$

$$= L\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$$

$\therefore$ The desired time is $t = \dfrac{L}{V_g} = \dfrac{L}{L\left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)} = \dfrac{t_1t_2}{t_1 + t_2}$

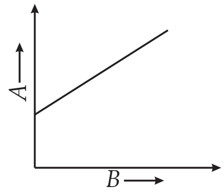
**Q. 6. The variation of quantity A with quantity B, plotted in describes the motions of a particle in a straight line.**

**(a) Quantity B may represent time.**

**(b) Quantity A is velocity if motion is uniform.**

**(c) Quantity A is displacement if motion is uniform.**

**(d) Quantity A is velocity if motion is uniformly accelerated.**




**[NCERT Exemp. Q. 3.7, Page 15]**

**Ans. Correct option:** (a), (c) and (d)

***Explanation:***

**(a)** Quantity B represents time.

**(b)** If motion is uniform and A is velocity then it should be parallel to time axis. Hence, it is incorrect.

**(c)** $A \rightarrow$ displacement, $\dfrac{dA}{dB}$ = velocity = constant

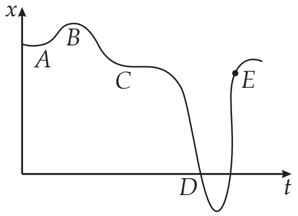
$\Rightarrow$ Motion is uniform.

**(d)** A $\rightarrow$ velocity

$\dfrac{dA}{dB}$ = acceleration = constant

$\Rightarrow$ Motion is uniformly accelerated.

**Q. 7. A graph of $x$ versus $t$ is shown in Choose correct alternatives from below.**




**(a) The particle was released from rest at $t = 0$.**

**(b) At B, the acceleration $a > 0$.**

**(c) At C, the velocity and the acceleration vanish.**

**(d) Average velocity for the motion between A and D is positive.**

**(e) The speed at D exceeds that at E.**

**[NCERT Exemp. Q. 3.8, Page 15]**

**Ans. Correct option:** (a), (c) and (e)

***Explanation:*** **(a)** At A, $t = 0$

(c) At c, stable position, so that $v = a = 0$

(e) slope of graph at D is greater than E.

So speed at D exceeds that at E.

**Q. 8. For the one-dimensional motion, described by $x = t - \sin t$**

**(a) $x(t) > 0$ for all $t > 0$.**

**(b) $v(t) > 0$ for all $t > 0$.**

**(c) $a(t) > 0$ for all $t > 0$.**

**(d) $v(t)$ lies between 0 and 2.**

**[NCERT Exemp. Q. 3.9, Page 15]**

**Ans. Correct option:** (a) and (d)

***Explanation:***

**(a)** $x(t) > 0$ for all $t > 0$

value of sin $t$ not greater than 1

then $x(t) > 0$ for all value of 1, 2, 3, 4, .......

**(d)** $v(t) = \dfrac{dx}{dt} = 1 - \cos t$

Here, $\cos t$ = minimum value = – 1

= maximum value = + 1

$v(t)_{min.} = 1 - 1 = 0$

$v(t)_{max} = 1 - (-1) = 2$

## Very Short Answer Type Questions (1 mark each)

**Q. 1. Give examples of a one-dimensional motion where**

**(a) the particle moving along positive $x$-direction comes to rest periodically and moves forward.**

**(b) the particle moving along positive $x$-direction comes to rest periodically and moves backward.**

**[NCERT Exemp. Q. 3.14, Page 16]**

**Ans. (a)** Example - $x(t) = t - \sin t$

**(b)** Example - $x(t) = \sin t$

**Q. 2. Give example of a motion where $x > 0$, $v < 0$, $a > 0$ at a particular instant.**

**[NCERT Exemp. Q. 3.15, Page 16]**

**Ans.** Example- $x(t) = A + Be^{-\gamma t}$

Here, A > B and $\gamma > 0$ and all are positive constants.

**Q. 3. In which of the following examples of motion, can a body be considered approximately a point object :**

**(a) a railway carriage moving without jerks between two stations.**

**(b) a monkey sitting on top of a man cycling smoothly on a circular track.**

**(c) a spinning cricket ball turns sharply on hitting the ground.**

**(d) a tumbling beaker that has slipped off the edge of a table.**

**[NCERT Ex. Q. 3.1, Page 55]**

**Ans. (a)** Since the motion of the train between two distant stations is smooth throughout, so keeping in view the long distance covered between the two stations in reasonable duration of time, the size of the train is neglected and it is considered as a point object.

**(b)** The distance covered by the monkey in reasonable duration of time is more so the monkey is considered as a point object. (Since its size is smaller.)

**(c)** Since, the turning of the ball is not smooth but sharp so the distance covered by it in reasonable duration of time is not large so this ball cannot be treated as a point object.

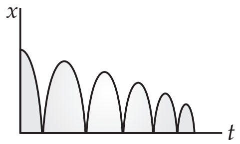
**(d)** Since the beaker is tumbling and then it slips off, So the distance covered by it in reasonable duration of time is not large. Thus, it is not treated as a point object.

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. A ball is dropped and its displacement vs time graph is as shown (displacement $x$ is from ground and all quantities are +ve upwards).**

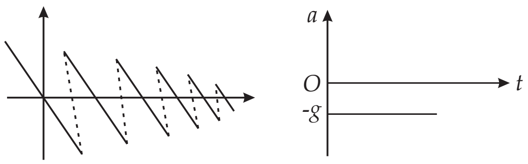
**(a) Plot qualitatively velocity vs time graph.**

**(b) Plot qualitatively acceleration vs time graph.**




**[NCERT Exemp. Q. 3.17, Page 17]**

**Ans.** Under gravity, the ball is released and falling. For short time intervals in which the ball collides with ground and when the impulsive force acts, a large acceleration produces, unless it is $-g$.




(a) (b)

**Q. 2. A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has a speed of 18 m/h while the other has the speed of 27 km/h. The bird starts moving from first car towards the other and is moving with the speed of 36 km/h and when the two cars were separated by 36 km. What is the total distance covered by the bird? What is the total displacement of the bird?**

**[NCERT Exemp. Q. 3.19, Page 17]**

**Ans.** Let first car be A and second be B,

Speed of car A, $v_A$ = 18 km/h

Speed of car B, $v_B$ = 27 km/h

Speed of bird, $v$ = 36 km/h

Relative speed of cars,

$v_{AB} = v_{BA} = v_A + v_B = 45$ km/h

Initial distance between cars, $x$ = 36 km

Time taken by cars to meet, $t = \frac{x}{v_{AB}} = \frac{36}{45}$ km/h

$= 0.8$ h

Distance covered by bird in the time,

$x' = v > t = 36 \times 0.8 = 28.8$ km

Displacement of bird = distance covered by car A in 0.8 h on straight line (because bird starts from car A)

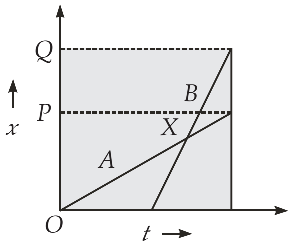
$\therefore x_A = v_A \times t = 18 \times 0.8 = 14.4$ km

Suppose man is at building A and wants to land on building B.

Vertical speed of man, $v_y = 0$

**Q. 3. The position time ($x-t$) graphs for two children A and B returning form their school O to their homes P and Q respectively are shown in fig.**

**Choose the correct entries in the brackets below:**




**(a) (A/B) lives closer to the school than (B/A)**

**(b) (A/B) starts from the school earlier than (B/A)**

**(c) (A/B) walks faster than (B/A)**

**(d) A and B reach home at the (same/different) time**

**(e) (A/B) overtakes (B/A) on the road (once/twice).**

**[NCERT Ex. Q. 3.2, Page 55]**

**Ans.** **(a)** A lives closer to the school than B, because B has to cover higher distance [OP < OQ].

**(b)** A starts from the school earlier than B because $t$ = 0 for A but B has some finite value of time.

**(c)** B walks faster than A because it covers more distance in less duration of time [slope of B is greater than that of A].

**(d)** A and B reach home at the same time.

**(e)** B overtakes A on the road once (at $x$, *i.e.*, the point of intersection).

**Q. 4. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the *x-t* graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.**

**[NCERT Ex. Q. 3.4, Page 56]**

**Ans.** Distance covered with 1 step = 1 m

Time taken = 1 s

Time taken to move first 5 m forward = 5 s

Time taken to move 3 m backward = 3 s

Net distance covered = 5 – 3 = 2 m

Net time taken to cover 2 m = 8 s

Drunkard covers 2 m in 8 s.

Drunkard covered 4 m in 16 s.

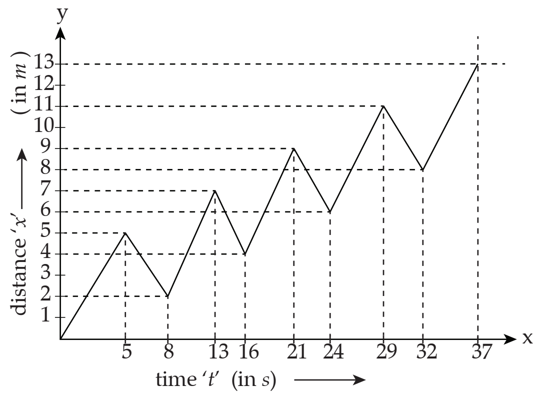
Drunkard covered 6 m in 24 s.

Drunkard covered 8 m in 32 s.

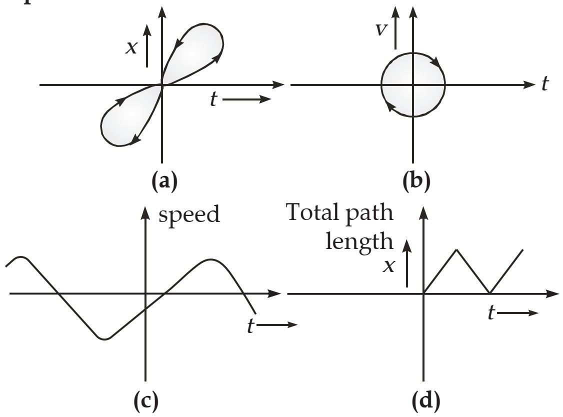
In the next 5 s, the drunkard will cover a distance of 5 m and a total distance of 13 m and falls into the pit.

Net time taken by the drunkard to cover 13 m = 32 + 5 = 37 s

The x-t graph of the drunkard's motion can be shown as:




**Q. 5. Look at the graphs (a) to (d) in figure carefully and state with reasons which of the following cannot possibly represent one dimensional motion of a particle.**




**[NCERT Ex. Q. 3.16, Page 57]**

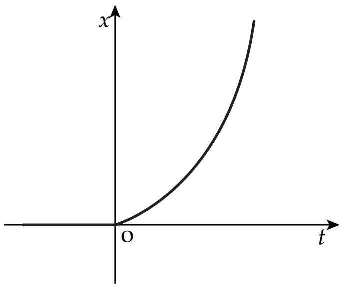
**Ans.** Figure (a) does not represent one dimensional motion of particle because the particle has two different positions at the same instant which is not the case of one dimensional motion.

Figure (d) also does not represent one dimensional motion of the particle because here the total path length is shown to decrease with time which is not possible in one dimensional motion.

Graph (b) does not represent one dimensional motion because at the same instant a particle cannot have positive and negative velocity if the motion is one dimensional.

Graph (c) does not represent one dimensional motion because speed cannot be negative as shown in graph.

**Q. 6. Figure shows the *x-t* plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$ ? If not, suggest a suitable physical context for this graph.**




**[NCERT Ex. Q. 3.17, Page 58]**

**Ans.** No, because the *x-t* graph does not represent the trajectory of the path followed by a particles. From the graph, a suitable physical context can be the particle thrown from the top of the tower at $t = 0$.

**Q. 7. A police van moving on a highway with a speed of 30 kmh$^{-1}$ fires a bullet at a thief's car speeding away in the same direction with a speed of 192 kmh$^{-1}$. If the muzzle speed of the bullet is 150 ms$^{-1}$, with what speed does the bullet hit the thief's car ? (Note: Obtain that speed which is relevant for damaging the thief's car).**

**[NCERT Ex. Q. 3.18, Page 58]**

**Ans.** Speed of the police van, $v_p$ = 30 km/h = 8.33 m/s

Muzzle speed of the bullet, $v_b$= 150 m/s

Speed of the thief 's car, $v_t$= 192km/h = 53.33m/s

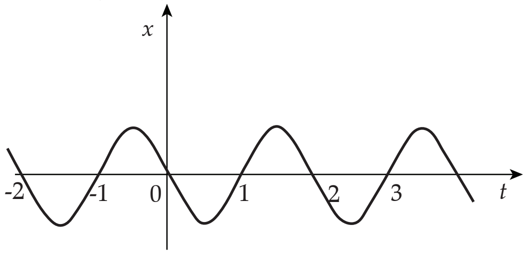
Since the bullet is fired from a moving van, its resultant speed can be obtained as:

(= 150 + 8.33 = 158.33 m/s)

Since both the vehicles are moving in the same direction, the velocity with which the bullet hits the thief's car can be obtained as:

$$v_{tb} = v_b - v_t$$
$$= 158.33 - 53.33 = 105 \text{ m/s}$$

**Q. 8. Figure gives the *x-t* plot of a particle executing one-dimensional simple harmonic motion. (You will learn about this motion in more detail in Chapter 14). Give the signs of position, velocity and acceleration variables of the particle at *t* = 0.3 *s*, *1.2 s*, *–1.2 s*.**




**[NCERT Ex. Q. 3.20, Page 58]**

**Ans.** Negative, Negative, Positive (at $t = 0.3$ s)

Positive, Positive, Negative (at $t = 1.2$ s)

Negative, Positive, Positive (at $t = 1.2$ s)

For simple harmonic motion (SHM) of a particle, acceleration (a) is given by the relation: angular frequency

$a = -\omega^2 x$ ...(i)

where, all symbols have their usual meanings.

t = 0.3 s

In this time interval, x is negative. Thus, the slope of the x-t plot will also be negative. Therefore, both position and velocity are negative. However, using equation (i), acceleration of the particle will be positive.

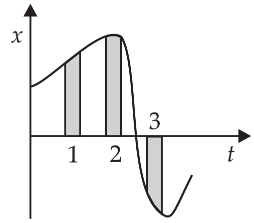
$t = 1.2$ s

In this time interval, x is positive. Thus, the slope of the x-t plot will also be positive. Therefore, both position and velocity are positive. However, using equation (i), acceleration of the particle comes to be negative.

$t = -1.2$ s

In this time interval, x is negative. Thus, the slope of the x-t plot will also be negative. Since both x and t are negative, the velocity comes to be positive. From equation (i), it can be inferred that the acceleration of the particle will be positive.

**Q. 9. Figure gives x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.**




**[NCERT Ex. Q. 3.21, Page 58]**

**Ans.** Average speed in small interval of time is equal to slope of x-t graph in same interval.

Average speed is greatest in interval 3 because slope is greatest in this interval.

Average speed is least in interval 2 because slope is least in this interval.

Average speed is positive in intervals 1 and 2 and negative in interval 3.

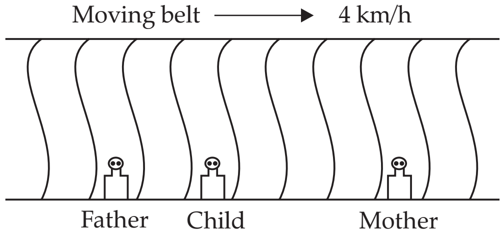
**Q. 10. On a long horizontally moving belt (in fig.), a child runs to and fro with a speed 9 kmh$^{-1}$ (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 kmh$^{-1}$. For an observer on a stationary platform outside, what is the-**

**(a) speed of the child running in the direction of motion of the belt?**

**(b) speed of the child running opposite to the direction of motion of the belt?**

**(c) time taken by the child in (a) and (b)?**

**Which of the answers alter if motion is viewed by one of the parents?**




**[NCERT Ad. Ex. Q. 3.25, Page 59]**

**Ans.** **(a)** speed of belt, $v_B = 4\ \text{kmh}^{-1}$

speed of child, $v_C = 9\ \text{kmh}^{-1}$

speed of child w.r.t. stationary observer

$$v_C' = v_C + v_B = 9 + 4 = 13\ \text{kmh}^{-1}$$

**(b)** speed of child w.r.t. stationary observer (in opposite direction)

$$v_C'' = v_C - v_B = 9 - 4 = 5\ \text{kmh}^{-1}$$

**(c)** Distance between the child's parents = 50 m

Speed of the child w.r.t parent = 9 km/h = 2.5 m/s

Time taken, $= t = \frac{50}{2.5} = 205$

If motion is viewed by one of the parents, case (a) and case (b) will get altered but case (c) remains unaltered.

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. It is a common observation that rain clouds can be at about a kilometer altitude above the grounds.**

**(a) If a rain drop falls from such a height freely under gravity, what will be its speed? Also calculate in km/h. ($g$ = 10 m/s$^2$)**

**(b) A typical rain drop is about 4 mm diameter. Momentum is mass x speed in magnitude. Estimate its momentum when it hits ground.**

**(c) Estimate the time required to flatten the drop.**

**(d) Rate of change of momentum is force. Estimate how much force such a drop would exert on you.**

**(e) Estimate the order of magnitude force on umbrella. Typical lateral separation between two rain drops is 5 cm.**

**(Assume that umbrella is circular and has a diameter of 1 m and cloth is not pierced through !!). [NCERT Exemp. Q. 3.23, Page 18]**

**Ans.** **(a)** $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1000} = 141\ \text{m/s}$

$= 507.6$ km/h.

**(b)** $m = \frac{4\pi}{3} r^3 \rho = \frac{4\pi}{3}(2 \times 10^{-3})^3 (10^3)$

$= 3.4 \times 10^{-5}$ kg.

$p = mv \approx 4.7 \times 10^{-3}$ kg m/s $\approx 5 \times 10^{-3}$ kg m/s.

**(c)** Diameter $\approx$ 4 mm

$\Delta t \approx d / v = 28\ \mu s \approx 30\ \mu s$

**(d)** $F = \frac{\Delta p}{\Delta t} = \frac{4.7 \times 10^{-3}}{28 \times 10^{-6}} \approx 168\ \text{N} \approx 1.7 \times 10^2$ N.

**(e)** Area of cross-section $= \pi d^2 / 4 \approx 0.8\ \text{m}^2$.

With average separation of 5 cm, no. of drops that will fall almost simultaneously is $\frac{0.8\ \text{m}^2}{(5 \times 10^{-2})^2} \approx 320$.

Net force $\approx$54000 N

(Practically drops are damped by air viscosity).

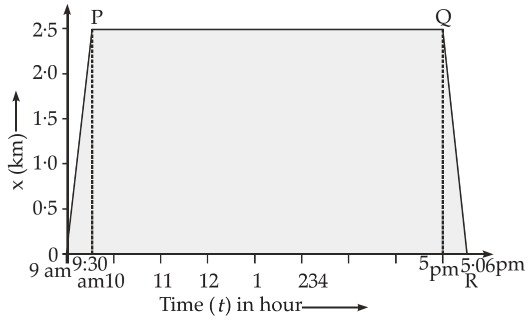
**Q. 2. A woman starts from her home at 9.00 a.m., walks with a speed of 5 kmh$^{-1}$ on a straight road up to her office 2.5 km away, stays at the office up to 5.00 p.m. and returns home by an auto with a speed of 25 kmh$^{-1}$. Choose suitable scales and plot the *x-t* graph of the motion.**

**[NCERT Exemp. Q. 3.3, Page 36]**

**Ans.** *x-t* graph of the motion of woman is shown in given figure.

Let $v_1$ = speed of woman while walking at 5 kmh$^{-1}$

$x$ = Distance covered by her = 2.5 km




If $t_1$ = time taken to reach office, then it can be calculated by using the formula

$$x = v_1 t_1$$

$$\Rightarrow \qquad t_1 = \frac{x}{v_1}$$

Hence, $t_1 = \frac{2.5}{5} = \frac{1}{2} h = 30$ minutes.

When O is regarded as origin for both time and distance, then at $t$ = 9.00 a.m., $x$ = 0, and at $t$ = 9.30 a.m., $x$ = 2.5 km and she reaches in her office. So, OP represents *x-t* graph of the motion when the woman walks from her home to office.

When she stays at her office from 9.30 a.m. to 5.00 p.m., she is at rest and her stay is represented by the straight line PQ in the graph.

On return, speed of auto,

$$v_2 = 25\ \text{km/h}.$$

Let, $t_2$ = time taken by her, *i.e.* by auto from office to her home, then

$$t_2 = \frac{x}{v_1} = \frac{2.5}{25} = \frac{1}{10}\ \text{hour}$$

$= 6$ minutes.

So, she reaches back to her home at 5.06 p.m.

Her motion on the return journey is shown by QR part of the graph.

Scale taken

Time on x-axis, 1 division = 1 hour.

Distance on y-axis, 1 division = 0.5 km.

**Q. 3. A jet airplane travelling at a speed of 500 kmh$^{-1}$, ejects its products of combustion at the speed of 1500 kmh$^{-1}$ relative to the jet plane. What is the speed of the later with respect to an observer on the ground ?**

**[NCERT Ex. Q. 3.5, Page 56]**

**Ans.** Suppose $\vec{v}_j, \vec{v}_g$ and $\vec{v}_o$ be the velocities of jet, ejected gases, *i.e.*, combustion products and observer on the ground respectively.

Suppose jet be moving toward right (positive direction). The ejected gases will move towards left (negative direction).

∴ According to the statement

$$\vec{v}_j = 500 \text{ kmh}^{-1}$$

Since, observer is at ground, i.e., at rest

$$\vec{v}_o = 0$$

Now, relative velocity of plane w.r.t. the observer is

$$= \vec{v}_j - \vec{v}_o$$

$$= 500 - 0 = 500 \text{ kmh}^{-1} \quad ...(i)$$

Relative velocity of the combustion products w.r.t. jet plane

$$\vec{v}_g - \vec{v}_j = -1500 \text{ kmh}^{-1} \text{ (given)} \quad ...(ii)$$

Negative sign shows that the combustion products move in a direction opposite to that of jet.

∴ Adding equations (i) and (ii), we get the speed of combustion products w.r.t. observer on the ground, *i.e.*,

$$(\vec{v}_j - \vec{v}_o) + (\vec{v}_g - \vec{v}_j) = 500 + (-1500)$$

$$\vec{v}_g - \vec{v}_o = -1000 \text{ kmh}^{-1}$$

Negative sign indicates that relative velocity of the ejected gases w.r.t. observer is towards left. It means negative direction, *i.e.*, in a direction opposite to the motion of the jet plane.

**Q. 4. Two towns *p* and *q* are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 kmh$^{-1}$ in the direction *p* to *q* notices that a bus goes past him every 18 min in the direction of his motion and every 6 min in opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road ?**

**[NCERT Ex. Q. 3.9, Page 56]**

**Ans.** Suppose the speed of each bus = $v_q$ kmh$^{-1}$

and speed of cyclist = $v_c$ = 20 kmh$^{-1}$

**Case I :** Relative speed of the buses plying in the direction of motion of cyclist, *i.e.*, from *p* to *q* = $v_q - v_c = (v_q - 20)$ kmh$^{-1}$.

Since, the bus goes past the cyclist every 18 minutes i.e., $\left(\frac{18}{60}h\right)$

∴ Distance covered by the bus w.r.t. the cyclist

$$= (v_q - 20) \times \frac{18}{60} \text{ km.} \quad ...(i)$$

As the bus leaves after every T minutes, hence the distance covered by the bus in T min i.e., $\left(\frac{T}{60}\text{h}\right)$ is given by

$$= v_q \times \frac{T}{60} \quad ...(ii)$$

∴ From equation (i) and (ii), we get

$$(v_q - 20) \times \frac{18}{60} = v_q \times \frac{T}{60}$$

or $$v_q - 20 = v_q \times \frac{T}{18} \quad ...(iii)$$

**Case II :** Relative speed of the bus coming from town *q* to *p* w.r.t. cyclist = $(v_q + 20)$ kmh$^{-1}$

(∴ cyclist is moving from *p* to *q*)

As the bus goes past the cyclist after every 6 minutes i.e., $\frac{6}{60}$h

∴ Distance covered by the bus w.r.t. the cyclist

$$= (v_q + 20) \times \frac{6}{60} \text{ km} \quad ...(iv)$$

Also distance covered by bus in T minutes is

$$= v_q \times \frac{T}{60} \text{ km} \quad ...(v)$$

∴ From equation (iv) and (v), we get

$$(v_q + 20) \times \frac{6}{60} = v_q \times \frac{T}{60}$$

or $$v_q + 20 = v_q \times \frac{T}{6} \quad ...(vi)$$

Dividing (vi) by (iii), we get

$$\frac{v_q + 20}{v_q - 20} = 3$$

or $$v_q + 20 = 3v_q - 60$$

or $$2v_q = 80$$

or $$v_q = 40 \text{ kmh}^{-1}$$

Putting the value of $v_q$ in equation (iii), we get

$$40 - 20 = 40 \times \frac{T}{18}$$

or $$20 = 40 \times \frac{T}{18}$$

or $$T = 20 \times \frac{18}{40} = 9 \text{ minutes}$$

∴ $$v_q = 40 \text{ kmh}^{-1}, T = 9 \text{ min.}$$

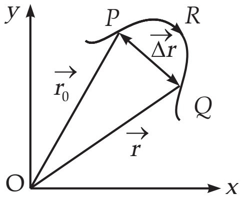
**Q. 5. Explain clearly, with examples, the distinction between :**

**(a) Magnitude of displacement (sometimes called distance) over an interval of time and the total length of path covered by a particle over the same interval.**

**(b) Magnitude of average velocity over an interval of time and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]. Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true ? [For simplicity, consider one dimensional motion only.]**

**[NCERT Ex. Q. 3.13, Page 57]**

**Ans. (a)** Magnitude of displacement in a given time interval is the difference between the initial and final positions of the body. It can be expressed as $|\vec{r} - \vec{r_0}|$.




The total length of the path travelled by a body is the length of the actual path covered by it. The total length of the path is either equal or greater than the magnitude of the displacement.

**(b)** Suppose a particle at times $t_1$ and $t_2$ be at the positions P and Q, respectively

∴ Average velocity between $t_1$ and $t_2$

$$= \frac{\overrightarrow{PQ}}{t_2 - t_1}$$

or $$\vec{v_a} = \underset{\Delta t \to 0}{\text{Lt}} \left(\frac{\Delta \vec{r}}{\Delta t}\right) = \frac{d\vec{r}}{dt}$$

where, $\Delta t = t_2 - t_1$

The average velocity does represent us whether the motion of the particle was actually along the path PQ or along PRQ and whether the motion between $t_1$ and $t_2$ was steady or not. Average speed is defined as the ratio of the total distance covered by the body to the total time taken, *e.g.*, suppose a person goes to a temple 1 km away from his house and comes back taking 1 hour to perform the journey both ways.

∴ Average speed of the man

$$= \frac{1+1}{1} = 2 \text{ km/h.}$$

But average velocity is zero because his displacement during the time of 1 hour is zero.

So, in either case (a) and (b) the second quantity is either greater than or equal to the first quantity. Equality sign is correct when displacement is equal to the distance. It holds when body moves along a straight path in a fixed direction.

**Q. 6. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 kmh$^{-1}$. Finding the market closed he instantly turns and walks back home with a speed of 7.5 kmh$^{-1}$. What is the**

**(a) magnitude of average velocity, and**

**(b) average speed of the man over the interval of time.**

**(i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min? [Note : You will appreciate from the exercise why it is better to define average speed as total path length divided by time, and not as magnitude of average velocity. You would not like to tell the tired man on his return home that his average speed was zero.]**

**[NCERT Ex. Q. 3.14, Page 57]**

**Ans. (a)** From relation average velocity

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

**(b)** and Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

**(i) For 0 to 30 min interval :**

Distance travelled in going to market

= 2.5 km

speed = 5 kmh$^{-1}$

∴ Time taken to go to market

$$= \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{2.5}{5} = \frac{1}{2}\text{ h} = 30 \text{ min.}$$

∴ **(a)** Magnitude of average velocity

$$= \frac{\text{Displacement}}{\text{Time}} = \frac{2.5 \text{ km}}{\frac{1}{2}\text{h}}$$

$= 5$ kmh$^{-1}$

**(b)** Average speed

$$= \frac{2.5 \text{ km}}{\frac{1}{2}\text{h}} = 5 \text{ kmh}^{-1}$$

**(ii) For 0 to 50 min. interval**

$$= \frac{50}{60}\text{h} = \frac{5}{6}\text{h}$$

Speed of return journey

= 7.5 kmh$^{-1}$

Distance to be covered to reach home

= 2.5 km

∴ Time taken in return journey

$$= \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{2.5}{7.5}$$

$$= \frac{1}{3}\text{ h} = 20 \text{ min.}$$

∴ Total time taken for the journey

= 30 + 20 = 50 min.

Now let displacement of the man is zero as he started from his home and came back home.

*i.e.,* $25 - 25 = 0$ km

$\therefore$ **(a)** Magnitude of average velocity

$$= \frac{0}{5/6} = 0$$

Total distance travelled during to whole journey

$$= 2.5 + 2.5$$
$$= 5 \text{ km}$$

$\therefore$ **(b)** Average speed $= \dfrac{5\text{km}}{(5/6)\text{h}} = 6 \text{ kmh}^{-1}$

**(iii) For 0 to 40 min. interval**

$$= \frac{2}{3}\text{h}$$

Since, the man takes 30 min. to go to market

$\therefore$ Time for which he performed return journey

$$= 40 - 30 = 10 \text{ min}$$

Distance travelled in 10 min.

= Velocity of return journey × time

$$= 7.5 \times \frac{10}{60} = 1.25 \text{ km}$$

$\therefore$ Net displacement made by the man

$$= 2.5 - 1.25 = 1.25 \text{ km}$$

$\therefore$ Average velocity $= \dfrac{1.25\text{km}}{\frac{2}{3}}$

$$= 1.25 \times \frac{3}{2}$$
$$= \frac{3.75}{2}$$
$$= 1.875 \text{ kmh}^{-1}$$

and

Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

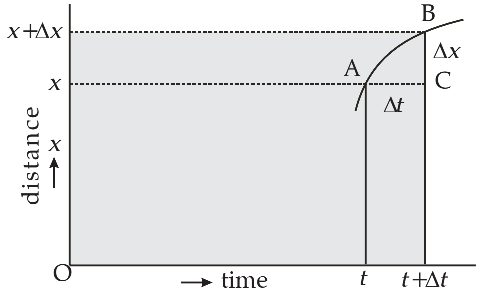
$$= \frac{2.5+1.25}{\frac{2}{3}} \text{ kmh}^{-1}$$
$$= 3.75 \times \frac{3}{2} \text{ kmh}^{-1}$$
$$= \frac{11.75}{2} = 5.625 \text{ kmh}^{-1}$$

Hence, from case (ii), we see that average velocity is zero but not the average speed, so it is better to define average speed as the ratio of the total distance covered to the total time taken and not as the magnitude of average velocity.

**Q. 7. In Exercises 5 and 6, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?**

**[NCERT Ex. Q. 3.15, Page 57]**

**Ans.** The instantaneous velocity is defined as the velocity of an object at a particular instant of time. The instantaneous speed is defined as the limiting value of the average speed. So, when time interval is very small, the magnitude of the displacement is effectively equal to the distance travelled by the object in the same small interval of time. Hence, both instantaneous velocity and instantaneous speed are equal in this case. This can be understood from the following as :




Let a small displacement over a time $\Delta t$ between time interval, $t$ and $t + \Delta t$.

$\therefore$ Average velocity in the interval

$$v = \frac{\Delta x}{\Delta t}$$

$\therefore$ Instantaneous velocity at instant $t$ is

$$= \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad ...(i)$$

Also,

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$
$$= \frac{\text{Arc AB}}{\Delta t}$$

$\therefore$ Instantaneous speed

$$= \underset{\Delta t \to 0}{\text{Lt}} \text{ average speed}$$
$$= \underset{\Delta t \to 0}{\text{Lt}} \left(\frac{\text{Arc AB}}{\Delta t}\right)$$
$$= \underset{\Delta t \to 0}{\text{Lt}} \left(\frac{\text{AB}}{\Delta t}\right) \quad ...(ii)$$

where, arc AB ≈ length of the line AB

Now from ΔABC,

$$AB^2 = AC^2 + BC^2$$

From equation (2), we get

Instantaneous speed

$$= \underset{\Delta t \to 0}{\text{Lt}} \left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

which is equal to magnitude of instantaneous velocity.

# TOPIC-2
## Uniformly Accelerated Motion

## Quick Review

- **Accelerated motion :** When an object is moving in non-uniform motion, the velocity is different at different instants *i.e.* the velocity keeps on changing with time. This motion is an accelerated motion.
- **Acceleration :** It is defined as the ratio of change in velocity & the corresponding time taken by the mirror object *i.e.*

(a) It is vector quantity.
(b) It is either positive or negative.
(c) Negative acceleration is called retardation.
(d) Unit-m/$s^2$ in SI & cm/$s^2$ in CGS system.
(e) Dimensional formula– $[LT^{-2}]$.

**(i) Uniform acceleration :** An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time.

**(ii) Variable acceleration :**

(a) An object is said to be moving with a variable acceleration when its velocity changes by unequal amounts in equal intervals of time.
(b) The velocity time graph of a body having variable acceleration is represented by a curve.

**(iii) Average acceleration :** When an object is moving with a variable acceleration, then the average acceleration of the body is defined as the ratio of the total change in velocity during motion to the total time taken, *i.e.*

$$\text{Average acceleration} = \frac{\text{Total change in velocity}}{\text{Total time taken}}$$

**(iv) Instantaneous acceleration :**

(a) If a body is moving with a variable acceleration, then the acceleration of a body at the given instant of time is called instantaneous acceleration.
(b) If at an instant $t$, a body while moving with a variable acceleration shows a change in velocity $\Delta\vec{v}$ in a small interval of time $\Delta t$, so that $\Delta t \to 0$, then

$$\text{Instantaneous acceleration} = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

where, $\frac{d\vec{v}}{dt}$ is the derivative of velocity $(\vec{v})$ w.r.t. time.

## Know the Terms

- **Total displacement of the body** in the given time is equal to the area which velocity time-graph encloses with time axis.
- **Uniformly accelerated object** in a given time-interval is represented by the slope on the velocity-time graph for a given time-interval.
- **Acceleration of object** is the slope of velocity-time graph of uniformly accelerated motion.
- **Reaction time** is that time which a person takes to observe, think & act.

## Know the Formulae

- Suppose

$u$ = initial velocity of body,
$a$ = uniform acceleration of the body,
$v$ = velocity of the body after time $t$,
$s$ = distance travelled by body in time $t$,

$s_n$ = distance travelled by body in $n^{th}$ second.

**(a) The equations of motion for accelerated body are :**

**(i)** $v = u + at$ **(ii)** $s = ut + \frac{1}{2}at^2$

**(iii)** $v^2 = u^2 + 2as$ **(iv)** $s_n = u + \frac{a}{2}(2n - 1)$

**(b) The equations of motion for retarded body (here, *a* is negative) are :**

**(i)** $v = u - at$ **(ii)** $s = ut - \frac{1}{2}at^2$

**(iii)** $v^2 = u^2 - 2as$ **(iv)** $s_n = u - \frac{a}{2}(2n - 1)$

**(c) The equations of motion for a body falling down under gravity (here, $a = +g$, $s = h$) are :**

**(i)** $v = u + gt$ **(ii)** $h = ut + \frac{1}{2}gt^2$

**(iii)** $v^2 = u^2 + 2gh$ **(iv)** $h_n = u + \frac{g}{2}(2n - 1)$

**(d) The equations of motion for a body going up under gravity (here $a = -g$, $s = h$) are :**

**(i)** $v = u - gt$ **(ii)** $h = ut - \frac{1}{2}gt^2$

**(iii)** $v^2 = u^2 - 2gh$ **(iv)** $h_n = u - \frac{g}{2}(2n - 1)$

**(e) The maximum height attained by a body thrown vertically upwards with initial velocity *u* is**

$$h_{max} = \frac{u^2}{2g}$$

**(f) Time taken to reach the maximum height is**

$$t = \frac{u}{g}$$

**(g) Total time taken by body in going up and coming down,**

$$T = 2t = \frac{2u}{g}$$

**(h) The initial velocity of body in order to attain height *h* is,**

$$u = \sqrt{2gh}$$

## Know the Links

- *www.vedantu.com*
- *https://schools.aglasem.com*
- *www.learncbse.in*

## MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct?**

**(a)** $x < 0, v < 0, a > 0$

**(b)** $x > 0, v < 0, a < 0$

**(c)** $x > 0, v < 0, a > 0$

**(d)** $x > 0, v > 0, a < 0$

**[NCERT Exemp. Q. 3.2, Page 14]**

**Ans. Correct option:** (a)

*Explanation:* Lift is coming downwards, so it's retarding. Thus, a is acting downwards, so $a > 0$ and the value of $x$ becomes less here negative, i.e., $x < 0$, velocity is downwards (i.e. negative) so $v < 0$.

**Q. 2. A spring with one end attached to a mass and the other to a rigid support is stretched and released.**

**(a) Magnitude of acceleration, when just released is maximum.**

**(b) Magnitude of acceleration, when at equilibrium position, is maximum.**

**(c) Speed is maximum when mass is at equilibrium position.**

**(d) Magnitude of displacement is always maximum whenever speed is minimum.**

**[NCERT Exemp. Q. 3.10, Page 15]**

**Ans. Correct option:** (a) and (c)

*Explanation:* It is a simple harmonic motion, in S.H.M. the maximum value of acceleration is extreme position and the maximum value of velocity is equilibrium position. So option (a) and (c) are correct.

**Q. 3. A ball is bouncing elastically with a speed 1 m/s between walls of a railway compartment of size 10 m in a direction perpendicular to walls. The train is moving at a constant velocity of 10 m/s parallel to the direction of motion of the ball. As seen from the ground,**

**(a) the direction of motion of the ball changes every 10 seconds.**

**(b) speed of ball changes every 10 seconds.**

**(c) average speed of ball over any 20 second interval is fixed.**

**(d) the acceleration of ball is the same as from the train.**

**[NCERT Exemp. Q. 3.11, Page 16]**

**Ans. Correct option:** (b), (c) and (d)

*Explanation:* In this question train is stable with respect to ball now ball seen from ground. option (b), (c) and (d) are correct.

## (B) True or False :

**Q. 4. Read each statement below carefully and state with reasons and examples, if it is correct or wrong. A particle in one-dimensional motion.**

**(a) with zero speed at an instant may have non-zero acceleration at that instant.**

**(b) with zero speed may have non-zero velocity.**

**(c) with constant speed must have zero acceleration.**

**(d) with positive value of acceleration must be speeding up.**

**[NCERT Exemp. Q. 3.11, Page 57]**

**Ans. (a)** True, Example, if the ball is in vertically upward motion, *i.e.*, thrown vertically upward, then it will be of zero speed at the highest point and an acceleration of 9.8 $ms^{-2}$ in downward direction.

**(b)** False, because speed is the magnitude of velocity.

**(c)** True, if the particle rebounds instantly with the same speed acceleration will be zero.

Example, elastic collision of a ball with an elastic floor.

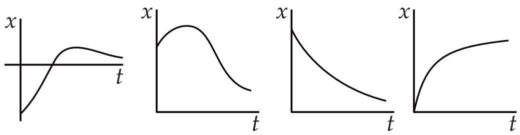
**(d)** False,

Example, if we take the positive direction of acceleration due to gravity downwards, a ball thrown vertically upwards will have its speed decreasing. However, the statement can be correct if the positive direction of acceleration is along the direction of motion.

# Very Short Answer Type Questions **(1 mark each)**

**Q. 1. Refer to the graphs given below in figure. Match the following**

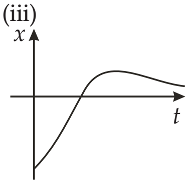
| Graph | Characteristics |
|---|---|
| **(a)** | **(i) has $v > 0$ and $a < 0$ throughout.** |
| **(b)** | **(ii) has $x > 0$ throughout and has a point with $v = 0$ and a point with $a = 0$.** |
| **(c)** | **(iii) has a point with zero displacement for $t > 0$.** |
| **(d)** | **(iv) has $v < 0$ and $a > 0$.** |




**(a) (b) (c) (d)**

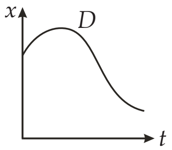
**[NCERT Exemp. Q. 3.12, Page 16]**

**Ans. (a)** – (iii) From graph, at point B, $x = 0$ For $t > 0$. Hence (a) → (iii)




**(b)** – (ii) From graph, throughout the motion $x > 0$. At point C, $\frac{dx}{dt} = v = 0$

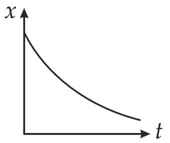
Since at D, curvature changes, hence $a = 0$



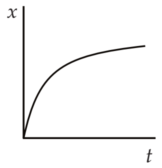

**(c)** – (iv) From graph, $\frac{dx}{dt}$ = negative ; So, $v < 0$;

$a = \frac{d^2x}{dt^2}$ is positive.

Since rate of change of negative velocity decreases. So, $a > 0$



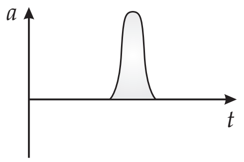

**(d)** – (i) This graph is reverse of the graph above $v > 0$ but $a < 0$.




**Q. 2. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time interval. Show the variation of its acceleration with time. (Take acceleration in the backward direction as positive).**

**[NCERT Exemp. Q. 3.13, Page 16]**

**Ans.** Variation of ball's acceleration with time-




**Q. 3. An object falling through a fluid is observed to have acceleration given by $a = g - bv$ where $g$ = gravitational acceleration and b is constant. After a long time of release, it is observed to fall with constant speed. What must be the value of constant speed?**

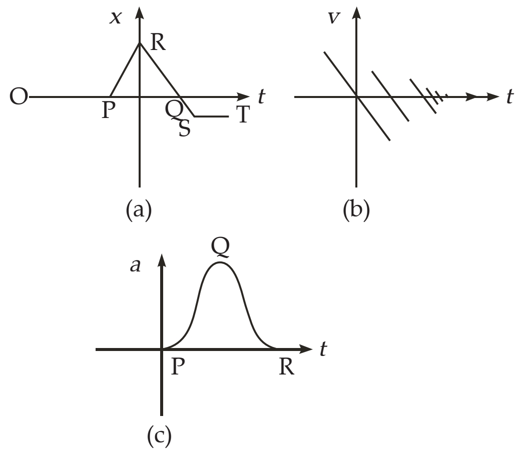
**[NCERT Exemp. Q. 3.16, Page 16]**

**Ans.** When $a = 0$, velocity of the particle becomes constant i.e., $v$ = constant = $v_C$

Then, from given equation $a = g - bv$,

$0 = g - bv_C \Rightarrow v_C = g/b$.

**Q. 4. Suggest a suitable physical situation for each of the following graphs :**




**[NCERT Ex. Q. 3.19, Page 58]**

**Ans.** **(a)** A ball at rest on a smooth floor is kicked, OP, shows it at rest position, at R the ball rebounds from the wall. RQ represents the bounced ball with reduced speed because the slope of RQ is less than that of PR. (The slope of *x-t* graph gives the speed of the moving body), ST shows the rest position of the ball.

**(b)** The graph shows the case of a ball thrown up with some initial velocity and rebounding from the floor with reduced speed after each hit every time on the floor, *i.e.*, after every bounce.

**(c)** It is the acceleration-time graph. The graph represents a uniformly moving cricket ball turned back after hitting the bat for a very short time interval (represented by the portion PQR of the graph).

## Short Answer Type Questions

**(2 or 3 marks each)**

**Q. 1. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed – time graph of its motion between $t$=0 to 12 s. (g=10ms$^{-2}$)**

**[NCERT Ex. Q. 3.12, Page 57]**

**Ans.** Given, $h = 90$ m, $u = 0$, $a = 10$ m/s$^2$,

$s = h = 90$ m

$$S = ut + \frac{1}{2}at^2$$

$$t_1 = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 90}{10}} = \sqrt{18} = 4.24 \text{ s.}$$

$$u_1 = \sqrt{2as} = \sqrt{2 \times 10 \times 90}$$

$$= 30\sqrt{2} \text{ m/s} = 42.43 \text{ m/s.}$$

In each collision, ball loses one tenth of its speed,

$$\therefore u_2 = \frac{9}{10}u_1 = \frac{9}{10} \times 30\sqrt{2} = 27\sqrt{2}$$

$$= 38.2 \text{ m/s}$$

Time taken to reach highest point

$$t_2 = \frac{u_2}{a} = \frac{27\sqrt{2}}{10}$$

$$= 3.82 \text{ s.}$$

Now,

$$u_3 = \frac{9}{10}u_2$$

$$= \frac{9}{10} \times 27\sqrt{2} = 24.3\sqrt{2}$$

$$= 34.37 \text{ m/s.}$$

$$\text{Total time } = t_1 + t_2$$

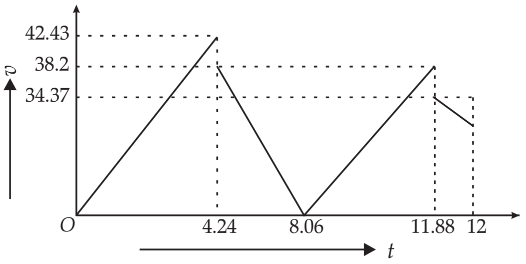
$$= 4.24 + 3.82$$

$$= 8.06 \text{ s}$$

Now, total time before upward motion of ball

$$= 8.06 + 3.82$$

$$= 11.88 \text{ s}$$




**Q. 2. A particle executes the motion described by $x(t) = x_0 (1 - e^{-\gamma t})$; $t \geq 0$, $x_0 > 0$.**

**(a) Where does the particles start and with what velocity?**

**(b) Find maximum and minimum values of $x(t)$, $v(t)$, $a(t)$. Show that $x(t)$ and $a(t)$ increase with time and $v(t)$ decreases with time.**

**[NCERT Exemp. Q. 3.18, Page 17]**

**Ans.** As $x(t) = x_0(1 - e^{-\gamma t})$

For velocity,

$$v(t) = \frac{dx(t)}{dt} = +x_0 \gamma e^{-\gamma t}$$

For acceleration,

$$a(t) = \frac{dv(t)}{dt} = -x_0 \gamma^2 e^{-\gamma t}$$

**(a)** When $t = 0; x(t) = x_0(1 - e^{-0}) = x_0(1-1) = 0$

$$v(t=0) = x_0 \gamma e^{-0} = x_0 \gamma(1) = \gamma x_0$$

**(b)** (i) $x(t)$ is maximum, when-

$t = \infty$, $x(t) = x_0$

$x(t)$ is minimum, when $t = 0$, $x(t) = 0$

(ii) $v(t)$ is maximum, when $t = 0$, $v(0) = x_0\gamma$

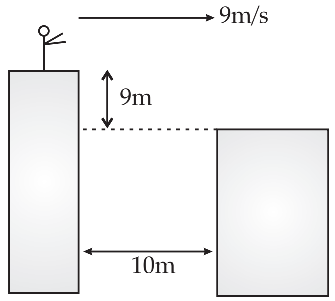
$v(t)$ is minimum, when $t = \infty$, $v(\infty) = 0$

(iii) $a(t)$ is maximum, when $t = \infty$, $a(\infty) = 0$

$a(t)$ is minimum, when $t = 0$, $a(0) = -x_0\gamma$

**Q. 3. A man runs across the roof-top of a tall building and jumps horizontally with the hope of landing on the roof of the next building which is of a lower height than the first. If his speed is 9 m/s, the (horizontal) distance between the two buildings is 10 m and the height difference is 9 m, will he be able to land on the next building? (take $g$ = 10 m/s$^2$). [NCERT Exemp. Q. 3.20, Page 17]**

**Ans.** Horizontal speed of man, $v_y$ = 9 m/s




Horizontal distance between two buildings, $x$ = 10 m

Height difference between heights of two buildings, $h$ = 9 m

$g$ = 10 m/s$^2$

Suppose $t$ is the time taken by the man to fall vertically downward by a height, $h$

$$h = v_y t + \frac{1}{2} g t^2$$

$$a = 0 \times t + \frac{1}{2} \times 10 \times t^2 \Rightarrow 5t^2 = 9$$

$$t = \sqrt{\frac{9}{5}} = 1.346 \text{ s}$$

If the distance covered by the man along $x$-axis during this time t is $x'$, then

$$x' = v_x t = 9 \times 1.34 = 12.06 \text{ m}$$

Here, $x' > x$

So, man will land successfully from building A to building B.

**Q. 4. A ball is dropped from a building of height 45 m. Simultaneously another ball is thrown up with a speed 40 m/s. Calculate the relative speed of the balls as a function of time.**

**[NCERT Exemp. Q. 3.21, Page 17]**

**Ans.** In motion under gravity, if the ball is released or dropped, that means its initial velocity is zero. Both balls are freely falling. Hence, there is no acceleration of one w.r.t. another.

Therefore, relative speed remains constant (= 40 m/s).

**OR**

Velocity of ball dropped after time $t$, $v_d = gt$ (downwards)

Velocity of ball thrown after time $t$, $v_t = 40 - gt$ (upwards)

$\therefore$ Relative speed of balls = $v_{dt} = v_{td} = v_d + v_t$

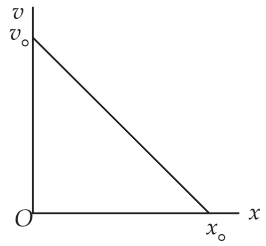
$$= gt + (40 - gt)$$

$$= 40 \text{ m/s.}$$

**Q. 5. The velocity-displacement graph of a particle is shown in Figure.**

**(a) Write the relation between $v$ and $x$.**

**(b) Obtain the relation between acceleration and displacement and plot it.**




**[NCERT Exemp. Q. 3.22, Page 17]**

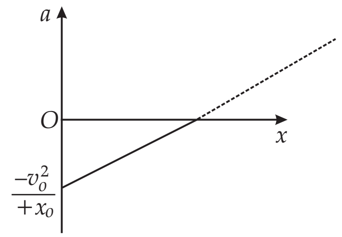
**Ans.** Initial Velocity = $v_0$

Let the distance travelled in time $t = x_0$

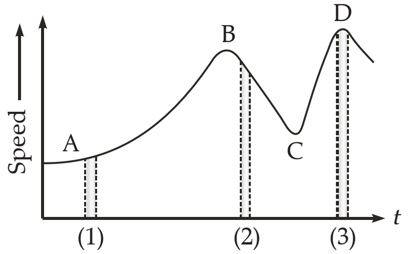
For graph, $\tan\theta = \frac{v_0}{x_0} = \frac{v_0 - v}{x}$

$v = (-v_0/x_0)\ x + v_0$, $a = (v_0/x_0)^2\ x - v_0^2/x_0$

The variation of a with $x$ is shown in the figure. It is a straight line with a positive slope and a negative intercept.




**Q. 6. Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude ? In which interval is the average speed greatest ? Choosing the positive direction as the constant direction of motions give the signs of $v$ and $a$ in the three intervals. What are the accelerations at the points A, B, C and D ?**




**[NCERT Ex. Q. 3.22, Page 59]**

**Ans.** **(i)** The magnitude of the average acceleration is given by $a_{av} = \frac{\text{Change in speed}}{\text{Time interval}}$

*i.e.*, average acceleration in a small interval of time is equal to the slope of $v$-$t$ graph in that time interval.

As the slope of $v$-$t$ graph is maximum in the interval 2 as compared to intervals 1 and 3, hence the magnitude of average acceleration is greatest in interval 2.

**(ii)** The average speed is greatest in the interval 3 as peak D is at maximum on speed axis.

**(iii)** $v > 0$, *i.e.*, positive in all the three intervals.

**(iv)** The slope is positive in intervals 1 and 3, so '$a$', *i.e.* acceleration is positive in these intervals while the slope is negative in interval 2, so acceleration is negative in it.

*i.e.*, $a > 0$, *i.e.*, positive in interval 1 and 3.

and, $a < 0$, *i.e.*, negative in interval 2.

**(v)** As speed is constant at points A, B, C and D, hence the acceleration is zero at all the four points.

**Q. 7. A three-wheeler starts from rest, accelerates uniformly with 1 ms$^{-2}$ on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the nth second (n = 1,2,3....) versus $n$. What do you expect this plot to be during accelerated motion : a straight line or a parabola ?**

**[NCERT Ad. Ex. Q. 3.23, Page 59]**

**Ans.** Straight line.

Distance covered by a body in $n$th second is given by the relation

$$D_n = u + \frac{a}{2}(2n - 1) \qquad ...(i)$$

Where,

$u$ = Initial velocity = 0

$a$ = Acceleration

$n$ = Time = 1, 2, 3, ...., $n$

In the given case,

$u = 0$ and $a = 1$ m/s$^2$

$$\therefore\ D_n = \frac{1}{2}(2n - 1) \qquad ...(ii)$$

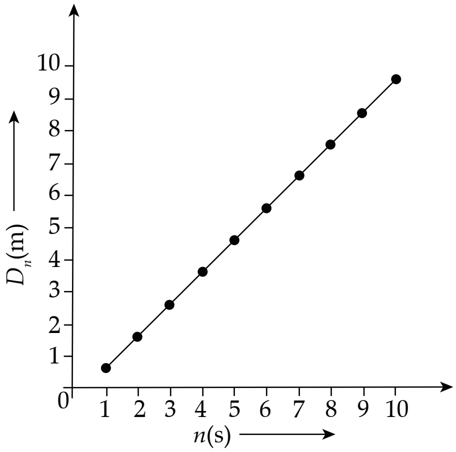
This relation shows that:

$$D_n \propto n \qquad ...(iii)$$

Now, substituting different values of n in equation $D_n = u + \frac{a}{2}(2n-1)$, we get the following table:

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $D_n$ | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 |

The plot between $n$ and $D_n$ will be a straight line shown in below figure:




**Q. 8. A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 ms$^{-1}$. How much time does the ball take to return to his hands ? If the lift starts moving up with a uniform speed of 5 ms$^{-1}$ and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?**

**[NCERT Ad. Ex. Q. 3.24, Page 59]**

**Ans.** **Case I :** If the lift is stationary :

Suppose $t$ = total time taken by ball in going vertically upward and coming down to the hands of the boy.

**Given :** $u = 49\ ms^{-1}$, $a = -9.8\ ms^{-2}$, $s$ = total displacement = 0

$\therefore$ Using the relation, $s = ut + \frac{1}{2}at^2$,

$$\Rightarrow \quad 0 = 49t + \frac{1}{2}(-9.8) \times t^2$$

$$\Rightarrow \quad 49t = 4.9\ t^2$$

$$\Rightarrow \quad t = \frac{49}{4.9} = 10\ s.$$

**Alter :** Suppose $t_1$ = Time taken by the ball to reach the maximum height = time of ascent

Again $u = 49\ ms^{-1}$, $v = 0$ at the highest point.

$$a = -9.8\ ms^{-2}$$

$\therefore$ From equation, $v = u + at$,

$$0 = 49 - 9.8\ t$$

or $$t = \frac{49}{9.8} = 5\ sec$$

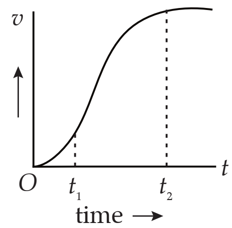
Also, time of descent = time of ascent

$\therefore$ Total time of flight = $2t = 2 \times 5 = 10$ s.

**Case II.** If the lift starts moving with uniform speed :

Since, the lift starts moving upwards with uniform speed of 5 $ms^{-1}$, there is no change in the relative velocity of the ball w.r.t. the boy which remains 49 $ms^{-1}$ due to the fact that there is no acceleration in the lift. Therefore, the ball will naturally return back to the boy's hand after 10 sec.

**Q. 9. The velocity–time graph of a particle in one-dimensional motion is shown in fig.**




**Which of the following formulae are correct for describing the motion of the particle over the time-interval $t_1$ to $t_2$:**

**(a)** $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + \left(\frac{1}{2}\right)a(t_2 - t_1)^2$

**(b)** $v(t_2) = v(t_1) + a(t_2 - t_1)$

**(c)** $v_{average} = \frac{[x(t_2) - x(t_1)]}{(t_2 - t_1)}$

**(d)** $a_{average} = \frac{[v(t_2) - v(t_1)]}{(t_2 - t_1)}$

**(e)** $x(t_2) = x(t_1) + v_{average}(t_2 - t_1) + \left(\frac{1}{2}\right)a_{average}(t_2 - t_1)^2$

**(f)** $x(t_2) - x(t_1)$ **= area under the** $v$-$t$ **curve bounded by the** $t$**-axis and the dotted line shown.**

**[NCERT Ad. Ex. Q. 3.28, Page 60]**

**Ans.** Slope of given graph over time interval $t_1$ to $t_2$ is not constant and not uniform

Therefore, acceleration is not constant.

So, (a), (b), (e) are not correct and (c), (d), (f) are correct.

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. A car moving along a straight highway with speed of 126 kmh$^{-1}$ is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop ?**

**[NCERT Ex. Q. 3.6, Page 56]**

**Ans.** Initial velocity of car,

$$u = 126\ kmh^{-1}$$

$$= 126 \times \frac{5}{18}\ ms^{-1}$$

$$= 35\ ms^{-1} \quad ...(i)$$

Since, the car finally comes to rest, $v = 0$

Distance covered, $s = 200$ m, $a = ?$, $t = ?$

From equation of motion

$$v^2 = u^2 - 2as$$

or, $$a = \frac{v^2 - u^2}{2s} \quad ...(ii)$$

Substituting the values from eq. (i) in eq. (ii)

$$a = \frac{0 - (35)^2}{2 \times 200} = -\frac{35 \times 35}{400}$$

$$= -\frac{49}{16}\ ms^{-2}$$

$$= -3.06\ ms^{-2}$$

Negative sign shows that acceleration is negative which is called retardation, *i.e.*, car is uniformly retarded at $-a = 3.06\ ms^{-2}$.

To find $t$, let us use the relation

$$v = u + at$$

$$\therefore \quad t = \frac{v - u}{a}$$

Here, $a = -3.06\ ms^{-2}$, $v = 0$, $u = 35\ ms^{-1}$.

$$\therefore \quad t = \frac{v-u}{a} = \frac{0-35}{-3.06} = 11.44\ s$$

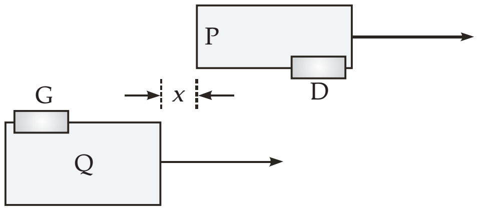
$$\therefore \quad t = 11.44\ sec.$$

**Q. 2. Two trains P and Q of length 400 m each are moving on two parallel tracks with a uniform speed of 72 kmh$^{-1}$ in the same direction, with P**

**ahead of Q. The driver of Q decides to overtake P and accelerates by 1 ms$^{-2}$. If after 50 s, the guard of Q just brushes past the driver of P, what was the original distance between them ?**

**[NCERT Ex. Q. 3.7, Page 56]**

**Ans.** Originally both the trains P and Q have the same velocities,




*i.e.*, if $u_P$ and $u_Q$ be the initial speeds of the two trains, then

$$u_P = u_Q = 72 \text{ kmh}^{-1}$$
$$= 72 \times \frac{5}{18} = 20 \text{ ms}^{-1}.$$

Let $x$ = original distance between the two trains :
Now, for train Q, $a = 1$ ms$^{-2}$, $t = 50$ s.
Let $s_Q$ be the distance travelled by train Q in 50 s, then,
Using the relation

$$s = ut + \frac{1}{2}at^2,$$

we get, $$s_Q = u_Q t + \frac{1}{2}at^2$$
$$= 20 \times 50 + \frac{1}{2} \times 1 \times (50)^2$$
$$= 1000 + 1250 \text{ m}$$
$$= 2250 \text{ m}.$$

Also, let $s_P$ be the distance covered by the train P during 50 s, then from the relation,

$$x = ut, \text{ we get } s_P = u_A \times t$$

or $s_P = 20 \times 50 = 1000$ m.

Original distance between the two trains

$$= s_Q - s_P - \text{length of two train}$$
$$s = 2250 - 1000 - 800$$
$$s = 450 \text{ m}.$$

**Q. 3. On a two lane road, car P is travelling with a speed of 36 kmh$^{-1}$. Two cars Q and R approach car P in opposite directions with a speed of 54 kmh$^{-1}$ each. At a certain instant, when the distance PQ is equal to PR both being 1 km, Q decides to overtake P before R does. What minimum acceleration of car Q is required to avoid an accident ?**

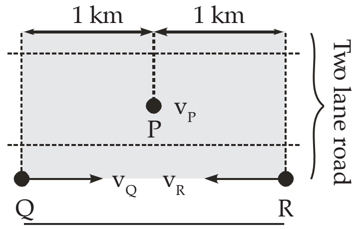
**[NCERT Ex. Q. 3.8, Page 56]**

**Ans.** Speed of car P,

$$v_P = 36 \text{ kmh}^{-1}$$
$$= 36 \times \frac{5}{18} = 10 \text{ ms}^{-1}$$

Let $v_Q$ and $v_R$ be the speeds of cars Q and R

$\therefore$ $$v_Q = v_R = 54 \text{ kmh}^{-1}$$
$$= 54 \times \frac{5}{18}$$
$$= 15 \text{ ms}^{-1} \text{ (given)}$$




$\therefore$ Relative speed of car Q w.r.t. car P, *i.e.*, $v_{QP}$ is given by

$$v_{PQ} = v_Q - v_P = 15 - 10$$
$$= 5 \text{ ms}^{-1}$$

Also, Relative speed of car R w.r.t. car P, *i.e.*, $V_{RP}$ is given by

$$v_{RP} = v_R - (-v_P) = v_R + v_P$$
$$= 15 + 10 = 25 \text{ ms}^{-1}$$

Also, $$PQ = PR = 1 \text{ km (given)}$$
$$= 1000 \text{ m}$$

Let $t$ = time taken by car PR to travel distance PR

$\therefore$ Using relation

$$s = ut$$

($\therefore$ car R is in uniform motion)

We get, $$t = \frac{s}{u} = \frac{PR}{v_{RP}} = \frac{1000}{25} = 40 \text{ s}.$$

Suppose $a$ = acceleration of car Q for $t = 40$ s; it will cover 1000 m in 40 s.

$\therefore$ Using the relation,

$$s = ut + \frac{1}{2}at^2,$$

we get $$PQ = u_{QP}\,t + \frac{1}{2}at^2$$

or $$1000 = 5 \times 40 + \frac{1}{2}a \times (40)^2$$
$$= 200 + a \times \frac{1600}{2}$$

or $$800a = 800$$

$\therefore$ $$a = 1 \text{ ms}^{-2}.$$

**Q. 4. A player throws a ball upwards with an initial speed of 29.4 ms$^{-1}$.**

**(a) What is the direction of acceleration during the upward motion of the ball ?**

**(b) What are the velocity and acceleration of the ball at the highest point of its motion ?**

**(c) Choose the $x = 0$ m and $t = 0$ s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of $x$-axis, and give the signs of position, velocity**

**and acceleration of the ball during its upward and downward motion.**

**(d) To what height does the ball rise and after how long does the ball return to the player's hands ? (Take $g = 9.8\ ms^{-2}$ and neglect air resistance.)**

**[NCERT Ex. Q. 3.10, Page 58]**

**Ans. (a)** As the ball is moving under the effect of gravity, the direction of acceleration due to gravity remains vertically downwards.

**(b)** If the ball is at the highest point of its motion, its velocity becomes zero and the acceleration is equal to the acceleration due to gravity = $9.8\ ms^{-2}$ in vertically downward direction.

**(c)** If the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of $x$-axis.

**For upward motion,** sign of position is negative, sign of velocity is negative and the sign of acceleration is positive, *i.e.*, $v < 0, a > 0$.

**For downward motion,** sign of position is positive, sign of velocity is positive and the sign of acceleration is also positive, *i.e.*, $v > 0, a > 0$.

**(d)** Suppose, $t$ = time taken by the ball to reach the highest point.

H = height of the highest point from the ground.

During vertically upward motion of the ball,

$\therefore$ Initial velocity,

$$u = -29.4\ ms^{-2},$$

$$a = g = 9.8\ ms^{-2},$$

Final velocity $v = 0, s = H = ?, t = ?$

Applying the relation,

$$v^2 - u^2 = 2as$$

$$0^2 - (29.4)^2 = 2 \times 9.8\ H$$

$$\text{or } H = -\frac{29.4 \times 29.4}{2 \times 9.8} = -44.1\ m$$

where negative sign indicates that the distance is covered in upward direction.

Using relation, $v = u + at$

$$0 = -29.4 + 9.8 \times t$$

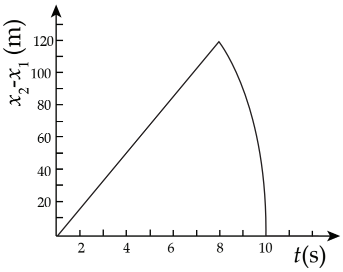
$$\therefore \quad t = \frac{29.4}{9.8} = 3\ s.$$

*i.e.*, time of ascent = 3 s.

It is also well known that when the object moves under the effect of gravity alone, the time of ascent is always equal to the time of descent.

$\therefore$ Total time after which the ball returns to the player's hand = $2t = 2 \times 3 = 6$ s.

**Q. 5. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m/s and 30 m/s. Verify that the graph shown in figure correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take $g = 10\ ms^{-2}$. Give the equations for the linear and curved parts of the plot.**




**[NCERT Ad. Ex. Q. 3.26, Page 59]**

**Ans. For first stone:**

Initial velocity, $u_i = 15$ m/s

Acceleration, $a = -g = -10\ m/s^2$

Using the relation,

$$x_1 = x_0 + u_1 t + \frac{1}{2} at^2$$

Where, height of the cliff, $x_0 = 200$ m

$$x_1 = 200 + 15t - 5t^2 \quad \ldots(i)$$

When this stone hits the ground, $x_1 = 0$

$\therefore -5t^2 + 15t + 200 = 0$

$t^2 - 3t - 40 = 0$

$t^2 - 8t + 5t - 40 = 0$

$t(t-8) + 5(t-8) = 0$

$t = 8$ s or $t = -5$ s

Here, t = –5 s is rejected as time is never negative

$\therefore t = 8$ s

**For second stone:**

Initial velocity, $u_2 = 30$ m/s

Acceleration, $a = -g = -10\ m/s^2$

Using the relation,

$$x_2 = x_0 + u_2 t + \frac{1}{2} at^2$$

$$= 200 + 30\,t - 5t^2 \quad \ldots(ii)$$

At the moment when this stone hits the ground; $x_2 = 0$

$-5t^2 + 30\,t + 200 = 0$

$t^2 - 6\,t - 40 = 0$

$t\,(t - 10) + 4\,(t - 10) = 0$

$(t - 10)\,(t + 4) = 0$

$t = 10$ s or $t = -4$ s

Here, t = –4 s is rejected as time is never negative

$\therefore\ t = 10s$

Subtracting equations (i) and (ii), we get

$x_2 - x_1 = (200 + 30t - 5t^2) - (200 + 15t - 5t^2)$

$x_2 - x_1 = 15t \quad \ldots(iii)$

Equation (iii) represents the linear path of both stones. Due to this linear relation between $(x_2 - x_1)$ and $t$, the path remains a straight line till 8 s.

Maximum separation between the two stones is at $t = 8$ s.

$(x_2 - x_1)_{max} = 15 \times 8 = 120$ m

This is in accordance with the given graph.

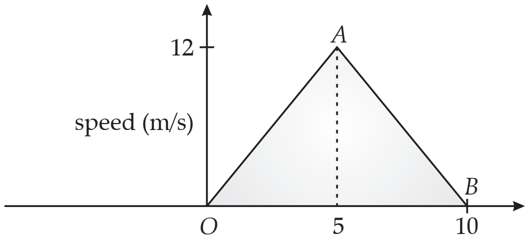
After 8 s, only second stone is in motion whose variation with time is given by the quadratic equation:

$x_2 - x_1 = 200 + 30t - 5t^2$

Hence, the equation of linear and curved path is given by

$x_2 - x_1 = 15t$ **(Linear path)**
$x_2 - x_1 = 200 + 30t - 5t^2$ **(Curved path)**

**Q. 6. The speed–time graph of a particle moving along a fixed direction is shown in fig. Obtain the distance traversed by the particle between (a) $t$=0 s to 10 s, (b) $t$=2 s to 6 s.**




**What is the average speed of the particle over the intervals in (a) and (b)?**

**[NCERT Ad. Ex. Q. 3.27, Page 60]**

**Ans.** (a) Distance travelled between $t$ = 0 s to 10 s

$$= \text{Area of triangle AOB}$$
$$= \frac{1}{2} \times (10 - 0) \times (12 - 0)$$
$$= \frac{1}{2} \times 10 \times 12 = 60 \text{ m}$$

$$\text{Average speed } = \frac{\text{Total distance}}{\text{Total time}} = \frac{60}{10} = 6 \text{ m/s}$$

(b) For, $t = 0$, $u = 0$, $t = 5$ s, $v = 12$ m/s at OA

$$\therefore v = u + at$$
$$12 = 0 + a \times 5 \Rightarrow a = \frac{12}{5} = 2.4 \text{ m/s}^2$$

Speed of particle at end of $t = 2$ s

$$v_1 = u + at$$
$$= 0 + 2.4 \times 2 = 4.8 \text{ m/s}$$

= Distance travelled by the particle between time 2s and 5s *ie*, in 3s.

Now, $s_1 = ut + \frac{1}{2}at^2$

$$= 4.8 \times 3 + \frac{1}{2} \times 2.4 \times (3)^2$$
$$s_1 = 25.2 \text{ m}$$

For part AB,
$t$= 5 s, $u$ = 12 m/s,
final speed, $t$ = 10 s, $v$ = 0

$$v = u + at$$
$$0 = 12 + a \times 5$$
$$a = -\frac{12}{5} = -2.4 \text{ m/s}^2$$

Distance travelled in 1 s

$$s_2 = ut + \frac{1}{2}at^2$$
$$= 12 \times 1 + \frac{1}{2}(-2.4) \times 1$$
$$= 12 - 1.2 = 10.8 \text{ m}$$

Total distance in $t$ = 2 s to $t$ = 6 s

$$s = s_1 + s_2$$
$$= (25.2 + 10.8) \text{ m} = 36.0 \text{ m}$$

$$\text{Average speed } = \frac{36 \text{ m}}{4 \text{ s}} = 9 \text{ m/s}.$$

**Q. 7. A motor car moving at a speed of 72 km/h can not come to a stop in less than 3.0 s while for a truck this time interval is 5.0 s. On a highway the car is behind the truck both moving at 72 km/h. The truck gives a signal that it is going to stop at emergency. At what distance the car should be from the truck so that it does not bump into (collide with) the truck. Human response time is 0.5 s.**

**(Comment : This is to illustrate why vehicles carry the message on the rear side "Keep safe Distance")**

**[NCERT Exemp. Q. 3.24, Page 18]**

**Ans.** Speed of car as well as truck=72 km/h

$$= 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$$

Time required to stop the truck = 5 s.
Car behind the truck

$$\text{Retardation of truck } = \frac{20}{5} = 4 \text{ ms}^{-2}$$

$$\text{Retardation of car } = \frac{20}{3} \text{ms}^{-2}$$

Let the truck be at a distance $x$ from the car when brakes are applied.
Distance of truck from A at $t > 0.5$ s is $x + 20t - 2t^2$.
Distance of car from A is

$$10 + 20(t - 0.5) - \frac{10}{3}(t - 0.5)^2.$$

If they meet

$$x + 20t - 2t^2 = 10 + 20t - 10 - \frac{10}{3}t^2 + \frac{10}{3}t - 0.25 \times \frac{10}{3}.$$

$$x = -\frac{4}{3}t^2 + \frac{10}{3}t - \frac{5}{6}.$$

To find $x_{min}$,

$$\frac{dx}{dt} = -\frac{8}{3}t + \frac{10}{3} = 0$$

which gives $t_{min} = \frac{10}{8} = \frac{5}{4}$s.

Therefore, $x_{min} = -\frac{4}{3}\left(\frac{5}{4}\right)^2 + \frac{10}{3} \times \frac{5}{4} - \frac{5}{6} = \frac{5}{4}.$

Therefore, $x > 1.25$ m.

**Second method :** This method does not require the use of calculus.
If the car is behind the truck,
$v_{car} = 20 - (20/3)(t - 0.5)$ for $t > 0.5$ s as car decelerate only after 0.5 s.
$v_{truck} = 20 - 4t$
Find ***t*** from equating the two or from velocity vs time graph. This yields $t = 5/4$ s.

In this time truck would travel truck,
$S_{truck} = 20(5/4) - (1/2)(4)(5/4)^2 = 21.875$ m
and car would travel.

$S_{car} = 20(0.5) + 20\ \frac{5}{4} - 0.5$

$-\frac{1}{2}\ (20/3) \times \frac{5}{4} - 0.52 = 23.125$ m

Thus $S_{car} - S_{truck} = 1.25$ m.
If the car maintains this distance initially, its speed after 1.25 s will be always less than that of truck and hence collision never occurs.

**Q. 8. A monkey climbs up a slippery pole for 3 seconds and subsequently slips for 3 seconds. Its velocity at time $t$ is given by $v(t) = 2t(3 - t)$; $0 < t < 3$ and $v(t) = -(t - 3)(6 - t)$ for $3 < t < 6$ s in m/s. It repeats this cycle till it reaches the height of 20 m.**

**(a) At what time is its velocity maximum?**

**(b) At what time is its average velocity maximum?**

**(c) At what time is its acceleration maximum in magnitude?**

**(d) How many cycles (counting fractions) are required to reach the top?**

**[NCERT Exemp. Q. 3.25, Page 18]**

**Ans.** Given, $v(t) = 2t(3-t) = 6t - 2t^2$

**(a)** For maximum velocity,

$$\frac{dv(t)}{dt} = 0$$

$$\Rightarrow \frac{d}{dt}(6t - 2t^2) = 0$$

$$\Rightarrow 6 - 4t = 0 \Rightarrow t = \frac{6}{4} = \frac{3}{2}s = 1.5s$$

**(b)** $\frac{ds}{dt} = 6t - 2t^2$ [s = displacement]

$ds = (6t - 2t^2)dt$

$\therefore$ Distance travelled in time interval 0 to 3s.

$$s = \int_0^3 (6t - 2t^2)dt$$

$$= \left[\frac{6t^2}{2} - \frac{2t^3}{3}\right]_0^3 = \left[3t^2 - \frac{2}{3}t^3\right]_0^3$$

$$= 3\times 9 - \frac{2}{3}\times 3\times 3\times 3$$

$$= 27 - 18 = 9 \text{ m.}$$

Average velocity $= \frac{9}{3} = 3$ m/s.

As,

$$v_{avg} = 6t - 2t^2 \Rightarrow 3 = 6t - 2t^2 \Rightarrow 2t^2 - 6t - 3 = 0$$

$$\Rightarrow t = \frac{6 \pm \sqrt{6^2 - 4\times 2\times 3}}{2\times 2} = \frac{6 \pm \sqrt{36 - 24}}{4}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Taking '+'ve sign we get $t_1 = \frac{3 - \sqrt{3}}{2} = 2.36$ sec.

Taking '–'ve sign we get $t_2 = \frac{3 - \sqrt{3}}{2} = 0.633$ sec

which is less than the least of the clock so it cannot be measured,

$\therefore$ rejecting $t_2$, the average velocity is maximum at 2.36 sec.

**(c)** Acceleration will be maximum when velocity is zero

$\therefore (6t - 2t^2) = 0$

$t(6 - 2t) = 0$

$t = 0, 3$ sec

**(d)**

$$s_1 = \int_0^3 (6t - 2t^2)dt$$

$$= \frac{6t^2}{2} - \frac{2t^3}{3}\Big|_0^3$$

$$= 9\text{m}$$

$$s_2 = \int_3^6 (-(t-3)(6-t))dt$$

$$= \int_3^6 -(6t - t^2 - 18 + 3t)dt$$

$$= \int_3^6 -(-t^2 + 9t - 18)dt$$

$$= \frac{t^3}{3} - \frac{9t^2}{2} + 18t\Big|_3^6$$

$$= -4.5\text{m}$$

Total distance travelled in one cycle is
$= 9 - 4.5$ m $= 4.5$ m

No. of cycles $= \frac{20}{4.5} = 4.44$ cycles $\approx 5$ cycles

**Q. 9. A man is standing on top of a building 100 m high. He throws two balls vertically, one at $t = 0$ and other after a time interval (less than 2 seconds). The later ball is thrown at a velocity of half the first. The vertical gap between first and second balls is +15 m at $t = 2$s. The gap is found to remain constant. Calculate the velocity with which the balls were thrown and the exact time interval between their throw.**

**[NCERT Exemp. Q. 2.26, Page 18]**

**Ans.** Let the speeds of two balls (1 & 2) be $v_1$ and $v_2$, if $v_1 = 2v$, $v_2 = v$

If $y_1$ and $y_2$ the displacement covered by the balls 1 and 2, respectively, then

$$y_1 = \frac{v_1^2}{2g} = \frac{4v^2}{2g} \text{ and } y_2 = \frac{v_2^2}{2g} = \frac{v^2}{2g}$$

since, $y_1 - y_2 = 15$ m,

$$\frac{4v^2}{2g} - \frac{v^2}{2g} = 15 \text{ m}$$

or $$\frac{3v^2}{2g} = 15 \text{ m}$$

$$v^2 = \sqrt{5\text{ m} \times (2 \times 10)}\text{ m/s}^2$$

$$v = 10\text{ m/s}$$

clearly, $v_1$ = 20 m/s, $v_2$ = 10 m/s.
Time interval = 1s.

**TIPS... & TRICKS...**

- *Understand Distance and displacement and difference between them.*
- *Understand speed and velocity and difference between them.*
- *Calculate speed as scalar method and velocity as vector method.*
- *Understand Acceleration and Deacceleration (retardation)*
- *Learn formulas of equation of motion and also for gravitational equation of motion.*
- *As acceleration is a vector quantity in calculation of acceleration its follows vectors law.*

## Some Commonly Made Errors

- Students ignore to learn the initial concept of solid state chemistry.
- Students get confused in determining the importance of negative and positive value of acceleration.
- Generally, students did not use the proper equation for acceleration and deacceleration.
- Students got confused between distance and displacement relation.

### EXPERT ADVICE

- *Always remember acceleration is a vector quantity so, negative value of acceleration states the retardation in acceleration.*
- *Graphical representation and interpretation of accelerated motion are very important.*
- *Thoroughly revise and practice the numerical questions as they are often asked in different forms.*

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CHAPTER

# 4 MOTION IN A PLANE

## Chapter Objective

**This chapter will help you understand :**

- ***Scalar and vector quantities :*** *Position and displacement vectors; general vectors and their notations; equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors; relative velocity.*
- ***Unit vector :*** *Resolution of a vector in a plane; rectangular components; scalar and vector products of vectors;*
- ***Motion in a plane :*** *Cases of uniform velocity and uniform acceleration; projectile motion; uniform circular motion.*

## TOPIC-1
## Scalar and Vector quantities



### Quick Review

- **Scalar :** A physical quantity which has only magnitude and no direction is called a scalar quantity or scalar.
- **Vector :** A physical quantity which constitutes of magnitude as well as direction is called a vector quantity or vector.
- **Unit vector :**

  (i) A unit vector of $\vec{A}$ is written as $\hat{A}$ and is given by $\hat{A} = \vec{A}/|A|$

  (ii) A unit vector is dimensionless quantity of magnitude equal to unity. Its magnitude is 1 and it can have any direction.

  (iii) In cartesian co-ordinates, $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along $x$, $y$ and $z$-axes, respectively.

- **Polar vectors :** These are those vectors which have a linear directional effect. For example, force, linear momentum, linear velocity etc.
- **Axial vectors or rotational vectors :** These vectors represent rotational effect. They are always directed along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque, angular momentum etc. are few examples of axial vectors.
- **Some vector laws :**

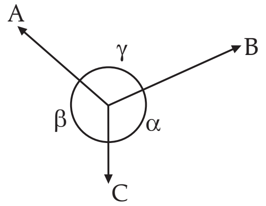
  **(1) General law for addition of vector :** It states that the vectors to be added are arranged in such a way so that the head of first vector coincides with the tail of second vector, whose head coincides with tail of third vector and so on.

  **(2) Triangle Law :** It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of triangle taken in opposite order.

  **(3) Parallelogram Law :** It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.

➢ **Lami's Theorem :** It states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces, *i.e.*

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$




## Know the Terms

➢ **Modulus of vector** is the magnitude of vector.
➢ **Equal vectors** are those vectors which have equal magnitude and same direction.
➢ **Negative vector** of a given vector is a vector of same magnitude but acting in a direction opposite to that of given vector.
➢ **Co-initial vectors** are those vectors which have common initial point.
➢ **Collinear vectors** are those vectors which are having equal or unequal magnitudes and are acting along parallel straight lines.
➢ **Coplanar vectors** are those vectors which are acting in the same plane.
➢ **Localised vector** is that vector whose initial point is fixed.
➢ **Non-Localised vector** is that vector whose initial point is not fixed.
➢ **Zero or Null vector** is that vector which has zero magnitude and an arbitrary direction and represented by $\vec{0}$.
➢ **Displacement vector** is that vector which tells how much and in which direction an object has changed its position in a given interval of time.
➢ **Resultant vector** is defined as that single vector which produces the same effect as is produced by two or more individual vectors together.
➢ **Equilibrant vector** is a single vector which balances two or more vectors acting on a body at the same time.

## Know the formulae

➢
$$\vec{R} = \vec{A} + \vec{B}$$
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

➢
$$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$
$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$
$$\tan\beta = \frac{B\sin(180-\theta)}{A + B\cos(180-\theta)} = \frac{B\sin\theta}{A - B\cos\theta}$$

➢ **Relative Velocity—**
$$\vec{V}_{BA} = \vec{V}_A - \vec{V}_B$$

➢
$$\vec{A} = A_x\hat{i} + A_y\hat{j} \text{ and } A_x = A\cos\theta,\ A_y = A\sin\theta$$

➢
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k},\ \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$
$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2},\ |B| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

➢
$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

➢ **Unit Vector of $\hat{A}$ is**
$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

➢
$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

- If two vectors are parallel to each other i.e., $\theta = 0°$

$$\vec{A} \cdot \vec{B} = AB \cos 0 = AB$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

- If two vectors are perpendicular to each other i.e., $\theta = 90°$

$$\vec{A} \cdot \vec{B} = AB \cos 90° = 0$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = 0$$

- If two vectors are parallel to each other i.e., $\theta = 0$
- $$\vec{A} \times \vec{B} = AB \sin \theta \; \hat{n}$$

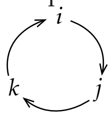
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

- If two vectors are perpendicular to each other i.e., $\theta = 90°$

$$\vec{A} \times \vec{B} = AB \sin 0° = 0$$

$$AB \sin 90° = AB$$

- Trick to remember Cross product




$\therefore$

$$\hat{i} \times \hat{j} = \hat{k},$$

$$\hat{j} \times \hat{k} = \hat{i},$$

$$\hat{k} \times \hat{i} = \hat{j}$$

and

$$\hat{i} \times \hat{k} = -\hat{j},$$

$$\hat{k} \times \hat{j} = -\hat{i},$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

- Area of triangle $= \frac{1}{2}|\vec{A} \times \vec{B}|$

- Area of parallelogram $= |\vec{A} \times \vec{B}|$

- Unit vector $\perp$ to A and B

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

where, $$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$

then $$\vec{A} \times \vec{B} = \vec{B} \times \vec{C} = \vec{C} \times \vec{A}$$

- $$\sin \theta = \frac{|\vec{A} \times \vec{B}|}{|A||B|}$$

- $$\cos \theta = \frac{\vec{A} \,.\, \vec{B}}{|A||B|}$$

- $$\tan \theta = \frac{\vec{A} \times \vec{B}}{\vec{A} \,.\, \vec{B}}$$

## Know the Links

- *www.vedantu.com*
- *www.learncbse.com*
- *www.byjus.com/ncert-solutions/*

## MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. Which one of the following statements is true ?**

**(a) A scalar quantity is the one that is conserved in a process**

**(b) A scalar quantity is the one that can never take negative values**

**(c) A scalar quantity is the one that does not vary from one point to another in space.**

**(d) A scalar quantity has the same value for observers with different orientations of the axes.**

**[NCERT Ex. Q. 4.2, Page 19]**

**Ans. Correct option:** (d)

*Explanation:* According to properties of scalar quantities the value of scalar quantity is same for all references of frames.

**Q. 2. The angle between A = $\hat{i} + \hat{j}$ and B = $\hat{i} - \hat{j}$ is**

**(a) 45** **(b) 90**

**(c) –90** **(d) 180**

**[NCERT Ex. Q. 4.1, Page 19]**

**Ans. Correct option:** (b)

*Explanation:* **Given :**

$$\vec{A} = \hat{i} + \hat{j},\ \vec{B} = \hat{i} - \hat{j}$$

$$\therefore \left|\vec{A}\right| = \sqrt{(1)^2 + (1)^2} = \sqrt{2},$$

$$\left|\vec{B}\right| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

$$\vec{A}.\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$$

$$\Rightarrow \cos\theta = \frac{\vec{A}.\vec{B}}{\left|\vec{A}\right|\left|\vec{B}\right|}$$

$$= \frac{\left(\hat{i} + \hat{j}\right)\left(\hat{i} - \hat{j}\right)}{\sqrt{2} \times \sqrt{2}} = 0$$

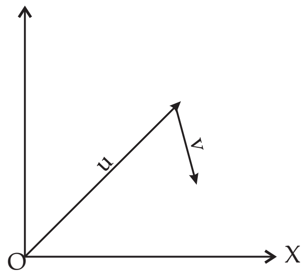
$$\cos\theta = \cos 90^\circ$$

$$\therefore \theta = 90^\circ$$

**Q. 3. Figure shows the orientation of two vectors u and v in the XY plane.**

**If $u = a\hat{i} + b\hat{j}$ and**

**$v = p\hat{i} + q\hat{j}$**




**Which of the following is correct?**

**(a) $a$ and $p$ are positive while $b$ and $q$ are negative.**

**(b) $a$, $p$ and $b$ are positive while $q$ is negative.**

**(c) $a$, $q$ and $b$ are positive while $p$ is negative.**

**(d) $a$, $b$, $p$ and $q$ are all positive.**

**[NCERT Ex. Q. 4.3, Page 19]**

**Ans. Correct option:** (b)

*Explanation:*

According to figure,in $\vec{u} = a\hat{i} + b\hat{j}$, both $a$ & $b$ are positive while in $\vec{v} = p\hat{i} + q\hat{j}$, $p$ is positive and $q$ is negative. Thus $a$, $b$ and $p$ are positive and $q$ is negative.

**Q. 4. Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are :**

**(a) Impulse, pressure and area**

**(b) Impulse and area**

**(c) Area and gravitational potential**

**(d) Impulse and pressure**

**[NCERT Ex. Q. 4.6, Page 20]**

**Ans. Correct option:** (b)

*Explanation:*

Impulse is a vector quantity. Area of a surface is a vector which is along normal to the surface in the outward direction.

**Q. 5. Three vectors A, B and C add up to zero. Find which is false.**

**(a) (A B) C is not zero unless B, C are parallel.**

**(b) (A B) C is not zero unless B, C are parallel.**

**(c) If A, B, C define a plane. (A B) C is in that plane.**

**(d) (A B).C = |A||B||C| → $C^2 = A^2 + B^2$.**

**[NCERT Ex. Q. 4.9, Page 21]**

**Ans. Correct option:** (c)

**Q. 6. It is found that |A+B|=|A|. This necessarily implies,**

**(a) B = 0.**

**(b) A, B are antiparallel.**

**(c) A, B are perpendicular.**

**(d) A•B ≤ 0**

**[NCERT Ex. Q. 4.10, Page 21]**

**Ans. Correct option:** (b)

*Explanation:*

If $|\vec{A}+\vec{B}| = |\vec{A}|$, then either $\vec{B} = 0$ or $\vec{B} = -2\vec{A}$ . Both are satisfied when $\vec{A}$ and $\vec{B}$ are anti-parallel.

**Q. 7. For two vectors A and B. |A+B|=|A–B| is always true when**

**(a) |A|=|B| ≠0**

**(b) A ⊥ B**

**(c) |A|=|B|≠0 and A and B are parallel or anti parallel**

**(d) When either |A| or |B| is zero.**

**[NCERT Ex. Q. 4.15, Page 22]**

**Ans. Correct option:** (b) and (d)

*Explanation:*

**(b)** $|A + B| = |A - B|$

From scalar product

$$(A + B)^2 = (A - B)^2$$

$$\Rightarrow \quad A^2 + B^2 + 2\vec{A}.\vec{B} = A^2 + B^2 - 2\vec{A}.\vec{B}$$

$$\Rightarrow \quad 4\vec{A}.\vec{B} = 0$$

$$\Rightarrow \quad \vec{A}.\vec{B} = 0 \Rightarrow \vec{A} \perp \vec{B}$$

**(d)** If $|A| = 0$ then $|B| = |-B| \Rightarrow \vec{B} = \vec{B}$

If $|B| = 0$ then $|A| = |A| \Rightarrow \vec{A} = \vec{A}$

## (B) True and False

**Q. 8. Which one of the following statements is true?**

**(a) A scalar quantity is the one that is conserved in a process.**

**(b) A scalar quantity is the one that can never take negative values.**

**(c) A scalar quantity must be dimensionless.**

**(d) A scalar quantity is the one that does not vary from one point to another is space.**

**(e) A scalar quantity has the same value for observes with different orientations of the axes.**

**[NCERT Ex. Q. 4.24, Page 87]**

**Ans.**

**(a)** False, because energy is not conserved during inelastic collision.

**(b)** False, because temperature can be negative.

**(c)** False, because total path length has the dimensions of length.

**(d)** False, because gravitational potential energy vary from point to point.

**(e)** True, as the R values of scalar does not change with orientation of axes.

**Q. 9. Read each statement below carefully and state with reasons, if it is true or false :**

**(a) The magnitude of a vector is always a scalar.**

**(b) Each component of a vector is always a scalar.**

**(c) The total path length is always equal to be magnitude of the displacement vector of a particle.**

**(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.**

**(e) Three vectors not lying in a plane can never add up to give a null vector.**

**[NCERT Ex. Q. 4.5, Page 85]**

**Ans. (a)** True, magnitude is pure number.

**(b)** False, each component is vector.

**(c)** True, only if particle moves along a straight line and in same direction.

Otherwise, this is false.

**(d)** True, because-

Total path length ≥ Magnitude of Displacement vector

**(e)** True, because these three vectors cannot represent three sides of triangle which taken in same order.

**Q. 10. Given $a + b + c + d = 0$ , which of the following statements are correct:**

**(a) a, b, c, and d must each be a null vector,**

**(b) The magnitude of (a + c) equals the magnitude of (b+ d),**

**(c) The magnitude of *a* can never be greater than the sum of the magnitudes of b, c, and d,**

**(d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear?**

**[NCERT Ex. Q. 4.7, Page 86]**

**Ans. (a)** Incorrect

In order to make, $\vec{a}+\vec{b}+\vec{c}+\vec{d}=\vec{0}$ it is not necessary to have all the four given vectors to be null vectors. There are many other combinations which can give the sum zero.

**(b)** Correct

$$\vec{a}+\vec{b}+\vec{c}+\vec{d}=\vec{0}$$

$$\vec{a}+\vec{c}=-(\vec{b}+\vec{d})$$

Taking modulus on both the sides, we get:

$$|\vec{a}+\vec{c}| = |-(\vec{b}+\vec{d})| = |\vec{b}+\vec{d}|$$

Hence, the magnitude of $(\vec{a}+\vec{c})$ is the same as the magnitude of $(\vec{b}+\vec{d})$

**(c)** Correct

$$\vec{a}+\vec{b}+\vec{c}+\vec{d}=\vec{0}$$

$$\vec{a}=-(\vec{b}+\vec{c}+\vec{d})$$

Taking modulus both sides, we get the magnitude of $\vec{a}$ to be equal to the magnitude of $(\vec{b}+\vec{c}+\vec{d})$ :

$$\vec{a}=|(\vec{b}+\vec{c}+\vec{d})|$$

$$|\vec{a}| \le |\vec{b}|+|\vec{c}|+|\vec{d}| \qquad \ldots(i)$$

Now, $(\vec{b}+\vec{c}+\vec{d})$ is the sum of vectors $\vec{b}$ , $\vec{c}$ and $\vec{d}$. Therefore, the magnitude of $(\vec{b}+\vec{c}+\vec{d})$ is less than, or equal to the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$. Hence, the magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$. Equation (*i*) shows that the magnitude of $\vec{a}$ is equal to or less than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$.

**(d)** Correct

For, $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$

$$\vec{a}+\left(\vec{b}+\vec{c}\right)+\vec{d}=0$$

The resultant sum of the three vectors $\vec{a}$, $\left(\vec{b}+\vec{c}\right)$, and $\vec{d}$ can be zero only if $\left(\vec{b}+\vec{c}\right)$ lie in the same plane as $\vec{a}$ and $\vec{d}$.

If $\vec{a}$ and $\vec{d}$ are collinear, then it implies that the vector $\left(\vec{b}+\vec{c}\right)$ is in the line of $\vec{a}$ and $\vec{d}$. This implication holds true in this scenario and the vector sum of all the vectors will be zero.

## Very Short Answer Type Questions (1 mark each)

**Q. 1. State, for each of the following physical quantities. If it is a scalar or a vector :**
**Volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity. [NCERT Ex. Q. 4.1, Page 85]**

**Ans.**

| Scalars | Vectors |
|---|---|
| volume | acceleration |
| mass | velocity |
| speed | displacement |
| density | angular velocity |
| number of moles | angular frequency |

**Q. 2. Pick out the two scalar quantities in the following list :**
**Force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.**
**[NCERT Ex. Q. 4.2, Page 85]**

**Ans.** Work, Current.

**Q. 3. Pick out the only vector quantity in the following list :**
**Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.**
**[NCERT Ex. Q. 4.2, Page 85]**

**Ans.** Impulse, as Impulse = Force × time × momentum

Here, force and momentum are vector quantities.

**Q. 4. A, B and C are three non–collinear, non co-planar vectors. What can you say about direction of $\vec{A}\times(\vec{B}\times\vec{C})$? [NCERT Ex. Q. 4.20, Page 23]**

**Ans.**

$$\vec{B}\times\vec{C}=BC\sin\theta\,\hat{n}\quad\left[\begin{array}{l}\hat{n}=\text{ unit vector}\\ \text{perpendicular to the}\\ \text{plane containing }\vec{B}\text{ and }\vec{C}\end{array}\right]$$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{A}\times\hat{n}(BC\sin\theta)$$

$$=(BC\sin\theta)A\sin\alpha\,\hat{P}\quad\left[\begin{array}{l}\hat{P}=\text{ unit vector}\\ \text{perpendicular to the}\\ \text{plane containing }\vec{A}\\ \text{ and}\left(\vec{B}\times\vec{C}\right)\end{array}\right]$$

Hence, $\vec{A}\times(\vec{B}\times\vec{C})$ will lie in the plane of $\vec{B}$ and $\vec{C}$, and is perpendicular to $\vec{A}$.

**Q. 5. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :**
**(a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.**
**[NCERT Ex. Q. 4.4, Page 85]**

**Ans. (a)** Yes, adding of two scalar quantities is meaningful only if they both represent the same physical quantity.

**(b)** No, adding a scalar to a vector of the same dimension is not meaningful as a scalar cannot be added to a vector.

**(c)** Yes, multiplying any vector by any scalar is meaningful in algebraic operation. It is because to when any vector is multiplied by any scalar, then we get a vector having magnitude equal to scalar number of times the magnitude of the given vector. For example, when acceleration $a$ is multiplied by mass $m$, we get force F = $ma$ which is a meaningful operation.

**(d)** Yes, it is clear product of two scalar gives a meaningful result. For example, when power P is multiplied by time $t$, then we get work done (W), *i.e.*, W = P$t$, which is a useful algebraic operation.

**(e)** yes, the addition of two vector quantities is meaningful only if they both represent the same physical quantity.

**(f)** yes, Addition of a component of a vector to the same vector can be done by the law of vector addition.

**Q. 6. Establish the following vector inequalities geometrically or otherwise:**

**(a) $|a + b| \le |a| + |b|$**

**(b) $|a - b| \ge ||a| - |b||$**

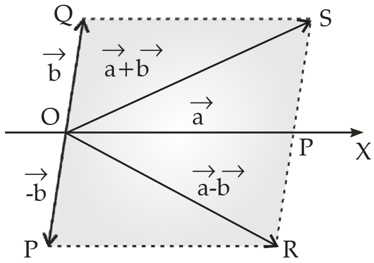
**(c)** $|\vec{a}-\vec{b}| \le |\vec{a}|+|\vec{b}|$

**(d)** $|\vec{a}-\vec{b}| \ge ||\vec{a}|-|\vec{b}||$

**When does the equality sign above apply?**

**[NCERT Ex. Q. 4.6, Page 85]**

**Ans. (a)** As-




Length of one side of triangle is always less than sum of lengths of other two sides.

$$|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}| \quad \text{(I)}$$

If the two vectors & are acting along same straight line and in same direction,

$$|\vec{a}+\vec{b}| = |\vec{a}|+|\vec{b}| \quad \text{(II)}$$

**(b)** From Δ OPS,

OS+PS>OP

OS > |OP – PS|, OS > |OP – OQ| (1)

Modulus of (OP−PS) has been taken, as LHS is positive but RHS may be negative :

If OP<PS, From eq.(III)

$$|\vec{a}+\vec{b}| > ||\vec{a}|-|\vec{b}|| \quad \text{(IV)}$$

If $\vec{a}$ and $\vec{b}$ are acting along straight line but in opposite direction,

$$|\vec{a}+\vec{b}| = ||\vec{a}|-|\vec{b}|| \quad \text{(III)}$$

from eq (II) &(III)

$$|\vec{a}+\vec{b}| \ge ||\vec{a}|-|\vec{b}||$$

**(c)** From ΔOPS, OR<OP+PR

So, $|\vec{a}-\vec{b}| < ||\vec{a}|+|\vec{b}||$ (I)

Now, two vectors are acting along straight line in opposite direction-

$$|\vec{a}-\vec{b}| < ||\vec{a}|+|\vec{b}|| \quad \text{(II)}$$

from eq. (I)&(II)

$$|\vec{a}-\vec{b}| \le |\vec{a}|+|\vec{b}|$$

**(d)** From ΔOPS,

$|\overrightarrow{OP}-\overrightarrow{OT}|$ has been taken, because LHS = positive,

RHS may be negative

if OP<OT,

$$\therefore \quad |\vec{a}-\vec{b}| > ||\vec{a}|-|\vec{b}||$$

**Q. 7. A vector has magnitude and direction. Does it have a location in space ? Can it vary with time ? Will two equal vectors $\vec{a}$ and $\vec{b}$ at different location in space necessarily have identical physical effects ? Give examples in support of your answer.**

**[NCERT Ad. Ex. Q. 4.26, Page 88]**

**Ans.** In addition to magnitude and direction, each vector also has a definite location in space. For example, a velocity vector has definite location of every point in uniform circular motion.

A vector can vary with time. For example, increase in velocity is produced by acceleration.

Two equal vectors $\vec{a}$ and $\vec{b}$ having different locations may not produce identical physical effects. For example, two equal forces (vectors) acting at two different points may not produce equal turning effects.

**Q. 8. A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector ? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector ?**

**[NCERT Ad. Ex. Q. 4.27, Page 88]**

**Ans.** No, anything that has both magnitude and direction is not necessarily a vector. It must obey the laws of vector addition.

Rotation is not generally considered a vector even though it has magnitude and direction because the addition of two finite rotations does not obey commutative law. However, infinitesimally small rotations obey commutative law and hence, an infinitesimally small rotation is considered as a vector.

**Q. 9. Can you associate vectors with**

**(a) the length of a wire bent into a loop,**

**(b) a plane area,**

**(c) a sphere? Explain.**

**[NCERT Ad. Ex. Q. 4.28, Page 88]**

**Ans. (a)** No.

One cannot associate a vector with the length of a wire bent into a loop. Because length of a loop does not have a definite direction.

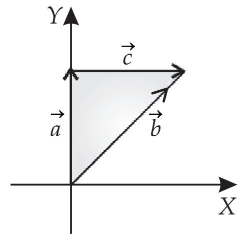
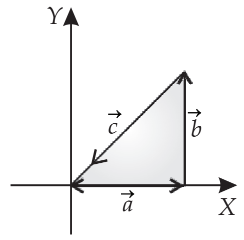
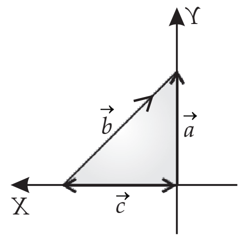
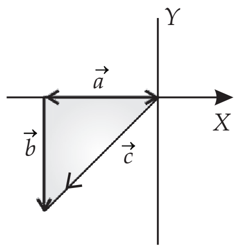
(b) Yes.

One can associate an area vector with a plane area. The direction of this vector is represented by a normal drawn outward to the area.

**(c)** No.

One cannot associate a vector with the volume of a sphere as it does not have a specific direction. However, a null vector can be associated with the area of a sphere.

**Q. 10. Given below in column I are the relations between vectors *a, b* and *c* and in column II are the orientations of *a, b* and *c* in the XY plane. Match the relation in column I to correct orientations in column II.**

| Column I | | Column II | |
|---|---|---|---|
| (a) | $\vec{a}+\vec{b}=\vec{c}$ | (i) |  |
| (b) | $\vec{a}-\vec{c}=\vec{b}$ | (ii) |  |
| (c) | $\vec{b}-\vec{a}=\vec{c}$ | (iii) |  |
| (d) | $\vec{a}+\vec{b}+\vec{c}=0$ | (iv) |  |

**[NCERT Exemp. Q. 4.26, Page 24]**

**Ans.**
**(a) (iv)** **(b) (iii)**
**(c) (i)** **(d) (ii)**

**Q. 11. If |A| = 2 and |B| = 4, then match the relations in column I with the angle θ between A and B in column II.**

| Column I | | Column II | |
|---|---|---|---|
| (a) | A.B = 0 | (i) | θ = 0 |
| (b) | A.B = +8 | (ii) | θ = 90° |
| (c) | A.B = 4 | (iii) | θ = 180° |
| (d) | A.B = −8 | (iv) | θ = 60° |

**[NCERT Exemp. Q. 4.27, Page 25]**

**Ans.**
**(a) (ii)** because cos 90°=0, hence $|A||B|\cos 90° = 2\times4\times0 = 0$
**(b) (i)** because cos 0°=1, hence $A||B|\cos 0° = 2\times4\times1 = +8$
**(c) (iv)** because cos 60°= $\frac{1}{2}$, hence $|A||B|\cos 60° = 2\times4\times\frac{1}{2} = +4$
**(d) (iii)** because cos 180°=−1, hence $|A||B|\cos 180° = 2\times4\times-1 = -8$

**Q. 12. If |A| = 2 and |B| = 4 , then match the relations in column I with the angle θ between A and B in column II.**

| Column I | | Column II | |
|---|---|---|---|
| (a) | $\|A\times B\|=0$ | (i) | θ = 30° |
| (b) | $\|A\times B\|=8$ | (ii) | θ = 45° |
| (c) | $\|A\times B\|=4$ | (iii) | θ = 90° |
| (d) | $\|A\times B\|=4\sqrt{2}$ | (iv) | θ = 0° |

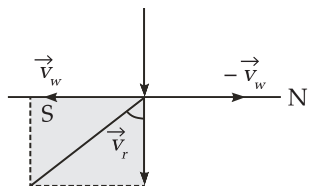
**[NCERT Exemp. Q. 4.28, Page 25]**

**Ans.**
**(a) (iv)** because sin θ=0°, hence $|A||B|\sin 0 = 2\times4\times0 = 0$
**(b) (iii)** because sin 90°=1, hence $|A||B|\sin 90° = 2\times4\times1 = 8$
**(c) (i)** because sin 30°= $\frac{1}{2}$,
hence $|A||B|\sin 30° = 2\times4\times\frac{1}{2} = 4$
**(d) (ii)** because $\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$,
hence, $|A||B|\sin 45° = 2\times4\times\frac{\sqrt{2}}{2} = 4\sqrt{2}$

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. Rain is falling vertically with a speed of 30 ms$^{-1}$. A woman rides a bicycle with a speed of 10 ms$^{-1}$ in the north to south direction. What is the direction in which she should hold her umbrella ?**
**[NCERT Ex. Q. 4.12, Page 86]**

**Ans.** Velocity of rain relative to the women of cycle is




$$\vec{v}_{rw} = \vec{v}_r - \vec{v}_w$$

$$\tan\theta = \frac{v_w}{v_r} = \frac{10}{30} = \frac{1}{3}$$

$$= 0.3333$$

$$\theta = 18°26'$$

with vertical towards south.

**Q. 2. A man can swim with a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?**

**[NCERT Ex. Q. 4.13, Page 86]**

**Ans.** Speed of the man $v_m = 4$ km/h

Width of the river = 1 km

$$\text{Time taken to cross the river} = \frac{\text{Width of the river}}{\text{Speed of the river}}$$

$$= \frac{1}{4}\text{h} = \frac{1}{4}\times 60 = 15\,\text{min}$$

Speed of the river, $v_r$ = 3 km/ h

Distance covered with flow of the river = $v_r \times t$

$$= 3\times\frac{1}{4} = \frac{3}{4}\text{km}$$

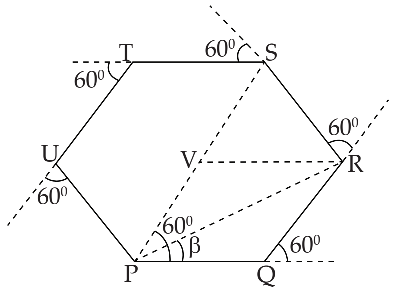
$$= \frac{3}{4}\times 1000 = 750\text{m}$$

## Long Answer Type Questions (5 marks each)

**Q. 1. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.**

**[NCERT Ex. Q. 4.10, Page 86]**

**Ans.** The path followed by the motorist is a regular hexagon with side 500 m, as shown in the given figure




Let the motorist start from point P.

The motorist takes the third turn at S.

∴ Magnitude of displacement = PS = PV + VS = 500 + 500 = 1000 m

Total path length = PQ + QR + RS = 500 + 500 +500 = 1500 m

The motorist takes the sixth turn at point P, which is the starting point.

∴ Magnitude of displacement = 0

Total path length = PQ + QR + RS + ST + TU + UP

= 500 + 500 + 500 + 500 + 500 + 500 = 3000 m

The motorist takes the eight turn at point R

∴ Magnitude of displacement = PR

$$= \sqrt{PQ^2 + QR^2 + 2(PQ).(QR)\cos 60^\circ}$$

$$= \sqrt{500^2 + 500^2 + (2\times 500\times 500\times \cos 60^\circ)}$$

$$= \sqrt{250000 + 250000 + \left(500000\times\frac{1}{2}\right)}$$

= 866.03 h

$$\beta = \tan^{-1}\left(\frac{500\sin 60^\circ}{500 + 500\cos 60^\circ}\right) = 30^\circ$$

Therefore, the magnitude of displacement is 866.03 m at an angle of 30° with PR.

Total path length = Circumference of the hexagon + PQ + QR

= 6 × 500 + 500 + 500 = 4000 m

The magnitude of displacement and the total path length corresponding to the required turns is shown in the given table

| Turn | Magnitude of displacement (m) | Total path length (m) |
|---|---|---|
| Third | 1000 | 1500 |
| Sixth | 0 | 3000 |
| Eighth | 866.03; 30° | 4000 |

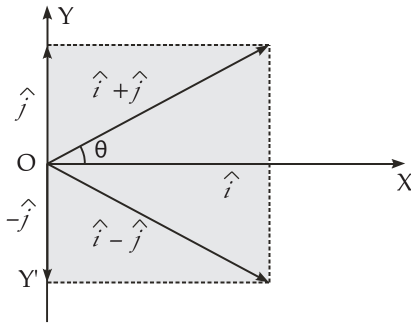
**Q. 2. (i) $\hat{i}$ and $\hat{j}$ are unit vector along *X*-and *Y*-axis respectively. What is the magnitude and direction of the vectors $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$ ?**

**(ii) What are the components of a vector A = $2\hat{i} + 3\hat{j}$ along the directions of $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$ ? You may use graphical method.**

**[NCERT Ex. Q. 4.27, Page 87]**

**Ans.** (i) Magnitudes of $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$ vectors: As $\hat{i}$ and $\hat{j}$ are the unit vectors along X-and Y-axis respectively.

$$|\hat{i}| = |\hat{j}| = 1$$




The magnitude of vector $\hat{i}+\hat{j}$ is given by

$$|\hat{i}+\hat{j}| = \sqrt{1^2+1^2+2.1.1\cos 90°}$$
$$= \sqrt{2+0} = \sqrt{2}$$
$$= 1.414 \text{ units.}$$

The magnitude of vector $\hat{i}-\hat{j}$ is given by

$$|\hat{i}-\hat{j}| = \sqrt{1^2+1^2-2.1.1\cos 90^o}$$
$$= \sqrt{2-0} = \sqrt{2}$$
$$= 1.414 \text{ units.}$$

Directions of $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$

Let θ be the angle made by $\hat{i}\ \ \hat{j}$ with $\hat{i}$, *i.e.*, with X-axis.

By definition of scalar product of two vectors

$$(\hat{i}+\hat{j}).\hat{i} = |\hat{i}+\hat{j}|.|\hat{i}|\cos\theta$$

or $$\hat{i}\cdot\hat{i}+\hat{j}\cdot\hat{i} = \sqrt{2}\times 1\times\cos\theta$$

or $$1+0 = \sqrt{2}\cos\theta$$

$(\because \hat{i}.\hat{i} = 1 \text{ and } \hat{i}.\hat{j} = 0)$

or $$\cos\theta = \frac{1}{\sqrt{2}} = \cos 45°$$

$$\theta = 45°$$

Also let α be the angle made by $\hat{i}-\hat{j}$ with $\hat{i}$, *i.e.*, with X-axis

Then $$(\hat{i}-\hat{j}).\hat{i} = |\hat{i}-\hat{j}|.|\hat{i}|\cos\alpha$$

or $$\hat{i}.\hat{i}-\hat{i}.\hat{j} = \sqrt{2}\times 1\times\cos\alpha$$

or $$\mathbf{1-0} = \sqrt{2}\cos\alpha$$

or $$\cos\alpha = \frac{1}{\sqrt{2}} = \cos 45°$$

$$\alpha = 45°$$

As $\hat{j}$, is – ve, so $\hat{i}-\hat{j}$ makes – 45° with $\hat{j}$.

Thus, with $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$ act at 45° and –45° with X-axis respectively.

**(ii)** **(a)** Let us now determine the component of $\vec{A} = 2\hat{i}+3\hat{j}$ in the direction of $(\hat{i}+\hat{j})$

Let $$\vec{B} = \hat{i}+\hat{j}$$

$$\therefore \vec{A}.\vec{B} = AB\cos\beta$$
$$= (A\cos\beta)B$$

or $$A\cos\beta = \frac{\vec{A}\cdot\vec{B}}{B}$$

Magnitude of the component of $\vec{A}$ in the direction of $\vec{B}$, *i.e.*, $(\hat{i}+\hat{j})$ is A cos β

$$= \frac{\vec{A}\cdot\vec{B}}{B} = \frac{(2\hat{i}+3\hat{j})(\hat{i}+\hat{j})}{|\hat{i}\ \ \hat{j}|}$$
$$= \frac{(2\hat{i}\cdot\hat{i}+3\hat{j}\cdot\hat{j})}{\sqrt{2}}$$

$(\because \hat{j}\cdot\hat{i} = \hat{i}\cdot\hat{j} = 0)$

$$= \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

If $\hat{n}_1$ = unit vector along $(\hat{i}+\hat{j})$, then

$$\hat{n}_1 = \frac{(\hat{i}+\hat{j})}{|\hat{i}+\hat{j}|} = \frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

∴ Component of A along $(\hat{i}+\hat{j})$ = Magnitude of the component of A along $\hat{i}+\hat{j}$

$$= \frac{5}{\sqrt{2}}\cdot\frac{(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{5}{2}(\hat{i}+\hat{j})$$

**(b)** Let us now determine the component of $\vec{A}$ along $\hat{i}-\hat{j}$.

Let $\hat{n}_2$ = unit vector acting along $\hat{i}-\hat{j}$

$$\hat{i}-\hat{j} = |\hat{i}-\hat{j}|\,\hat{n}_2 = \sqrt{2}\hat{n}_2$$

$$\therefore \hat{n}_2 = \frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

∴ Magnitude of the component of $\vec{A}$ along $(\hat{i}-\hat{j})$

$$= \vec{A}.\hat{n}_2 = (2\hat{i}+3\hat{j})\cdot\frac{\hat{i}-\hat{j}}{\sqrt{2}}$$
$$= \frac{2-3}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

∴ Component of $\vec{A}$ along $(\hat{i}-\hat{j})$

$$= (\vec{A}\cdot\hat{n}_2)\hat{n}_2$$
$$= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$$
$$= \frac{\hat{i}-\hat{j}}{\sqrt{2}} = \frac{-1}{2}(\hat{i}-\hat{j})$$

# TOPIC-2
## Projectile Motion

## Quick Review

- **Projectile :** Projectile is defined as a body thrown with some initial velocity and then allowed to move under the action of gravity alone, without being propelled by an engine or fuel or any source. The path followed by a projectile is known as its **trajectory**.
- **Centripetal force :**
  (a) It is the force required to move the body in circular path with a constant angular velocity.
  (b) The centripetal force acts on the particle along the radius which is directed towards the center of circular path.
  (c) The centripetal force does not increase the kinetic energy and angular momentum of the particle moving in a circular path, therefore the work done by the centripetal force is zero.
  (d) The centripetal force is provided in different ways, in different types of circular motions.

## Know the Terms

- **Angular displacement** of the object moving around a circular path is defined as the angle traced out by radius vector at the center of circular path in given time. It is vector quantity.
- **Angular velocity** of an object in circular motion is defined as the time rate of change of its angular displacement.
- **Angular acceleration** of an object in circular motion is defined as the time rate of change of its angular velocity.
- **Uniform circular motion** is the motion when a point object is moving on a circular path with a constant speed.

## Know the Formulae

- **For motion along $X$-axis,** $v_x = u_x + a_x t$ and $x = x_0 + u_x t + \frac{1}{2}a_x t^2$
- **For motion along $Y$-axis,** $v_y = u_y + a_y t$ and $y = y_0 + u_y t + \frac{1}{2}a_y t^2$
- **Velocity of projectile at an instant of its flight is**

$$v = \sqrt{v_x^2 + v_y^2}$$

and

$$\tan\beta = \frac{v_y}{v_x}$$

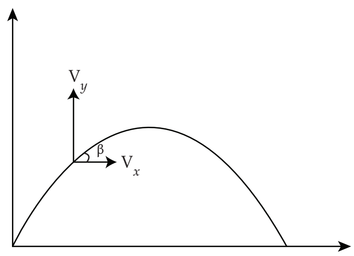
- **Angular projection of projectile :**

**(i) Time of flight,** $T = \frac{2u\sin\theta}{g}$

**(ii) Maximum height,** $h = \frac{u^2\sin^2\theta}{2g}$

**(iii) Horizontal range,** $R = \frac{u^2\sin 2\theta}{g}$

**(iv) Maximum horizontal range** $R_{max} = \frac{u^2}{g}$




- **Circular Motion**
  - $\omega = \theta/t$
  - $v = l/t$

- $\omega = 2\pi\nu = 2\pi/t$
- $a_c = \omega^2 r = \omega v = v^2/r$
- $a_T = r\alpha$

where, $a_c$ = Centripetal acceleration
$a_T$ = Tangential acceleration
$\omega$ = angular velocity
$\nu$ = frequency
$v$ = velocity

## Know the Links

☞ *www.vedantu.com*
☞ *www.learncbse.com*
☞ *www.byjus.com/ncert_solutions/*

## MCQ/Fillups/True or False **(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. The component of a vector *r* along x-axis will have maximum value if**

**(a) $\vec{r}$ is along positive Y-axis**
**(b) $\vec{r}$ is along positive X-axis**
**(c) $\vec{r}$ makes and angle of 45 with the X-axis**
**(d) $\vec{r}$ is along negative Y-axis**

**[NCERT Exemp. Q. 4.7, Page 20]**

**Ans. Correct option:** (b)

*Explanation:*

If $\vec{r}$ is at angle θ with x-axis, then component $\vec{r}$ of along x-axis= $r\cos\theta$. It will be maximum if $\cos\theta$ =maximum=1 or θ = 0 i.e., is along positive x-axis.

**Q. 2. The horizontal range of a projectile fired at an angle of 15 is 50 m. If it is fired with the same speed at an angle of 45°, its range will be**

**(a) 60 m** **(b) 71 m**
**(c) 100 m** **(d) 141 m**

**[NCERT Exemp. Q. 4.5, Page 20]**

**Ans. Correct option:** (c)

*Explanation:*

$$R = \frac{u^2 \sin 2\theta}{g}; R \propto \sin 2\theta$$

$$\therefore \frac{R_1}{R_2} = \frac{\sin 2\times 15^\circ}{\sin 2\times 45^\circ} = \frac{\sin 30^\circ}{\sin 90^\circ} = \frac{1}{2}$$

$$R_2 \Rightarrow 2R_1 \Rightarrow 2\times 50\,\text{m} = 100\,\text{m}$$

**Q. 3. In a two dimensional motion, instantaneous speed $v_0$ is a positive constant. Then which of the following are necessarily true?**

**(a) The average velocity is not zero at any time.**
**(b) Average acceleration must always vanish.**
**(c) Displacements in equal time intervals are equal.**
**(d) Equal path lengths are traversed in equal intervals.**

**[NCERT Exemp. Q. 4.7, Page 20]**

**Ans. Correct option:** (d)

*Explanation:*

In, two dimensional motion, when instantaneous speed is positive constant then, equal path lengths are travelled in equal intervals of time.

**Q. 4. In a two dimensional motion, instantaneous speed $v_0$ is a positive constant. Then which of the following are necessarily true?**

**(a) The acceleration of the particle is zero.**
**(b) The acceleration of the particle is bounded.**
**(c) The acceleration of the particle is necessarily in the plane of motion.**
**(d) The particle must be undergoing a uniform circular motion.**

**[NCERT Exemp. Q. 4.8, Page 20]**

**Ans. Correct option:** (c)

*Explanation:*

In two dimensional motion, when instantaneous speed is a positive constant then the acceleration of the particle is necessary in the plane of motion.

**Q. 5. Two particles are projected in air with speed $v_0$ at angels $\theta_1$ and $\theta_2$ (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then tick the right choices.**

**(a) angle of projection : $\theta_1 > \theta_2$**
**(b) time of flight : $T_1 > T_2$**
**(c) horizontal range : $R_1 > R_2$**
**(d) total energy : $U_1 > U_2$.**

**[NCERT Exemp. Q. 4.11, Page 21]**

**Ans. Correct option:** (a) and (b)

*Explanation:*

(a) from $h = \frac{v^2 \sin^2\theta}{2g} \Rightarrow h \propto \sin^2\theta \Rightarrow h \propto \theta$ ($\because \theta \le 90^\circ$

$\because h_1 > h_2 \therefore \theta_1 > \theta_2$

**(b)** from $T = \dfrac{2v\sin\theta}{g} \Rightarrow T \propto \sin\theta, \Rightarrow T \propto \theta$

If $\theta_1 > \theta_2$ then $T_1 > T_2$

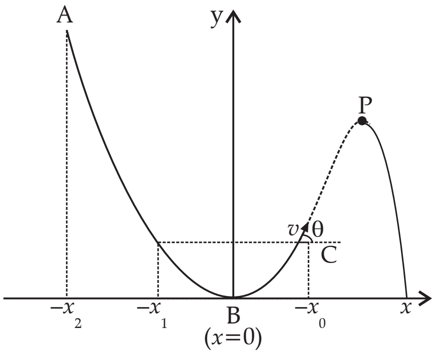
**Q. 6. A particle slides down a frictionless parabolic ($y = x^2$) track (A – B – C) starting from rest at point A. Point B is at the vertex of parabola and point C is at a height less than that of point A. After C, the particle moves freely in air as a projectile. If the particle reaches highest point at P, then**

**(a) KE at P = KE at B**

**(b) height at P = height at A**

**(c) total energy at P = total energy at A**

**(d) time of travel from A to B = time of travel from B to P.**




**[NCERT Exemp. Q. 4.12, Page 21]**

**Ans. Correct option:** (c)

*Explanation:* According to law of Mechanical energy conservation —

total energy at P = total energy at B

= total energy at A

**Q. 7. Following are four different relations about displacement, velocity and acceleration for the motion of a particle in general. Choose the incorrect one (s) :**

**(a)** $\mathbf{v}_{av} = \frac{1}{2}[\mathbf{v}(t_1) + \mathbf{v}(t_2)]$

**(b)** $\mathbf{v}_{av} = \dfrac{r(t_2) - r(t_1)}{t_2 - t_1}$

**(c)** $r = \frac{1}{2}(v(t_2) - v(t_1))(t_2 - t_1)$

**(d)** $a_{av} = \dfrac{v(t_2) - v(t_1)}{t_2 - t_1}$

**[NCERT Exemp. Q. 4.13, Page 21]**

**Ans. Correct option:** (a) and (c)

*Explanation:*

**(a)** $v_{av} = \frac{1}{2}[v(t_1) + v(t_2)]$ is not correct. Correct formula is

$$v_{av} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{v(t_1) \times t_1 + v(t_2) \times t_2}{t_1 + t_2}$$

**(c)** $r = \frac{1}{2}[v(t_2) - v(t_1)](t_2 - t_1)$ is not correct, correct formula $r = vd_2 \times t_2 - vd_1 \times t_1$

**Q. 8. For a particle performing uniform circular motion, choose the correct statement(s) from the following :**

**(a) Magnitude of particle velocity (speed) remains constant.**

**(b) Particle velocity remains directed perpendicular to radius vector.**

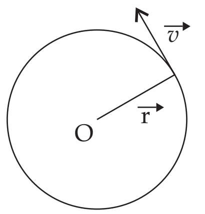
**(c) Direction of acceleration keeps changing as particle moves.**

**(d) Angular momentum is constant in magnitude but direction keeps changing.**

**[NCERT Exemp. Q. 4.14, Page 22]**

**Ans. Correct option:** (a), (b) and (c)

*Explanation:*




**(a)** In the uniform circular motion speed and magnitude of velocity are same and constant.

**(b)** In the figure shows the $\vec{v} \perp \vec{r}$

**(c)** In circular motion the acceleration towards to centre of circle. So direction of acceleration changes every time.

**(B) True and False**

**Q. 9. Read each statement below carefully and state, with reasons, if it is true or false :**

**(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.**

**(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.**

**(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.**

**[NCERT Ex. Q. 4.19, Page 87]**

**Ans.**

**(a) False—**The net acceleration of a particle in circular motion is towards the center only if its speed is constant.

**(b) True—**A particle released at any point of its path will always move along the tangent to the path at the point.

**(c) True—**For any two diametrically opposite points on the circumference, the acceleration vectors are equal and opposite. Hence, the acceleration vector average over one completely cycle is null vector.

**Q. 10. For any arbitrary motion in space, which of the following relations are true:**

(a) $v_{average} = \left(\frac{1}{2}\right)(v(t_1) + v(t_2))$

(b) $v_{average} = \frac{[r(t_2) - r(t_1)]}{(t_2 - t_1)}$

(c) $v(t) = v(0) + at$

(d) $r(t) = r(0) + v(0)t + \left(\frac{1}{2}\right)at^2$

(e) $a_{average} = \frac{[v(t_2) - v(t_1)]}{(t_2 - t_1)}$

**(The 'average' stands for average of the quantity over the time interval $t_1$ to $t_2$ )**

**[NCERT Exemp. Q. 4.23, Page 87]**

**Ans.**

**(a)** False.

It is given that the motion of the particle is arbitrary. Therefore, the average velocity of the particle cannot be given by this equation.

**(b)** True.

The arbitrary motion of the particle can be represented by this equation.

**(c)** False.

The motion of the particle is arbitrary. The acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of the particle in space.

**(d)** False.

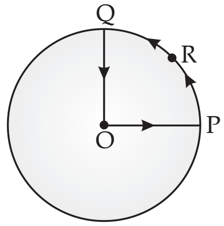
The motion of the particle is arbitrary; acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of particle in space.

**(e)** True.

The arbitrary motion of the particle can be represented by this equation.

## Very Short Answer Type Questions (1 mark each)

**Q. 1. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path OPRQO as shown Fig. If he maintains constant speed of 10 ms$^{-1}$, what is his acceleration at point R in magnitude and direction?**



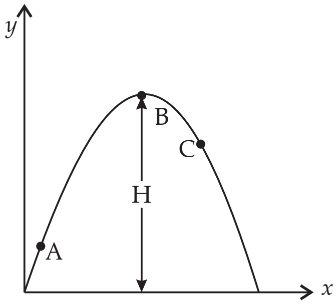

**[NCERT Exemp. Q. 4.16, Page 22]**

**Ans.** Centripetal acceleration at R is given by the relation,

$$a_c = \frac{v^2}{r}$$

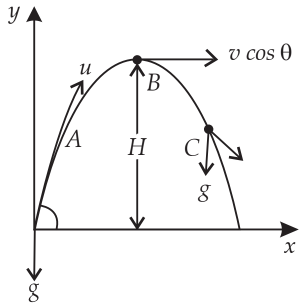
$$\Rightarrow a_c = \frac{v^2}{R} = \frac{(10)^2}{1000} = \frac{100}{10^3} = 0.1 \text{ m/s}^2 \quad \text{along } RO.$$

**Q. 2. A particle is projected in air at some angle to the horizontal, moves along parabola as shown in Fig. where x and y indicate horizontal and vertical directions, respectively. Show in the diagram, direction of velocity and acceleration at points A, B and C.**




**[NCERT Exemp. Q. 4.17, Page 22]**

**Ans.** Direction of the velocities at point A, B and C are along the tangent of the path. Acceleration at each point is the acceleration due to gravity vertically downwards.




**Q. 3. A ball thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during motion, does the ball have**

**(a) greatest speed.**

**(b) smallest speed.**

**(c) greatest acceleration.**

**Explain.**

**[NCERT Exemp. Q. 4.18, Page 23]**

**Ans. (a)** It has greatest speed just before it hits the ground.

**(b)** It has smallest speed at the highest point reached.

**(c)** Acceleration is always constant throughout the journey and is vertically downward equal to g.

**Q. 4. A football is kicked into the air vertically upwards. What is its**

**(a) acceleration, and (b) velocity at the highest point?**

**[NCERT Exemp. Q. 4.19, Page 23]**

**Ans.** **(a)** Acceleration will always be vertical downward and is called acceleration due to gravity ($g$).

**(b)** When football reaches the highest point it is momentarily at rest velocity, $v = 0$. As, it is continuously retarded by acceleration due to gravity ($g$).

## Short Answer Type Questions

**(2 or 3 marks each)**

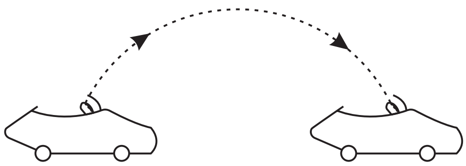
**Q. 1. A boy travelling in an open car moving on a levelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.**

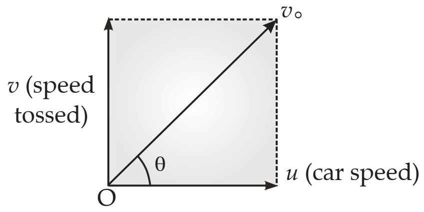
**[NCERT Exemp. Q. 4.21, Page 23]**

**Ans.** The horizontal speed of the ball is same as that of the car, therefore ball as well as car travels equal horizontal distance. $u$ = speed of car, $v$ = vertical speed of ball

$$v_0 = \sqrt{u^2 + v^2}$$

$$\theta = \tan^{-1}\left(\frac{v}{u}\right)$$






For a ground-based observer, the ball is a projectile with speed $v_0$ and the angle of projection θ with horizontal in as shown above.

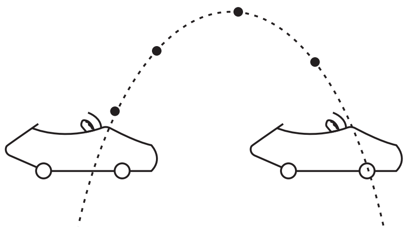
**Q. 2. A boy throws a ball in air at an angle 60° to the horizontal along a road with a speed of 10 m/s (36km/h). Another boy setting in a passing by car observes the ball. Sketch the motion of the ball as observed by the boy in the car, if car has a speed of (18km/h). Give explanation to support your diagram.** **[NCERT Exemp. Q. 4.22, Page 23]**

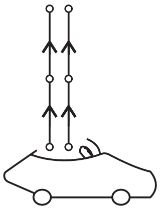
**Ans.** Horizontal component of velocity $u_x$=10 cos θ

Vertical component of velocity $u_y$=10 sin θ

$$u_x = 10\cos 60° = 10 \times \frac{1}{2} = 5 \text{ m/s}$$

$$u_y = 10\sin 60° = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ m/s}$$





Since the speed of car matches with the horizontal speed of the projectile, boy sitting in the car will see only vertical component of motion as shown in Fig (b).

As, Horizontal Component of velocity

$u_x$ = 10 cos θ

$u_x = 10 \cos 60° = 10 \times \frac{1}{2} = 5$ m/s

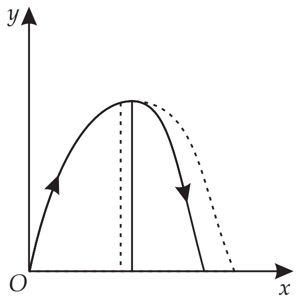
Vertical Component of velocity, $u_y$ = 10 sin θ

$u_y = 10 \sin 60° = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ m/s.

**Q. 3. In dealing with motion of projectile in air, we ignore effect of air resistance on motion. This gives trajectory as a parabola as you have studied. What would the trajectory look like if air resistance is included? Sketch such a trajectory and explain why you have drawn it that way.**

**[NCERT Exemp. Q. 4.23, Page 23]**

**Ans.** If air resistance is included the horizontal component of velocity will not be constant and obviously, trajectory will change.




Due to air resistance, particle energy as well as horizontal component of velocity keep on decreasing making the fall steeper as shown in the figure.

**Q. 4. (a) Earth can be thought of as a sphere of radius 6400 km. Any object (or a person) is performing circular motion around the axis of earth due to earth's rotation (period 1 day). What is acceleration of object on the surface of the earth (at equator) towards its centre? What is it at latitude θ ? How does these accelerations compare with g = 9.8 m/s²?**

**(b) Earth also moves in circular orbit around sun once every year with on orbital radius of 1.5 × $10^{11}$ m. What is the acceleration of earth (or any object on the surface of the earth) towards the centre of the sun? How does this acceleration compare with g = 9.8 m/s²?**

$$\left(Hint: acceleration = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}\right)$$

**[NCERT Exemp. Q. 4.25, Page 23]**

**Ans. (a)** Radius of earth, R = 6400 km = 6.4 × $10^6$m

Time period, T = 1 day=24 ×60×60 = 26400 s

$$a_c = \frac{4\pi^2 R}{T^2}$$

$$= \frac{4\times(22/7)^2\times 6.4\times10^6}{(24\times60\times60)^2}$$

$$= \frac{4\times484\times64\times10^6}{49\times(24\times3600)^2} = 0.034\,\text{m/s}^2$$

At equator, latitude θ = 0°

$$\therefore \frac{a_c}{g} = \frac{0.034}{9.8} = \frac{1}{288}$$

**(b)** Orbital radius=1.5×$10^{11}$m

$$T = 1\,\text{year} = 365\text{ days} = 365\times26400$$

$$= 3.15\times10^7\,\text{s}$$

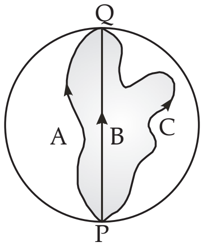
$$a_c = \frac{4\pi^2 R}{T^2}$$

$$= \frac{4\times\left(\frac{22}{7}\right)^2\times1.5\times10^{11}}{(3.15\times10^7)^2}$$

$$= 5.97\times10^{-3}\,\text{m/s}^2$$

$$\therefore \frac{a_c}{g} = \frac{5.9\times10^{-3}}{9.8} = \frac{1}{1642}$$

**Q. 5. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Figure. (i) What is the magnitude of the displacement vector for each ? (ii) For which girl is this equal to the actual length of path skate ?**




**[NCERT Ex. Q. 4.8, Page 86]**

**Ans. (i)** PQ = diameter

= displacement for each girl

= $2r$ = 2 × 200 = 400 m

Since, displacement vector does not depend upon the actual path length and it is the shortest distance between initial and final position, so in the case of each girl the displacement is 400 m.

**(ii)** For girl B, the displacement is equal to the actual length of path skate.

**Q. 6. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is**

**(a) the average speed of the taxi,**

**(b) the magnitude of average velocity? Are the two equal? [NCERT Ex. Q. 4.11, Page 86]**

**Ans. (a)** Total distance travelled = 23 km

Total time taken = 28 min = $\frac{28}{60}$h

$$\therefore \text{ Average speed of the taxi} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$\Rightarrow \frac{23}{\left(\frac{28}{60}\right)} = 49.29\text{ km/h}$$

**(b)** Distance between the hotel and the station = 10 km = Displacement of the car

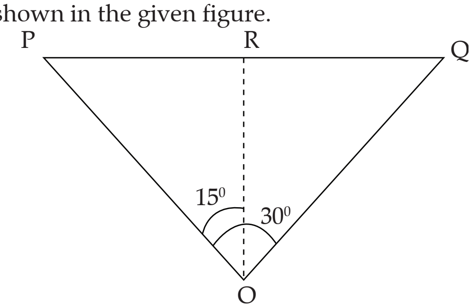
$$\therefore \text{Average velocity} = \frac{10}{\left(\frac{28}{60}\right)} = 21.43\text{ km/h}$$

Therefore, the two physical quantities (average speed and average velocity) are not equal.

**Q. 7. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft?**

**[NCERT Ex. Q. 4.25, Page 87]**

**Ans.** The positions of the observer and the aircraft are shown in the given figure.



Height of the aircraft from ground, OR = 3400 m
Angle subtended between the positions, ∠POQ = 30°
Time = 10 s
In ΔPRO:

$$\tan 15° = \frac{PR}{OR}$$

$PR = OR \tan 15° = 3400 \times \tan 15°$
ΔPRO is similar to ΔRQO.
∴ PR = RQ
PQ = PR + RQ
= 2PR = 2 x 3400 tan 15°
= 6800 x 0.268= 1822.4 m

∴ Speed of the aircraft = $\frac{1822.4}{10}$ =182.24 m/s

**Q. 8. A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 km/h. At what angle of sight (w. r. t. horizontal) when the target is seen where should the pilot drop the bomb in order to attack the target?**

**[NCERT Ad. Ex. Q. 4.24, Page 23]**
**[NCERT Exemp. Q. 4.24, Page 23]**

**Ans.** $g=9.8\ m/s^2$, H=1.5 Km, $v_0$=720 km/h

H=1500 m, $v_0$=200 m/s

$$R = v_0\sqrt{\frac{2H}{g}} = 200\sqrt{\frac{2\times1500}{9.8}} = 3.49\,\text{km}$$

$$\theta = \tan^{-1}\left(\frac{H}{R}\right)$$

$$= \tan^{-1}\left(\frac{H}{v_0}\sqrt{\frac{g}{2H}}\right)$$

$$\theta = 23°42'$$

## Long Answer Type Questions (5 marks each)

**Q. 1. A hill is 500 m high. Supplies are to be sent across the hill using a cannon that can hurl packets at a speed of 125 m/s over the hill. The cannon is located at a distance of 800m from the foot of hill and can be moved on the ground at a speed of 2 m/s; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take $g = 10 m/s^2$. [NCERT Exemp. Q. 4.29, Page 25]**

**Ans.** Given: Speed of packets= 125 m/s
Height of hill= 500m
d = 800 m

To cross the hill in shortest time, then the vertical velocity should be such that it just crosses the height of hill.

The minimum vertical velocity required for crossing the hill is given by

$$v_\perp^2 > 2gh = 10{,}000$$
$$v_\perp > 100\,\text{m/s}$$

As cannon can haul packets with a speed of 125m/s, so the maximum value of horizontal velocity, $v_{||}$ will be

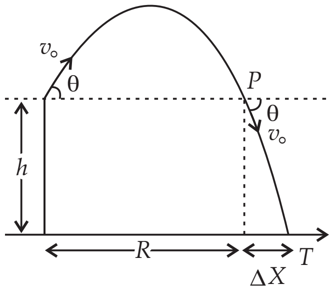
$$v_{||} = \sqrt{125^2 - 100^2} = 75\,\text{m/s}$$

The time taken to reach the top of the hill with velocity $v_{||}$ is given by

$$\frac{1}{2}gT^2 = h$$

putting values, $T = 10\ s$
In 10 s the horizontal distance covered 10 × 75 = 750 m.
So cannon has to be moved through a distance of 50 m on the ground.
So total time taken (shortest) by the packet to reach ground across the hill = $\frac{50}{2}s + 10s + 10s = 45\,s$.

**Q. 2. A gun can fire shells with maximum speed $v_0$ and the maximum horizontal range that can be achieved is $R = \frac{v_0^2}{g}$.**




**If a target farther away by distance Δx ( beyond R) has to be hit with the same gun (Fig), show that it could be achieved by raising the gun to a height at least**

$$h = \Delta x\left[1 + \frac{\Delta x}{R}\right]$$

**(Hint: This problem can be approached in two different ways:**
**(i) Refer to the diagram: target T is at horizontal distance and below point of projection y=−h.**
**(ii) From point P in the diagram: Projection at speed $v_0$ at an angle ∠ below horizontal with height h and horizontal range Δx.)**

**[NCERT Exemp. Q. 4.30, Page 25]**

**Ans.** As, $R = \frac{v_0^2}{g}$ ...(1)

Displacement along y-axis,

$$-h = (v_0 \sin\theta)t + \frac{1}{2}(-g)t^2 \quad ...(2)$$

Displacement along x-axis

$$t=\frac{(R+\Delta x)}{v_0 cos\theta} \quad ...(3)$$

Substituting t in eq.(3),

$$h=(-v_0\sin\theta)\times\left(\frac{R\times\Delta x}{v_0\cos\theta}\right)+\frac{1}{2}g\left(\frac{R+\Delta x}{v_0\cos\theta}\right)^2$$

$$h=-(R+\Delta x)\tan\theta+\frac{1}{2}g\frac{(R+\Delta x)^2}{v_0^2\cos^2\theta}$$

*As* θ = 45°,

$$h=-(R+\Delta x)\times 1+\frac{1}{2}g\frac{(R+\Delta x)^2}{v_0^2\left(\frac{1}{2}\right)}$$

$$h=-(R+\Delta x)+\frac{(R+\Delta x)^2}{R} \qquad \because R=\left(\frac{v_0^2}{g}\right)$$

$$=-R-\Delta x+\left(R+\frac{\Delta x^2}{R}+2\Delta x\right)=\Delta x+\frac{\Delta x^2}{R}$$

$$h=\Delta x\left(1+\frac{\Delta x}{R}\right).$$

**Q. 3. A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal (Fig.).**

**(a) Find an expression of range on the plane surface (distance on the plane from the point of projection at which particle will hit the surface).**

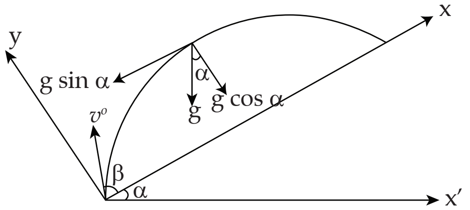
**(b) Time of flight.**

**(c) β at which range will be maximum.**

**[Hint: This problem can be solved in two different ways:**

**(i) Point P at which particle hits the plane can be seen as intersection of its trajectory (parabola) and straight line. Remember particle is projected at an angle (α + β) w.r.t. horizontal.**

**(ii) We can take x-direction along the plane and y-direction perpendicular to the plane. In that case resolve g (acceleration due to gravity) two different components, $g_x$ along the plane and $g_y$ perpendicular to the plane. Now the problem can be solved as two independent motions in x and y directions respectively with time as a common parameter.)**




**[NCERT Exemp. Q. 4.31, Page 26]**

**Ans. Part(b) solved first:**

**(b)** From A to B,

y=0, $u_y$=$v_0$sin β,

$a_y$= −g cos α, t=T

$$y=u_y t+\frac{1}{2}a_y t^2$$

$$0=v_0 sin\,\beta.T+\frac{1}{2}(-g\,cos\alpha)T^2$$

$$\Rightarrow T=\frac{2v_0 sin\,\beta}{g\,cos\alpha}$$

**(a)** x=L, $u_x$ =cosβ

$a_x$= −g sin α

$$x=u_x t+\frac{1}{2}a_x t^2$$

$$L=v_0\cos\beta T+\frac{1}{2}(-g\ \sin\alpha)T^2$$

$$L=T\left[v_0\cos\beta-\frac{1}{2}g\ \sin\alpha\times\frac{2v_0\sin\beta}{g\ cos\alpha}\right]$$

$$=\frac{2v_0^2\sin\beta}{g\ cos^2\alpha}[\cos\beta\ cos\alpha-\sin\alpha.\sin\beta]$$

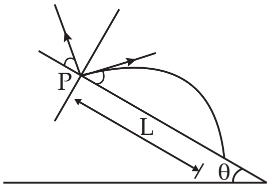
$$\Rightarrow L=\frac{2v_0^2\sin\beta}{g\ cos^2\alpha}\cos(\alpha+\beta)$$

**(c)** For maximum range, sin β **cos** (α + β) should be maximum. Now, by applying formula, we get

$$\sin(2\beta+\alpha)=1\Rightarrow 2\beta+\alpha=\frac{\pi}{2}$$

$$\beta=\frac{\pi}{4}-\frac{\alpha}{2}$$

**Q. 4. A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed $v_0$ and rebounds elastically (Fig). Find the distance along the plane where it will hit second time.**




**(Hint: (i) After rebound, particle still has speed $v_0$ to start.**

**(ii) Work out angle particle speed has with horizontal after it rebounds.**

**(iii) Rest is similar to if particle is projected up the incline.)**

**[NCERT Exemp. Q. 4.32, Page 26]**

**Ans.** As θ=0, $T=\frac{2v_0\cos\theta}{g\cos\theta}\Rightarrow\frac{2v_0}{g}$

Considering motion along x-axis,

X=L, $u_x$=$v_0$ sinθ, $a_x$ = g sinθ, t=T=2$v_0$/g

From kinematics equation-

$$x=u_x t+\frac{1}{2}a_x t^2$$

$$L=v_0\sin\theta t+\frac{1}{2}g\sin\theta t^2$$

$$=(v_0\sin\theta)(T)+\frac{1}{2}g\sin\theta T^2$$

$$=(v_0\sin\theta)\left(\frac{2v_0}{g}\right)+\frac{1}{2}g\sin\theta\times\left(\frac{2v_0}{g}\right)$$

$$=\frac{2v_0{}^2}{g}\sin\theta+\frac{1}{2}g\sin\theta+\frac{4v_0{}^2}{g^2}$$

$$\text{Distance}, L=\frac{4v_0{}^2}{g}\sin\theta.$$

**Q. 5. A girl riding a bicycle with a speed of 5m/s towards north direction, observes rain falling vertically down. If she increases her speed to 10m/s, rain appears to meet her at 45° to the vertical. What is the speed of the rain? In what direction does rain fall as observed by a ground based observer?**

**(Hint: Assume north to be $\hat{i}$ direction and vertically downward to be $-\hat{j}$. Let the rain velocity $v_r$ be $a\hat{i}+b\hat{j}$. The velocity of rain as observed by the girl is always $v_r - v_{girl}$. Draw the vector diagram/s for the information given and find a and b. You may draw all vectors in the reference frame of ground based observer.)**

**[NCERT Exemp. Q. 4.33, Page 27]**

**Ans.** Let $v_r = a\hat{i}+b\hat{j}$

Velocity of girl = $v_g = (5 \text{ m/s})\hat{i}$

Velocity of rain w.r.t. girl

$$v_{rg} = v_r - v_g = \left(a\hat{i}+b\hat{j}\right)-5\hat{i}$$

$$=(a-5)\hat{i}+b\hat{j}$$

Hence, $a-5=0 \Rightarrow a=5$

Now, $v_g = (10 \text{ m/s})\hat{i}$

$$v_{rg} = v_r - v_g$$

$$=\left(a\hat{i}+b\hat{j}\right)-10\hat{i}$$

$$=(a-10)\hat{i}+b\hat{j}$$

As, angle appear $45^\circ$,

$$\therefore \tan 45^\circ = \frac{b}{a-10}=1$$

$$\Rightarrow b = a-10 = 5-10 = -5$$

Hence, velocity of rain = $a\hat{i}+b\hat{j}$

$$\Rightarrow v_r = 5\hat{i}-5\hat{j}$$

Speed of rain

$$=|v_r| = \sqrt{(5)^2+(-5)^2}$$

$$=\sqrt{50}=5\sqrt{2}\text{ m/s.}$$

**Q. 6. A river is falling due east with a speed 3m/s. A swimmer can swim in still water at a speed of 4m/s (Fig.)**

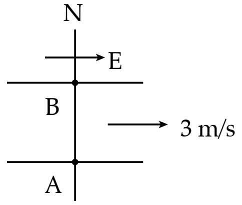
**(a) If swimmer starts swimming due north, what will be his resultant velocity (magnitude and direction)?**

**(b) If he wants to start from point A on south bank and reach opposite point B on north bank,**

**(a) which direction should he swim ?**

**(b) what will be his resultant ?**

**(c) From two different cases as mentioned in (a) and (b) above, in which case will he reach opposite bank shorter time?**




**[NCERT Exemp. Q. 4.34, Page 27]**

**Ans. (a)** Given. Speed of river, $v_r$=3m/s (east)

Velocity of summer (w.r.t. river), $v_s$ =4m/s (north)

Y-component of his resultant velocity = 4m/s (north)

x-component (produced by river flow)= 3m/s (east)

Resultant velocity, $v=\sqrt{v_r^2+v_s^2}$

$$=\sqrt{(3)^2+(4)^2}=\sqrt{9+16}$$

$$=\sqrt{25}=5m/s.$$

$$\therefore \tan\theta=\frac{v_r}{v_s}=\frac{3}{4}=0.75 = \tan 36°54'$$

θ = 36°54' = θ = 37°N

**(b)** The swimmer should swim at an angle of approx. of 37° north. (If he wants to reach on the point directly opposite to him).

Resultant speed of swimmer,

$$v=\sqrt{v_s^2-v_r^2}=\sqrt{4^2-3^2}$$

$$=\sqrt{16-9}=\sqrt{7}\text{m/s.}$$

$$\tan\theta=\frac{v_r}{v}=\frac{3}{\sqrt{7}} \Rightarrow \theta=\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)\text{with north}$$

**(c)** From (a),

Time taken by swimmer (to cross river), $t_1=\frac{d}{v_s}=\frac{d}{4}$s

From (b),

Time taken to cross river, $t_1=\frac{d}{v}=\frac{d}{\sqrt{7}}$

As, $\frac{d}{4}<\frac{d}{\sqrt{7}}$, $\therefore t_1<t_2$

Hence, in case (a), the swimmer will cross the river in shorter time.

**Q. 7. A cricket fielder can throw the ball with a speed $v_0$. If he throws the ball while running with a speed *u* at an angle θ to the horizontal, find**

**(a) the effective angle to the horizontal at which the ball is projected in air as seen by a spectator.**

**(b) what will be time of flight?**

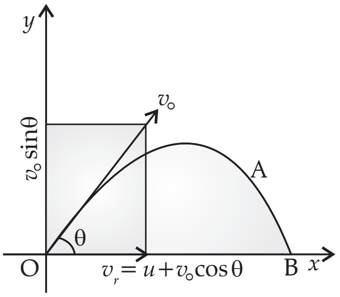
**(c) what is the distance (horizontal range) from the point of projection at which the ball will land?**

**(d) find $\theta$ at which he should throw the ball that would maximise the horizontal range as found in (iii).**

**(e) how does $\theta$ for maximum range change if u> $v_o$ , u= $v_o$ , u< $v_o$ ?**

**(f) how does $\theta$ in (v) compare with that for u=0 (i.e. 45° )? [NCERT Exemp. Q. 4.35, Page 27]**

**Ans. (a)** The angle of projection with horizontal seen by spectator will be




$$\tan\theta = \frac{u_y}{u_x} = \frac{v_0 \sin\theta}{u + v_0 \cos\theta}$$

**(b)** $y = u_y t + \frac{1}{2} a_y t^2$, as $y = 0$,

$$T = \frac{2v_0 \sin\theta}{g}$$

**(c)** $R = (u + v_0 \cos\theta)T = (u + v_0 \cos\theta)\frac{2v_0 \sin\theta}{g}$

**(d)** $\frac{dR}{dv} = 0,$

$$\frac{v_0}{g}[2u\cos\theta + v_0 \cos 2\theta \times 2] = 0,$$

$$\cos\theta = \frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{v_0}{u}\right)$$

(e) If $u = v_0$

$$\cos\theta = \frac{-v_0 \pm \sqrt{v_0^2 + 8v_0^2}}{4v_0} = \frac{-1+3}{4} = \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$

If $u << v_0$, then $8v_0^2 + u^2 \approx 8v_0^2$

$$\theta_{max} = \cos^{-1}\left[\frac{1}{\sqrt{2}} - \frac{u}{4v_0}\right] = \frac{\pi}{4} \qquad \text{(if } u << v_0\text{)}$$

If $u << v_0$ so neglecting $\frac{u}{4v_0}$, then

$$\theta_{max} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$= 45^\circ$

If $u > v_0$ and $u >> v_0$

$$\theta_{max} = \cos^{-1}\left[\frac{v_0}{u_0}\right] = 0 \qquad \because \left[\frac{v_0}{u} \to 0\right]$$

$$\Rightarrow \theta_{max} = 90^\circ$$

$$\cos^{-1}\left(\frac{-v_0}{u_0}\right) = \frac{\pi}{2} \qquad \text{(if } u << v_0\text{)}$$

(f) If $u = 0$, $\theta_{max} = \cos^{-1}\frac{-0 \pm \sqrt{8v_0^2}}{4v_0}$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

Hence, when $u = 0$, $\theta = 45$.

**Q. 8. Motion in two dimensions, in a plane can be studied by expressing position, velocity and acceleration as vectors in Cartesian co-ordinates $A = A_x\hat{i} + A_y\hat{j}$ where $\hat{i}$ and $\hat{j}$ are unit vector along x and y directions, respectively and $A_x$ and $A_y$ are corresponding components of A (Fig. 4.9). Motion can also be studied by expressing vectors in circular polar co-ordinates as $A = A_r\hat{r} + A_\theta\hat{\theta}$ where $\hat{r} = \frac{\vec{r}}{|r|} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$ and $\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ are unit vectors along direction in which 'r' and '$\theta$' are increasing.**

**(a) Express $\hat{i}$ and $\hat{j}$ in terms of $\hat{r}$ and $\hat{\theta}$ .**

**(b) Show that both $\hat{r}$ and $\hat{\theta}$ are unit vectors and are perpendicular to each other.**

**(c) Show that $\frac{d}{dt}(\hat{r}) = \omega\hat{\theta}$ where $\omega = \frac{d\theta}{dt}$ and $\frac{d}{dt}(\hat{\theta}) = -\omega\hat{r}$**

**(d) For a particle moving along a spiral given by $r = a\theta\hat{r}$, where a =1 (unit), find dimensions of 'a'.**

**(e) Find velocity and acceleration in polar vector representation for particle moving along spiral described in (d) above.**

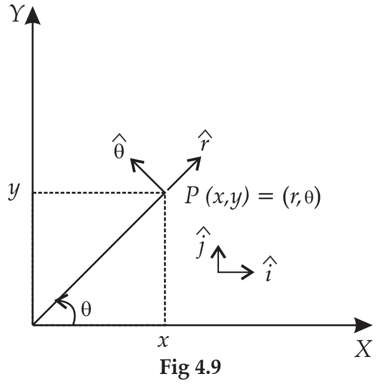



Fig 4.9

**[NCERT Exemp. Q. 4.36, Page 28]**

**Ans. (a)** unit vector, $\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$ (1)

$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ (2)

Multiplying eq. (1) by sinθ and eq. (2) by cosθ,

Adding $\Rightarrow \hat{r}\sin\theta + \hat{\theta}\cos\theta = \hat{j}$

Now, multiplying eq. (1) by cosθ and eq. (2) by θsinθ

$$n\left(\hat{r}\cos\theta - \hat{\theta}\sin\theta\right) = \hat{i}$$

**(b)** $\hat{r}.\hat{\theta} = \left(\cos\theta\hat{i} + \sin\theta\hat{j}\right).\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right)$

$= -\cos\theta.\sin\theta + \sin\theta.\cos\theta = 0$

$\Rightarrow \theta = 90^\circ$

**(c)** $\dfrac{dr}{dt} = \dfrac{d}{dt}\ (\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$= \omega(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) \qquad \because \left(\dfrac{d\theta}{dt} = \omega\right)$

**(d)** As, $r = a\theta\hat{r}$

$\Rightarrow [a] = L = \left[M^0L^1T^0\right]$

**(e)** $v = \dfrac{dr}{dt} = \dfrac{d\theta}{dt}\hat{r} + \theta\dfrac{d\hat{r}}{dt}$

$= \dfrac{d\theta}{dt}\hat{r} + \theta\left[\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right)\dfrac{d\theta}{dt}\right]$

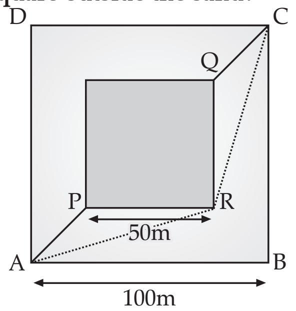
$v = \dfrac{d\theta}{dt}\hat{r} + \theta\dot{\theta}\omega = \omega\hat{r} + \omega\theta\hat{\theta}$

$a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left[\omega\hat{r} + \omega\theta\hat{\theta}\right] = \dfrac{d}{dt}\left[\dfrac{d\theta}{dt}\hat{r} + \dfrac{d\theta}{dt}\left(\theta\hat{\theta}\right)\right]$

$= \dfrac{d^2\theta}{dt^2}\hat{r} + \omega^2\hat{\theta} + \dfrac{d^2\theta}{dt^2} \times \theta\hat{\theta} + \omega^2\hat{\theta} + \omega^2\theta\left(-\hat{r}\right)$

$= \left(\dfrac{d^2\theta}{dt^2} - \omega^2\theta\right)\hat{r} + \left(2\omega^2 + \dfrac{d^2\theta}{dt^2}\hat{\theta}\right)$

**Q. 9. A man wants to reach from A to the opposite corner of the square C (Fig.). The sides of the square are 100 m. A central square of 50 m is filled with sand. Outside this square, he can walk only at a speed of 1 m/s. In the central square, he can walk only at a speed of v m/s (v<l). What is smallest value of v for which he can reach faster via a straight path through the sand than any path in the square outside the sand?**




**[NCERT Exemp. Q. 4.37, Page 28]**

**Ans.** Consider the straight line path APQC through the sand. Time taken to go from A to C via this path

$$T_{sand} = \frac{AP + QC}{1} + \frac{PQ}{v}$$

$$= \frac{25\sqrt{2} + 25\sqrt{2}}{1} + \frac{50\sqrt{2}}{v}$$

$$= 50\sqrt{2}\left[\frac{1}{v} + 1\right]$$

The shortest path outside the sand will be ARC.

$$T_{outside} = \frac{AR + RC}{1}\ s$$

$$AR = \sqrt{75^2 + 25^2} = \sqrt{75 \times 75 + 25 \times 25}$$

$$= 5 \times 5\sqrt{9 + 1} = 25\sqrt{10}m$$

$$RC = AR = \sqrt{75^2 + 25^2} = 25\sqrt{10}m$$

$$T_{outside} = 2AR = 2 \times 25\sqrt{10} = 50\sqrt{10}s$$

For $T_{sand} < T_{outside}$

$$50\sqrt{2}\left(\frac{1}{v} + 1\right) < 2 \times 25\sqrt{10}$$

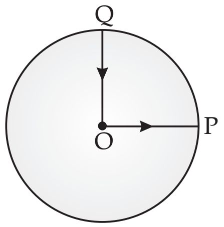
$$\frac{2\sqrt{2}}{2}\left(\frac{1}{v} + 1\right) = \sqrt{10}$$

$$\Rightarrow \left(\frac{1}{v} + 1\right) < \frac{2\sqrt{10}}{2\sqrt{2}} = \frac{\sqrt{5}}{2} \times 2 = \sqrt{5}$$

$$\Rightarrow \frac{1}{v} < \frac{\sqrt{5}}{2} \times 2 - 1 \Rightarrow \frac{1}{v} < \sqrt{5} - 1$$

$$\Rightarrow v > \frac{1}{\sqrt{5} - 1} \approx 0.81m/s \Rightarrow v > 0.81m/s$$

**Q. 10. A cyclist starts from the center O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity and (c) average speed of the cyclist ?**




**[NCERT Ex. Q. 4.9, Page 86]**

**Ans.** Given, radius of circular park = 1 km

**(a)** As cyclist returns to its initial state, therefore the net displacement of the cyclist is zero.

**(b)** Average velocity $= \dfrac{\text{Total displacement}}{\text{Total time taken}}$

$= \dfrac{0}{\text{Total time taken}}$

$= 0$

**(c)** Total distance travelled by the cyclist

= OP + Actual path length of PQ + QO

$$= r+\left(\frac{1}{4}\times 2\pi r\right)+r$$

$$= 1+\left(\frac{1}{2}\times\frac{22}{7}\times 1\right)+1$$

$$= 2+\frac{11}{7}$$

$$= \frac{25}{7}\text{ km}$$

Total time taken = 10 min

$$= \frac{10}{60}\text{h} = \frac{1}{6}\text{h}$$

∴ Average speed of the cyclist

$$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{25/7}{1/6} = \frac{150}{7}$$

= 21.43 km/h

**Value :** Physical exercises are always good for health.

**Q. 11. A bullet fired at an angle of 30° with the horizontal hits the ground 3.0 km a way. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the muzzle speed to the fixed, and neglect air resistance**.

**[NCERT Ad. Ex. Q. 4.29, Page 88]**

**Ans.** No.

Range, $R$ = 3 km

Angle of projection, $\theta$ = 30°

Acceleration due to gravity, $g$ = 9.8 m/s$^2$

Horizontal range for the projection velocity $u_0$, is given by the relation:

$$R = \frac{u_0^2 \sin 2\theta}{g}$$

$$3 = \frac{u_0^2}{g}\sin 60^0$$

$$\frac{u_0^2}{g} = 2\sqrt{3} \quad \ldots(i)$$

The maximum range ($R_{max}$) is achieved by the bullet when it is fired at an angle of 45° with the horizontal, that is,

$$R_{max} = \frac{u_0^2}{g} \quad \ldots(ii)$$

On comparing equations (*i*) and (*ii*), we get:

$R_{max} = 2\sqrt{3}$ = 2 × 1.732 = 3.46 km

Hence, the bullet will not hit a target 5 km away

**Q. 12. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?**

**[NCERT Ad. Ex. Q. 4.31, Page 88]**

**Ans.** Speed of the cyclist, $v$ = 21 km/ h = 7.5 m/ s

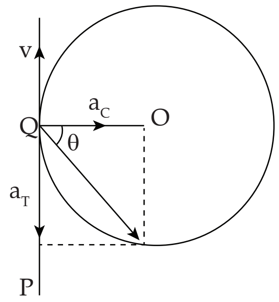
Radius of the circular turn, $r$ = 80 m

Centripetal acceleration is given as:

$$a_c = \frac{v^2}{r}$$

$$= \frac{(7.5)^2}{80} = 0.7\text{m/s}^2$$

The situation is shown in the given figure:




Suppose the cyclist begins cycling from point P and moves toward point Q. At point Q, he applies the breaks and decelerates the speed of the bicycle by 0.5 m/s$^2$.

This acceleration is along the tangent at Q and opposite to the direction of motion of the cyclist.

Since the angle between $a_c$ and $a_r$ is 90°, the resultant acceleration $a$ is given by:

$$a = \sqrt{a_c^2 + a_r^2}$$

$$= \sqrt{(0.7)^2 + (0.5)^2}$$

$$= \sqrt{0.74} = 0.85\text{m/s}$$

$$\tan\theta = \frac{a_c}{a_r}$$

Where θ is the angle of the resultant with the direction of velocity

$$\tan\theta = \frac{0.7}{0.5} = 1.4$$

$$q = \tan^{-1}(1.4)$$

$$= 54.46^0$$

**Q. 13. Show that for a projectile the angle between the velocity and the *x*-axis as a f unction of time is given by**

$$\boldsymbol{\theta(t) = \tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)}$$

**Show that the projection angle $\theta_0$ for a projectile launched from the origin is given by**

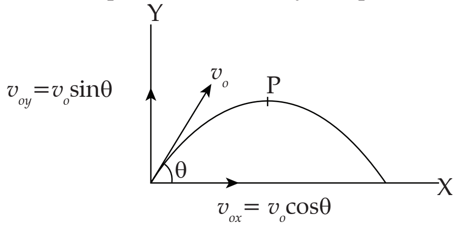
$$\boldsymbol{\theta_0(t) = \tan^{-1}\left(\frac{4h_m}{R}\right)}$$

**where the symbols have their usual meaning.**

**[NCERT Ad. Ex. Q. 4.32, Page 88]**

**Ans.** Let $v_{ox}$ and $v_{oy}$ respectively be the initial components of the velocity of the projectile along horizontal ($x$) and vertical ($y$) directions.

Let $v_x$ *and* $v_y$ respectively be the horizontal and vertical components of velocity at a point P.




Time taken by the projectile to reach point P = $t$

(a) Applying the first equation of motion along the vertical and horizontal directions, we get:

$v_y = v_{oy} - gt$

*And* $v_x = v_{ox}$

$$\therefore \tan\theta = \frac{v_y}{v_x} = \frac{v_{oy} - gt}{v_{ox}}$$

$$= \theta = \tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$$

(b) Maximum vertical height, $h_m = \frac{u_0^2 \sin^2 2\theta}{2g}$ ...($i$)

Horizontal range, $R = \frac{u_0^2 \sin^2 2\theta}{g}$ ...($ii$)

Solving equations ($i$) and ($ii$), we get:

$$\frac{h_m}{R} = \frac{\sin^2\theta}{2\sin^2\theta}$$

$$= \frac{\sin\theta \times \sin\theta}{2 \times 2\sin\theta\cos\theta}$$

$$= \frac{1}{4}\frac{\sin\theta}{\cos\theta} = \frac{1}{4}\tan\theta$$

$$\tan\theta = \left(\frac{4h_m}{R}\right)$$

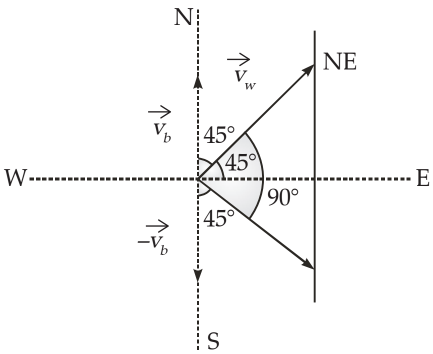
$$\therefore \theta = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

**Q. 14. In a harbour, wind is blowing at the speed of 72 kmh$^{-1}$ and the flag on the mast of a boat anchored in the harbour flutters along N-E direction. If the boat starts moving at the speed of 51 kmh$^{-1}$ to the north, what is the direction of the flag on the mast of the boat ?**

**[NCERT Ex. Q. 4.14, Page 87]**

**Ans.** Let $v_w$ = velocity of wind along NE direction
= 72 kmh$^{-1}$ along NE

$\vec{v}_b$ = velocity of boat along N direction
= 51 kmh$^{-1}$ along north




When the boat is anchored in the harbour, the flag flutters along the NE direction, so the velocity of the wind is along NE direction. When the boat starts moving along north direction, the flag will flutter along the direction of relative velocity of wind with respect to the boat.

Here, $\vec{v}_w$ = 72 kmh$^{-1}$ along NE

$\vec{v}_b$ = 51 kmh$^{-1}$ along N

Thus, if $\vec{v}_{wb}$ = relative velocity of wind w.r.t. boat, $\vec{v}_{wb}$ can be determined by bringing the boat to rest by imposing an equal and opposite velocity on boat as well as on the wind.

**(i)** Its own velocity, $\vec{v}_{wb}$ = 72 kmh$^{-1}$ along NE direction.

**(ii)** Imposed velocity, $\vec{v}_b$ = 51 kmh$^{-1}$ along S direction.

*i.e.*, thus $\vec{v}_{wb}$ is the resultant of these two velocities

*i.e.*, $\vec{v}_{wb} = \vec{v}_w + \left(-\vec{v}_b\right)$

It is clear from the figure that angle between $\vec{v}_w$ and $\vec{v}_b$ is 45° + 90° = 135°.

if β be the angle mode by $\vec{v}_{wb}$ with $\vec{v}_w$, then according to the parallelogram law of vector addition, the direction of flutter of flag is given by

$$\tan\beta = \frac{v_b \sin 135°}{v_w + v_b \cos 135°} \quad ...(i)$$

Now, $\sin 135° = \sin(180° - 45°)$

$= \sin 45° = \frac{1}{\sqrt{2}}$

$\cos 135° = \cos(180° - 45°)$

$= -\cos 45° = -\frac{1}{\sqrt{2}}$

$$\therefore \tan\beta = \frac{51 \times \frac{1}{\sqrt{2}}}{72 + \left[-\frac{1}{\sqrt{2}}\right] \times 51}$$

$$= \frac{\frac{51}{\sqrt{2}}}{72 - \frac{51}{\sqrt{2}}} = \frac{51}{72\sqrt{2} - 51}$$

$$= \frac{51}{101.52 - 51} = \frac{51}{50.52} = 1.009$$

or $\tan\beta = \tan 45°6'$

$\beta = 45°6'$

$= 45.1°$ w.r.t. NE direction

Thus, the flag with flutter at 45.1° w.r.t. NE direction, *i.e.*, direction of wind flow.

*or* (45.1° – 45°) = 0.1° w.r.t. east direction,

*i.e.*, almost due east.

**Q. 15. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms$^{-1}$ can go without hitting the ceiling of the hall ? [NCERT Ex. Q. 4.15, Page 87]**

**Ans.** **Step 1** : Using

$$H = \frac{u^2 \sin^2\theta}{2g}$$

when $H = 25$ m, $u = 40$ m/s

and $g = 9.8$ m/s$^2$

$$25 = \frac{40^2 \sin^2\theta}{2 \times 9.8}$$

*i.e.,* $$\sin^2\theta = \frac{490}{40^2}$$

$$\sin\theta = \frac{\sqrt{490}}{40} = 0.5534$$

*i.e.*, $\theta = 33.6°$

**Step 2 :** $$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{40^2 \sin 2(33.6)}{9.8} \text{ m}$$

$$= \frac{40^2 \sin 67.2}{9.8} \text{ m}$$

$$= \frac{40^2 \times 0.9219}{9.8} \text{ m}$$

$= 150.514$ m

**Q. 16. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ? [NCERT Ex. Q. 4.16, Page 87]**

**Ans.** From the formula the horizontal range is given by

$$R = \frac{u^2 \sin 2\theta}{g} \quad ..(i)$$

For $R = R_{max}$, $\theta = 45°$, *i.e.*, $\sin 2\theta = \sin 90° = 1$

Putting the given value is eq. (i), we

$$\therefore R_{max} = \frac{u^2}{g}$$

$$\Rightarrow 100 = \frac{u^2}{g} \quad (\because R_{max} = 100 \text{ given}) \; ...(ii)$$

Suppose H = height upto which the ball goes when the cricketer throws it with velocity $u$. Since the final velocity of the ball, $v = 0$.

$\therefore$ Applying the relation, $v^2 - u^2 = 2as$,

($\because$ here, $v = 0$, $a = -g$, $s = H$)

or $$H = \frac{u^2}{2g}$$

$$H = \frac{1}{2}\left(\frac{u^2}{g}\right)$$

$$= \frac{1}{2} \times (100) \quad \text{[by using (ii)]}$$

$H = 50$ m.

**Q. 17. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ? [NCERT Ex. Q. 4.17, Page 87]**

**Ans.** Given, Radius of the horizontal circle, $r = 80$ cm $= 0.80$ m

Angular speed of revolution of the stone is given by

$$\omega = \frac{\theta}{t} = \frac{2\pi\nu}{t} = 2\pi\left(\frac{\nu}{t}\right)$$

$$\Rightarrow \omega = 2 \times \frac{22}{7} \times \left(\frac{14}{25}\right)$$

$$\Rightarrow \omega = \frac{88}{25} \text{ rad/sec.}$$

$\therefore$ Magnitude of acceleration produced in the stone will be equal to the magnitude of centripetal acceleration.

$$a_c = r\omega^2$$

$$= 0.80 \times \left(\frac{88}{25}\right)^2$$

$$= 0.80 \times \frac{88}{25} \times \frac{88}{25}$$

$= 9.90$ ms$^{-2}$

We know that, the direction of the acceleration is towards the centre of the circle along its radius.

**Q. 18. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.**

**[NCERT Ex. Q. 4.18, Page 87]**

**Ans.** Given : $r = 1$ km $= 1000$ m;

$v = 900$ kmh$^{-1}$

$$= 900 \times \frac{1000}{3600} \text{ ms}^{-1}$$

$= 250$ ms$^{-1}$

The centripetal acceleration of the aircraft is

$$a = \frac{v^2}{r} = \frac{(250)^2}{1000}$$

$$= \frac{62500}{1000} = 62.5 \text{ ms}^{-2}$$

Acceleration due to gravity,

$$g = 9.8 \text{ ms}^{-2}$$

$$\therefore \frac{\text{Centripetal acceleration}}{\text{Acceleration due to gravity}} = \frac{a}{g}$$

$$= \frac{62.5}{9.8}$$

or $\frac{a}{g} = 6.38$

**Q. 19. The position vector of a particle is given by**

$$\boldsymbol{r = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k} \text{ m}}$$

**where $t$ is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.**

**(a) Find the $\vec{v}$ and $\vec{a}$ of the particle ?**

**(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s ?**

**[NCERT Ex. Q. 4.20, Page 87]**

**Ans.** The position vector ($\vec{r}$) of the particle is

$$\vec{} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k} \text{ m} \quad ...(i)$$

(a) velocity $\vec{v}$ (t) of the particle is given by

$$\vec{v}\,(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(\vec{r})$$

$$= \frac{d}{dt}(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k})$$

$$= 3i - 4t\,\hat{j} + 0 \quad ....(ii)$$

Also, acceleration $\vec{a}$ ($t$) of the participle is given by

$$\vec{a}\,(t) = \frac{d\vec{v}(t)}{dt} = \frac{d}{dt}\vec{v}(t)$$

$$= \frac{d}{dt}(3\hat{i} - 4t\,\hat{j}) \text{ [by using (ii)]}$$

$$= 0 - 4\hat{j}$$

$$\vec{a}\,(t) = -4\hat{j} \text{ ms}^2 \quad ....(iii)$$

**(b)** At time $t$, the velocity of the particle is given by using to equation (ii).

$$\vec{v}\,(t) = 3.0\hat{i} - 4t\,\hat{j} \text{ [by using (ii)]}$$

$\therefore$ At $t = 2$s,

$$v = 3.0\hat{i} - 4 \times 2\,\hat{j}$$

$$= 3.0\hat{i} - 8.0\,\hat{j}$$

$\therefore$ Its magnitude is

$$v = \sqrt{3^2 + (-8)^2}$$

$$= \sqrt{9 + 64}$$

$$= \sqrt{73} = 8.544 \text{ ms}^{-1}$$

and, direction of $v$ is given by

$$\therefore \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$= \tan^{-1}\left(\frac{-8}{3}\right)$$

$$= \tan^{-1}(-2.667)$$

$$= -69.45° \text{ with } x\text{-axis}$$

**Q. 20. A particle starts from the origin at $t$ = 0 s with a velocity of 10.0 $j$ and moves in the $x$-$y$ plane with a constant acceleration of (8.0 $\hat{i}$ +2.0 $\hat{j}$ ) ms$^{-2}$**

**(a) At what time is the x-coordinate of the particle 16 m? What is the $y$-coordinate of the particle at that time?**

**(b) What is the speed of the particle at the time?**

**[NCERT Ex. Q. 4.21, Page 87]**

**Ans. (a)** Velocity of the particle, $\vec{u} = 10.0\,\hat{j}$ m/s

Acceleration of the particle $\vec{a} = (8.0\hat{i} + 2.0\hat{j})$

Also,

But, $\vec{a} = \frac{d\vec{v}}{dt} = 8.0\hat{i} + 2.0\hat{j}$

$$d\vec{v} = (8.0\hat{i} + 2.0\hat{j})dt$$

Integrating both sides:

$$\vec{v}(t) = 8.0\hat{i}\,t + 2.0\,\hat{j}t + \vec{u}$$

Where,

$\vec{u}$ = Velocity vector of the particle at $t = 0$

$\vec{v}$ = Velocity vector of the particle at time $t$

But $\vec{v} = \frac{d\vec{r}}{dt}$

$$d\vec{r} = \vec{v}\,\text{dt} = (8.0\hat{i} + 2.0\hat{j} + \vec{u})dt$$

Integrating the equations with the conditions: at $t = 0$; $r = 0$ and at $t = t$; $r = r$

$$\vec{r} = \vec{u}t + \frac{1}{2}8.0\,t^2\,\hat{i} + \frac{1}{2}2.0\,t^2\hat{j}$$

$$= \vec{u}t + 4.0t^2\hat{i} + t^2\hat{j}$$

$$= (10.0\,\hat{j})t + 4.0\,t^2\hat{i} + t^2\,\hat{j}$$

$$x\,\hat{i} + y\,\hat{j} = 4.0\,t^2\hat{i} + (10t + t^2)\,\hat{j}$$

Since the motion of the particle is confined to the $x$-$y$ plane, on equating the coefficients of $\hat{i}$ and $\hat{j}$

,

we get:

$x = 4t^2$

$$t = \left(\frac{x}{4}\right)^{1/2}$$

And $y = 10t + t^2$

When $x = 16m$ :

$$t = \left(\frac{16}{4}\right)^{\frac{1}{2}} = 2s$$

$$\therefore y = 10 \times 2 + (2)^2 = 24 \text{ m}$$

**(b)** Velocity of the particle is given by:

$\vec{v}(t) = 8.0t\hat{i} + 2.0t\hat{j} + \vec{u}$

at $t = 2s$

$\vec{v}(t) = 8.0 \times 2\hat{i} + 2.0 \times 2\hat{j} + 10\hat{j}$

$= 16\hat{i} + 14\hat{j}$

$\therefore$ Speed of the particle:

$|\vec{v}| = \sqrt{(16)^2 + (14)^2}$

$= \sqrt{256 + 196} = \sqrt{452}$

$= 21.26$ m/s

**TIPS... & TRICKS...**

- *Understands about scalar and vector quantities.*
- *Study various type of vectors.*
- *Learn Addition and subtraction of vectors.*
- *Study and understand properties of scalar and vector products.*
- *Understand about Motion in a plane.*
- *Use formula of projectile motion as Time of Flight, Maximum hight and Range.*
- *In uniform circular motion the speed and magnitude of velocity are same.*

## Some Commonly Made Errors

- Students generally made mistakes during addition and subtraction of vectors, remember vectors has both magnitude and direction.

### EXPERT ADVICE

- *The kinematic equations for uniform acceleration do not apply to the cause of uniform circular motion since in this case the magnitude of acceleration is constant but its direction is changing.*
- *Resolving vector into components is an important tool used while solving numerically.*
- *Use differentiation and integration as a basic tool wherever required.*
- *Good Grip on vectors is very important to excel while solving numericals.*

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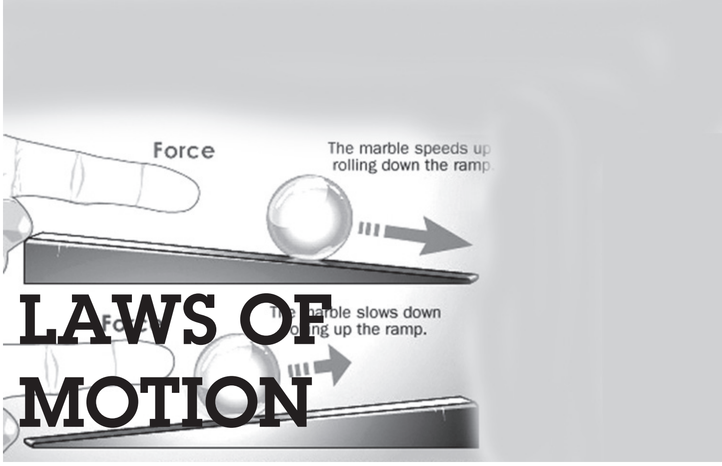
CHAPTER

# 5 LAWS OF MOTION




## Chapter Objective

**This chapter will help you understand :**

- *Basic concept of force, Inertia, Newton's first law of motion, Momentum and Newton's second law of motion, Impulse, Newton's third law of motion. Law of conservation of linear momentum and its applications. Equilibrium of concurrent forces.*
- *Static and Kinetic friction, Law of friction, Rolling friction, Lubrication, Dynamics of uniform circular motion, Centripetal force, Example of circular motion (vehicle on a level circular road, vehicle on a banked road).*

## TOPIC-1
## Newton's Laws of Motion



### Quick Review

- **Newton's first Law of motion :** A body continues to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some non-zero external force to change the state. This law defines forces and is also called law of inertia.
- **Inertia** of a body is of three types as follows :

**(i)** **Inertia of rest** of a body is inability to change by itself, its state on its own.

**(ii)** **Inertia of motion** of a body is the inability to change by itself its state of uniform motion *i.e.* body in uniform motion can neither accelerate nor retard on its own and comes to rest.

**(iii)** **Inertia of direction** of a body is inability to change by itself its direction of motion, *i.e.*, body continues to move along the same straight line unless compelled by some external force to change it.

- **Linear momentum :** Linear momentum ( $\vec{p}$ ) of a body is measured by the product of the mass ($m$) of the body and its velocity ( $\vec{v}$ ) *i.e.*,

$$\vec{p} = m\vec{v}$$

Linear momentum is a vector quantity. Its direction is same as the direction of velocity of the body. The S.I. unit of linear momentum is $kgms^{-1}$.

- **Newton's second Law of motion :** The rate of change of linear momentum ( $\vec{p} = m\vec{v}$ ) of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force, *i.e.*,

$$F \propto \frac{d\vec{p}}{dt}$$

or

$$F = k\frac{d\vec{p}}{dt} = k\frac{d}{dt}(m\vec{v})$$

$$= km\left(\frac{d\vec{v}}{dt}\right) = km\vec{a}$$

where $\frac{d\vec{v}}{dt} = \vec{a}$, which is called acceleration of the body. Force can be defined in such a way that $k = 1$, then Newton's second law is written as

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

➢ **Newton's third Law of motion :** To every action, there is always an equal and opposite reaction. The action and reaction act on different bodies, so they never cancel each other.

$$F_{AB} = -F_{BA}$$

$F_{AB}$ = Force exerted on A by B

$F_{BA}$ = Force exerted on B by A

Hence, $\vec{F}_{AB} + \vec{F}_{AB} = \vec{0}$

**(i) Principle of conservation of linear momentum :** From this principle, in an isolated system, the vector sum of the linear momentum of all the bodies of the system is conserved and is unaffected due to their mutual action and reaction. The total linear momentum of all the bodies in the system is given by

$$\vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n = M\vec{v}_{c.m} = \text{constant}$$

where, M is that total mass of the system and $\vec{v}_{c.m}$ is the velocity of the centre of mass of the system.

**(ii) Rocket propulsion :** The propulsion of a rocket is based on the principle of conservation of linear momentum of Newton's third law of motion.

Suppose,

$M_0$ = Initial mass of rocket,

$\frac{\Delta M}{\Delta t}$ = Rate of ejection of fuel,

M = Mass of rocket at any instant,

$\vec{v}$ = Relative velocity of ejected gases w.r.t. rocket.

Then, thrust on the rocket in the absence of gravity $= \frac{\Delta M}{\Delta t} \times \vec{v}$

Acceleration of the rocket in the absence of gravity $= \frac{\Delta M}{\Delta t} \times \frac{\vec{v}}{M}$

Thrust on the rocket in the presence of gravity,

$$F = \frac{\Delta M}{\Delta t} \times \vec{v} - Mg$$

Acceleration of the rocket in the presence of gravity,

$$\vec{a} = \frac{\Delta M}{\Delta t} = \frac{\vec{v}}{M} - \vec{g}$$

## Know the Terms

➢ **Force** is an external effort in the form of push or pull which can try to produce motion in a body at rest, or stops or try to stop a moving body or can change or try to change the direction of motion of the body.

➢ **Inertia** is the inherent property of a body, by virtue of which, the body doesn't change its state of rest or of uniform motion along a straight line, on its own. It depends upon the mass of the body.

➢ **Impulse :** When a large force acts on a body for a short time, then the measure of the total effect of force is called impulse of force. It can be found out

$$\text{Impulse} = \text{Force} \times \text{Time} = \vec{F}_{av} \times \Delta t$$

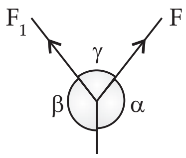
➢ **Inertial frame** is that in which the law of inertia is valid.

➢ **Non-inertial frame** is that in which the law of inertia is not valid.

➢ **Net force** is the vector sum of forces acting on an object.

## Know the Formulae

- **Force Unit :** Newton in SI, dyne in cgs.
  **Dimensional Formula :** $F = [MLT^{-2}]$
- **Impulse Unit :** N-S in SI, dyne-S in cgs
  **Dimensional Formula :** $\vec{I} = [MLT^{-1}]$
- **Linear Momentum :** $\vec{p} = m\vec{v}$
- **Force** $= \frac{d\vec{p}}{dt} = m\vec{a}$
- **Impulse** = Force × time
  $\vec{I} = \vec{F} \times t = m(v - u)$
- **Weight of person** = $mg$.
- **Principle of Conservation of Linear Momentum**
  $$m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n = \text{Constant}$$
- **Recoil velocity of gun** $(\vec{v}_2) = \frac{-m_1\vec{v}_1}{m_2}$
  $m_2$ = Mass of gun
  $m_1$ = Mass of bullet
  $v_1$ = Velocity of bullet
- **Lami's Theorem :**
  Three forces acting on body in equilibrium, then
  $$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$




## Know the Links

- *www.vedantu.com*
- *www.learncbse.in*
- *https://byjus.com*
- *https://schools.aglasem.com*

## MCQ/Fillups/True or False **(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. A ball is travelling with uniform translatory motion. This means that**

**(a) It is at rest.**

**(b) The path can be a straight line or circular and the ball travels with uniform speed.**

**(c) All parts of the ball have the same velocity (magnitude and direction) and the velocity is constant.**

**(d) The centre of the ball moves with constant velocity and the ball spins about its centre uniformly.**

**[NCERT Exemp. Q. 5.1, Page 29]**

**Ans. Correct option:** (c)

*Explanation:* According to definition of translatory motion. If every particle of the body is displaced same in same time that called translatory motion. So, all part of ball have the same velocity.

**Q. 2. A metre scale is moving with uniform velocity. This implies**

**(a) The force acting on the scale is zero, but a torque about the centre of mass can act on the scale.**

**(b) The force acting on the scale is zero and the torque acting about centre of mass of the scale is also zero.**

**(c) The total force acting on it need not be zero but the torque on it is zero.**

**(d) Neither the force nor the torque need to be zero.**

**[NCERT Exemp. Q. 5.2, Page 29]**

**Ans. Correct option:** (b)

*Explanation:* At uniform velocity acceleration and force both are zero. Then torque are also zero because

$$\tau = r \times F = r \times 0 = 0$$

**Q. 3. A cricket ball of mass 150 g has an initial velocity $u = (3\hat{i} + 4\hat{j})$ ms$^{-1}$ and a final velocity $v = -(3\hat{i} + 4\hat{j})$ ms$^{-1}$ after being hit. The change in momentum (final momentum-initial momentum) is (in kg m s$^{-1}$)**

**(a) Zero**

**(b) $-(0.45\hat{i} + 0.6\hat{j})$**

**(c) $-(0.9\hat{i} + 1.2\hat{})$**

**(d) $-5(\hat{i} + \hat{j})$. [NCERT Exemp. Q. 5.3, Page 30]**

**Ans. Correct option:** (c)

*Explanation:*

Here, $m = 150\ g = 0.15$ kg

$u = \left(3\hat{i} + 4\hat{j}\right)$ m/s

$v = -3i + 4j$ m/s

Initial momentum, $P_i = mu$

$$p_i = (0.15kg)\left(3\hat{i} + 4\hat{j}\right)\text{m/s}$$

$$= \left(0.45\hat{i} + 0.6\hat{j}\right)\text{kg m/s}$$

Final momentum, $p_f = (0.15\text{ kg})\left(-3\hat{i} - 4\hat{j}\right)$ m/s

$$= \left(-0.45\hat{i} - 0.6\hat{j}\right)\text{ kg m/s}$$

Change in momentum,

$$\Delta p = p_f - p_i = \left(-0.45\hat{i} - 0.6\hat{j}\right)\text{kg m/s} - \left(0.45\hat{i} + 0.6\hat{j}\right)\text{kg m/s}$$

$$= -\left(0.9\hat{i} + 1.2\hat{j}\right)\text{kg m/s}$$

**Q. 4. In the previous problem Q. 3, the magnitude of the momentum transferred during the hit is**

**(a) Zero**

**(b) 0.75 kg ms$^{-1}$**

**(c) 1.5 kg ms$^{-1}$**

**(d) 14 kgms$^{-1}$**

**[NCERT Exemp. Q. 5.4, Page 30]**

**Ans. Correct option:** (c)

*Explanation:*

By previous solution, $\Delta p = -\left(0.9\hat{i} + 1.2\hat{j}\right)$

$$\text{Magnitude} = |\Delta p| = \sqrt{(0.9)^2 + (1.2)^2} = \sqrt{0.81 + 1.44} = 1.5\text{kg m/s}$$

**Q. 5. Conservation of momentum in a collision between particles can be understood from**

**(a) Conservation of energy.**

**(b) Newton's first law only.**

**(c) Newton's second law only**

**(d) Both newton's second and third law.**

**[NCERT Exemp. Q. 5.5, Page 30]**

**Ans. Correct option:** (d)

*Explanation:* Force acting on a body, $F = \dfrac{\Delta p}{\Delta t}$

(IInd law)

if F = 0 then $\dfrac{dp}{dt} = 0$ or $P = mv$ = constant

At collision particles acts forces one another equal and opposite direction.

**Q. 6. A body of mass 2 kg travels according to the law $x(t) = pt + qt^2 + rt^3$ where $p = 3$ ms$^{-1}$, $q = 4$ ms$^{-2}$, and $r = 5$ ms$^{-3}$.**

**The force acting on the body at $t = 2$ seconds is**

**(a) 136 N (b) 134 N**

**(c) 158 N (d) 68 N**

**[NCERT Exemp. Q. 5.7, Page 30]**

**Ans. Correct option:** (a)

*Explanation:*

Here, $x(t) = pt + qt^2 + rt^3$

where $p = 3$ m/s, $q = 4$ m/s$^2$ and $r = 5$ m/s$^3$

$m = 2$ kg

$$\text{velocity, } v = \frac{dx}{dt} = \frac{d}{dt}\left(pt + qt^2 + rt^3\right) = p + 2qt + 3rt^2$$

$$\text{Acceleration, } a = \frac{dv}{dt} = 2q + 6rt$$

$$x = 2\left(4\text{ m/s}^2\right) + 6\left(5\text{ m/s}^3\right) \times (2\text{ s})$$

$$\left(\frac{dv}{dt}\right)_{t=2} = 8\text{ m/s}^2 + 60\text{ m/s}^2 = 68\text{ m/s}^2$$

∴ Force acting on the body of mass 2 kg is F = ma = (2 kg)(68 m/s$^2$)

= 136 N

**Q. 7. A body with mass 5 kg is acted upon by a force, $F = (-3\hat{i} + 4\hat{j})$ N. If its initial velocity at $t = 0$ is $u = (6\hat{i} - 12\hat{j})$ ms$^{-1}$, the time at which it will just have velocity along the $y$-axis is**

**(a) Never (b) 10 s**

**(c) 2 s (d) 15 s**

**[NCERT Exemp. Q. 5.8, Page 31]**

**Ans. Correct option:** (b)

*Explanation:* Given

$m = 5$ kg, $\vec{F} = -\left(3\hat{i} + 4\hat{j}\right)N$, $u = 6\hat{i} - 12\hat{j}$ m/s

The acceleration of the body is

$$\vec{a} = \frac{\vec{F}}{m} = \frac{-\left(3\hat{i} + 4\hat{j}\right)N}{5\text{ kg}},\ = -\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)\text{ m/s}^2$$

Velocity of the body along x-axis at any time t is

$$v_x = u_x + a_x t = 6 - \frac{3}{5}t$$

As the body will have a velocity along $y$-axis, When its velocity along $x$-axis will be zero.

i.e. $v_x = 0$

or $6 - \frac{3}{5}t = 0$ or $t = \frac{30}{3} = 10$ s

**Q. 8. A car of mass m starts from rest and acquires a velocity along east $v = v\hat{i}$ ($v > 0$) in two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is**

**(a) $\frac{mv}{2}$ eastward and is exerted by the car engine.**

**(b) $\frac{mv}{2}$ eastward and is due to the friction on the tyres exerted by the road.**

**(c) More than $\frac{mv}{2}$ eastward exerted due to the engine and overcomes the friction of the road.**

**(d) $\frac{mv}{2}$ Exerted by the engine.**

**[NCERT Exemp. Q. 5.9, Page 31]**

**Ans. Correct option:** (b)

*Explanation:*

Given, mass of the car = $m$

Initial velocity, $u = 0$ (As car starts from rest)

Final velocity, $\vec{v} = vi$ along east

$t = 2$s

$v = u + at$

$$v\vec{i} = 0 + \vec{a} \times 2 \text{ or } \vec{a} = \frac{v}{2}i$$

Force exerted on the car is

$$\vec{F} = m\vec{a} = \frac{mv}{2}\hat{i} = \frac{mv}{2} \text{ eastward}$$

This is because of friction on the tyres exerted by the road.

**Q. 9. The motion of a particle of mass $m$ is given by $x = 0$ for $t < 0$ s, $x(t) = A \sin 4\pi t$ for $0 < t < (1/4)$ s ($A > 0$), and $x = 0$ for $t > (1/4)$s. Which of the following statements is true?**

**(a) The force at $t = (1/8)$ s on the particle is $-16\pi^2 A$ m.**

**(b) The particle is acted upon by an impulse of magnitude $4\pi^2 A$ m at $t = 0$ s and $t = (1/4)$s.**

**(c) The particle is not acted upon by any force.**

**(d) The particle is not acted upon by a constant force.**

**(e) There is no impulse acting on the particle.**

**[NCERT Exemp. Q. 5.10, Page 31]**

**Ans. Correct option:** (a), (b) and (d)

*Explanation:*

**(a)** Given,

$$x(t) = A\sin 4\pi t, \quad 0 < t < \frac{1}{4}s$$

$$\therefore v(t) = \frac{dx}{dt} = A(\cos 4\pi t)4\pi$$

$$a(t) = \frac{dv}{dt} = 4\pi A(-\sin 4\pi t)4\pi$$

$$= -16\pi^2 A \sin(4\pi t)$$

At $t = \frac{1}{8}s, \ a(t) = -16\pi^2 A \sin\frac{\pi}{2}$

$$= -16\pi^2 A$$

$$\therefore F = ma = -16\pi^2 Am$$

**(b)** Impulse $= F\Delta t = -16\pi^2 Am\left[\frac{1}{4}s - 0s\right]$

$$= -4\pi^2 Am$$

**(d)** For $t > (1/4)$s, $x = 0, v = 0, a = 0$, F = 0

$\therefore$ Particle is not acted upon by a constant force.

**Q. 10. Two billiard balls A and B, each of mass 50 g and moving in opposite directions with speed of 5 ms$^{-1}$ each, collide and rebound with the same speed. If the collision lasts for $10^{-3}$ s, which of the following statements are true?**

**(a) The impulse imparted to each ball is 0.25 kg m s$^{-1}$ and the force on each ball is 250 N.**

**(b) The impulse imparted to each ball is 0.25 kg m s$^{-1}$ and the force exerted on each ball is $25 \times 10^{-5}$N.**

**(c) The impulse and the force on each ball is 0.5 Ns.**

**(d) The impulse and the force on each ball are equal in magnitude and opposite in direction.**

**[NCERT Exemp. Q. 5.14, Page 32]**

**Ans. Correct option:** (c) and (d)

*Explanation:*

$m_A = m_B = 50 \text{ g} = 50 \times 10^{-3}$ kg

$v_A = v_B = 5$ m/s

Impulse imported to each ball on collision = change in momentum of each ball

$= 2\, m_A v_A = 2\, m_B v_B$

$= 2 \times 50 \times 10^{-3} \times 5 = 0.5$ Ns

$$\text{Force} = \frac{\text{Impulse}}{\text{Time}} = \frac{\text{change in momentum}}{10^{-3}}$$

$$= \frac{0.5}{10^{-3}} = 500 \text{ N}$$

As direction of change of momentum of A is opposite to that of B, so the impulse and the force on each ball are equal in magnitude and opposite in direction.

**Q. 11. A body of mass 10 kg is acted upon by two perpendicular forces, 6 N and 8 N. The resultant acceleration of the body is**

**(a) 1 ms$^{-2}$ at an angle of $\tan^{-1}\left(\frac{4}{3}\right)$ w. r. t. 6 N force.**

**(b) 0.2 ms$^{-2}$ at an angle of $\tan^{-1}\left(\frac{4}{3}\right)$ w. r. t. 6 N force.**

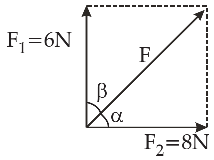
**(c) 1 ms$^{-2}$ at an angle of $\tan\left(-\right)$ w. r. t. 8 N force.**

**(d) 0.2 ms$^{-2}$ at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ w. r. t. 8 N force.**

**[NCERT Exemp. Q. 5.15, Page 32]**

**Ans. Correct option:** (a) and (c)

*Explanation:*




Given situation is shown in figure

$m$ =5 kg

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{6^2 + 8^2} = 10 \text{ N}$$

∴ Acceleration of body,

$$a = \frac{F}{m} = \frac{10}{10} = 1 \text{m/s}^2$$

direction of acceleration

$$\tan\alpha = \frac{F_1}{F_2} = \frac{6}{8} = \frac{3}{4}$$

$\therefore \alpha = \tan^{-1}\left(\frac{3}{4}\right)$ with respect to 8 N of force.

$$\tan\beta = \frac{F_2}{F_1} = \frac{8}{6} = \frac{4}{3}$$

$\therefore \beta = \tan^{-1}\left(\frac{3}{4}\right)$ with respect to 6 N of force.

## Very Short Answer Type Questions (1 mark each)

**Q. 1. A girl riding a bicycle along a straight road with a speed of 5 ms$^{-1}$ throws a stone of mass 0.5 kg which has a speed of 15 ms$^{-1}$ with respect to the ground along her direction of motion. The mass of the girl and bicycle is 50 kg. Does the speed of the bicycle change after the stone is thrown? What is the change in speed, if so?**

**[NCERT Exemp. Q. 5.16, Page 33]**

**Ans.** Mass of girl and bicycle = 50 kg

Mass of stone = 0.5 kg

$u_1$ = 5 m/s forward

$v_2$ = 15 m/s forward

$v_1$ = ?

where, $u_1$ = speed of cycle + girl before throwing stone

$v_1$ = speed of cycle + girl after throwing

$v_2$ = speed of stone

According to law of conservation of momentum

Initial momentum = Final momentum

$(m_1 + m_2)\, v_1 = m_1 v_1 + m_2 v_2$

$(50 + 0.5) \times 5 = 50 \times v_1 + 0.5 \times 15$

$50\, v_1 = 252.5 - 7.5$

$= 245.0$

$$v_1 = \frac{245.0}{50} = 4.9 \text{ m/s}$$

Hence, the speed of cycle decreased by 5 – 4.9 = 0.1 m/s

**Q. 2. A person of mass 50 kg stands on a weighing scale on a lift. If the lift is descending with a downward acceleration of 9 ms$^{-2}$, what would be the reading of the weighing scale?**

**($g$ = 10 ms$^{-2}$)**

**[NCERT Exemp. Q. 5.17, Page 33]**

**Ans.** If lift is descending, then acceleration = a, the apparent weight decreases on weighing scale,

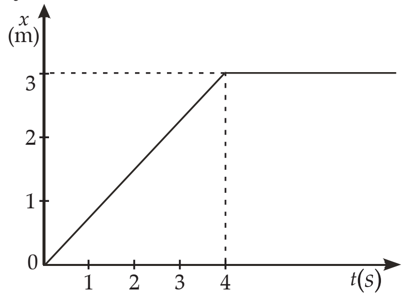
$$W_1 = R = (mg - ma) \text{ or } m(g - a)$$

Apparent weight due to reaction force,

$$W_1 = 50(10 - 9)$$
$$= 50 \text{ N}$$

$$\therefore \text{ Reading of weighing scale} = \frac{R}{g} = \frac{50}{10} = 5 \text{ kg}.$$

**Q. 3. The position time graph of a body of mass 2 kg is as given in figure. What is the impulse on the body at $t$ = 0 s and $t$ = 4 s.**




**[NCERT Exemp. Q. 5.18, Page 33]**

**Ans.** From graph, $\tan\theta = \frac{3}{4} = 0.75$ m/s .

At $t \geq 4$, slope of graph = 0, so $v_3 = 0$

$$\text{Impulse} = \vec{F}.t = \frac{d\vec{p}}{dt}.dt = d\vec{p} = m\vec{v}$$

At $t$ = 0, Impulse

$$= m\overrightarrow{v_2} - m\overrightarrow{v_1} = 2[0.75 - 0] = 1.50 \text{ kg m/s}$$

At $t$ = 4, Impulse

$$= m\left(\overrightarrow{v_3} - \overrightarrow{v_2}\right) = 2[0 - 0.75] = -1.50 \text{ kg m/s}$$

**Q. 4. A person driving a car suddenly applies the breaks on seeing a child on the road ahead. If he is not wearing seat belt, he falls forward and his head against the steering wheel. Why?**

**[NCERT Exemp. Q. 5.19, Page 34]**

**Ans.** When a person applies breaks suddenly, the only retarding force that acts on him, if he is not using a seat belt comes from the friction exerted by the seat. This is not enough to prevent him from moving forward when the vehicle is brought to a sudden halt.

**Q. 5. The velocity of a body of mass 2 kg as a function of t is given by $v(t) = 2t\,\hat{i} + t^2\,\hat{j}$. Find the momentum**

**and the force acting on it, at time $t$ = 2s.**

**[NCERT Exemp. Q. 5.20, Page 34]**

**Ans.** As, $v(t) = 2t\hat{i} + t^2\hat{j}$

$\vec{v}$ at 2sec, $\vec{v}(2) = 2 \times 2\hat{i} + (2)^2\hat{j}$

$$v = 4\hat{i} + 4\hat{j}$$

momentum, $\vec{p} = m\vec{v} = 2\left(4\hat{i} + 4\hat{j}\right)$

$$= 8\hat{i} + 8\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2t\hat{j}$$

$$(\vec{a})_{t=2} = (2\hat{i} + 4\hat{j})\text{m/s}^2$$

$$F = m\vec{a} = 2\left(2\hat{i} + 4\hat{j}\right) = \left(4\hat{i} + 8\hat{j}\right)N$$

**Q. 6. Why are porcelain objects wrapped in paper or straw before packing for transportation?**

**[NCERT Exemp. Q. 5.23, Page 23]**

**Ans.** Porcelain objects are brittle in nature and can crack with even small jerk on it.

In transportation, the vehicle may need to halt suddenly. To bring a fragile material, like porcelain object to a sudden halt means applying a large force and this is likely to damage the object. If it is wrapped up in straw, the object can travel some distance as the straw is soft before coming to a halt. The force needed to achieve this is less, thus reducing the possibility of damage.

**Q. 7. Why does a child feel more pain when she falls down on a hard cement floor, than when she falls down on the soft muddy ground in the garden?**

**[NCERT Exemp. Q. 5.23, Page 34]**

**Ans.** Effect of force, $F = ma$, if mass is constant for a system to decrease force,

The body of the child is brought to a sudden halt when she falls on a cement floor. The mud floor yields and the body travels some distance before it comes to rest, which takes some time. This means the force which brings the child to rest is less for the fall on a mud floor, as the change in momentum is brought about over a longer period.

**Q. 8. A woman throws an object of mass 500 g with a speed of 25 ms$^1$.**

**(a) What is the impulse imparted to the object?**

**(b) If the object hits a wall and rebounds with half the original speed, what is the change in momentum of the object?**

**[NCERT Exemp. Q. 5.24, Page 34]**

**Ans.** **(a)** mass of object, $m$ = 500 g = 0.5 kg

$u = 0$, $v = 25$ m/s

$$\text{Impulse} = F.dt = \frac{d\vec{p}}{dt}.dt = m\vec{v} - m\vec{u}$$

$$I = \Delta\vec{p} = 0.5(25 - 0) = 12.5 \text{ N-s}$$

**(b)** $m = 0.5$ kg, $u = +25$ m/s

$$v = \frac{-25}{2}\text{ m/s} = 12.5\text{ N} - \text{s}$$

$$\Delta p = 0.5\left(\frac{-25}{2} - 25\right)$$

$$= 0.5[-12.5 - 25] = 0.5 \times (-37.5)$$

$$\Delta p = -18.75 \text{ kg m/s or N-s}$$

Force is opposite to initial velocity of ball.

**Q. 9. Given the magnitude and direction of the net force acting on**

**(a) a drop of rain falling down with a constant speed.**

**(b) a cork of mass 10 g floating on water.**

**(c) a kite skillfully held stationary in the sky.**

**(d) a car moving with a constant velocity of 30 km/h on a rough road.**

**(e) a high-speed electron in space far from all material objects, and free of electric and magnetic fields.**

**[For Simplicity in numerical calculations, take $g$ = 10 ms$^{-2}$].** **[NCERT Ex. Q. 5.1, Page 109]**

**Ans.** **(a)** acceleration, $a = 0$, net force $F = ma = 0$

**(b)** Net force, $F = 0$, as its weight is being balanced by the upthrust.

**(c)** From Newton's first law, as kite is stationary,

So net force, $F = 0$.

**(d)** As, velocity $v$ constant

$\therefore$ acceleration, $a = 0$.

$F = ma = 0$.

**(e)** Net force, $F = 0$ because there is no fields.

**Q. 10. A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the new force on the pebble:**

**(a) during its upward motion.**

**(b) during its downward motion.**

**(c) at the highest point where it is momentarily at rest. Does your answer alter if the pebble was thrown at an angle of 45° with horizontal direction. Ignore air resistance.**

**[NCERT Ex. Q. 5.2, Page 109]**

**Ans.** **(a)** Force $F = mg = 0.05 \times 9.9$

$= 0.49 \text{ N} \downarrow$

**(b)** In this situation also

$F = mg = 0.5 \times 9.8$

$= 0.49 \text{ N} \downarrow$

**(c)** Again, $F = mg = 0.5 \times 9.8$

$= 0.49 \text{ N} \downarrow$

No, the velocity component has no effect on force.

**Q. 11. Give the magnitude and direction of the net force acting on a stone of mass 0.1 kg.**

**(a) just after it is dropped from the window of a stationary train.**

**(b) just after it is dropped from the window of a train running at a constant velocity of 36 km/h.**

**(c) just after it is dropped from the window of a train accelerating with 1 m/s².**

**(d) lying on the floor of a train which is accelerating with 1 ms⁻² the stone being at rest relative to the train. Neglect air resistance throughout.**

**[NCERT Ex. Q. 5.3, Page 109]**

**Ans.** **(a)** Net force, $F$ = mg.

= 0.1 × 10 = 1.0 N

Direction – vertically downwards.

**(b)** Speed of train is constant, so acceleration = 0.

No force acts on stone because of this motion.

∴ Force on stone is same i.e. 1.0 N.

**(c)** As, acceleration = 1 m/s²

∴ $F_1$ = ma = 0.1 × 1 = 0.1 N

Net force on stone –

F = mg = 0.1 × 10

= 1 N

It is vertically downwards.

**(d)** Stone is lying on the floor of train,

So, acceleration is same for stone and train.

∴ F = ma

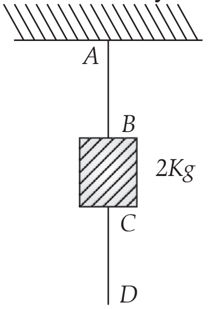
= 0.1 × 1

= 0.1 N

Along direction of motion of the train.

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. A mass of 2 kg is suspended with thread AB in figure. Thread CD of the same type is attached to the other end of 2 kg mass. Lower thread is pulled gradually, harder and harder in the downward direction so as to apply force on AB. Which of the threads will break and why?**




**[NCERT Exemp. Q. 5.26, Page 34]**

**Ans.** Thread AB will break.

AB, because force on the upper thread will be equal to sum of the weight of the body and the applied force.

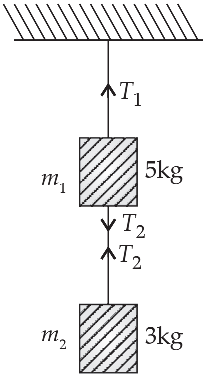
**Q. 2. In the above given problem if the lower thread is pulled with a jerk, what happens?**

**[NCERT Exemp. Q. 5.27, Page 34]**

**Ans.** Thread CD will break.

If the force is large and sudden, thread CD breaks because as CD is jerked, the pull is not transmitted to AB instantaneously (transmission depends on the elastic properties of the body). Therefore, before the mass moves, CD breaks.

**Q. 3. Two masses of 5 kg and 3 kg are suspended with the help of massless inextensible strings as shown in Fig. 5.6. Calculate $T_1$ and $T_2$ when whole system is going upwards with acceleration = 2 ms⁻² (use $g$ = 9.8 ms⁻²).**




**[NCERT Exemp. Q. 5.28, Page 34]**

**Ans.** $m_1 = 5$ kg, $m_2 = 3$ kg, $g = 9.8$ m/s²

Force on mass $m_1$,

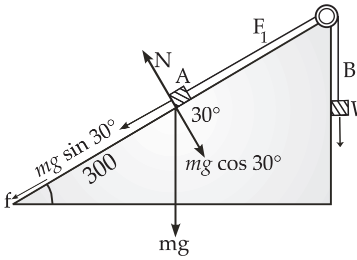
$$T_1 - T_2 - m_1 g = m_1 a$$
$$T_1 - T_2 - 5g = 5a$$
$$T_1 - T_2 = 59.0 \text{ N}$$

Force on mass $m_2$,

$$T_2 - m_2 g = m_2 a$$
$$T_2 = m_2(g + a)$$
$$= 3(9.8 + 2)$$
$$T_2 = 35.4N$$

$$T_1 = T_2 + 59.0$$
$$\text{or } T_1 = 35.4 + 59.0$$
$$= 94.4 \text{ N}$$

**Q. 4. Block A of weight 100 N rests on a frictionless inclined plane of slope angle 30° (Fig. 5.7). A flexible cord attached to A passes over a frictionless pulley and is connected to block B of weight W. Find the weight W for which the system is in equilibrium.**




**[NCERT Exemp. Q. 5.29, Page 35]**

**Ans.** $mg \sin 30^{\circ} = F$

$$\frac{1}{2}mg = F$$

$$F = \frac{1}{2} \times 100\ N$$

$$= 50\ N$$

For B (at rest), $W = F = 50\ N$.

**Q. 5. A 100 kg gun fires a ball of 1 kg horizontally from a cliff of height 500 m. It falls on the ground at a distance of 400 m from the bottom of the cliff. Find the recoil velocity of the gun. (acceleration due to gravity= 10 ms$^{-2}$)**

**[NCERT Exemp. Q. 5.31, Page 35]**

**Ans.** Applying 2$^{nd}$ kinematic equation of motion

$$s = ut + \frac{1}{2}at^2$$

If, $u = 0, g = 10\ \text{m/s}^2,\ s = 500$ m
or $t = 10$ sec.

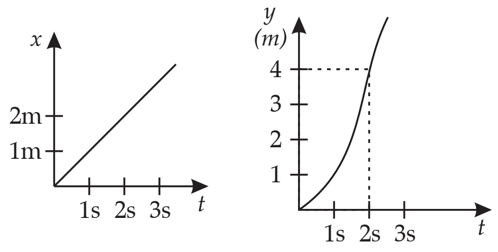
Horizontal range = $u \times 10$
$400 = u \times 10$ or $u = 40$ m/s

From law of conservation of momentum,

$$m_b u_b + m_G v_G = m_b v_b + m_G v_G$$
$$m_b \times 0 + m_G \times 0 = 1 \times 40 + 100 v_G$$
$$100 v_G = -40$$

or $v_G = 0{\cdot}4$ m/s

**Q. 6. Figure shows ($x$, $t$), ($y$, $t$) diagram of a particle moving in 2- dimensions.**




(a) (b)

**If the particle has a mass of 500 g. find the force (direction and magnitude) acting on the particle.**

**[NCERT Exemp. Q. 5.32, Page 35]**

**Ans.** From graph (a)

As, $v_x = \frac{dx}{dt} = \frac{2}{2} = 1$ m/s

$$a_x = \frac{dv_x}{dt} = 0$$

from (b), $y = t^2$

$$v_y = \frac{dy}{dt} = 2t$$

$$a_y = \frac{dv_y}{dt} = 2$$

$$F_y = ma_y$$

$$= 0.5 \times 2 = 1N \text{ (toward Y-axis)}$$

$$F_x = 0.5 \times 0 = 0 \text{ N}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{0^2 + 1^2}$$

$F = 1$ N (towards Y-axis)

**Q. 7. A person in an elevator accelerating upwards with an acceleration of 2 ms$^{-2}$, tosses a coin vertically upwards with a speed of 20 ms$^{-1}$. After how much time will the coin fall back into his hand? (g = 10 ms$^{-2}$)**

**[NCERT Exemp. Q. 5.33, Page 35]**

**Ans.** As, $v = u + at$

$$0 = 20 - 12\,t$$

$$t = \frac{20}{12} = \frac{5}{3} s$$

∵ Time of ascent=time of descent

∴ Total time after which coin falls back

$$t = \frac{2u}{g+a} = \frac{2 \times 20}{10+2} = \frac{40}{12} = \frac{10}{3} = 3.33\,s.$$

**Q. 8. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms$^{-1}$. How long does the body take to stop ?**

**[NCERT Exemp. Q. 5.5, Page 110]**

**Ans.** Force $\quad F = ma$

$$a = \frac{F}{m} = -\frac{50}{20}$$

$$= -2.5\ \text{ms}^{-2}$$

Using, $\quad v = u + at$

When, $\quad u = 15\ \text{ms}^{-1}, v = 0$

We get, $\quad 0 = 15 - 2.5 \times t$

$$t = \frac{15}{2.5}$$

or $\quad t = 6$ s.

**Q. 9. A constant force acting on a body of mass 3 kg changes its speed from 2 ms$^{-1}$ to 3.5 ms$^{-1}$ in 25 s. The direction of motion of the body remains unchanged. Find magnitude and direction of force. [NCERT Ex. Q. 5.6, Page 110]**

**Ans.** **Step 1.** Using $\quad v = u + at$

When $v = 3.5\ \text{ms}^{-1}$, $u = 2\text{ms}^{-1}$, $t = 25$ s

We get, $\quad 3.5 = 2 + a \times 25$

$$a = \frac{3.5 - 2}{25}$$

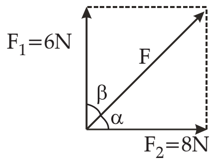
$= 0.06 \text{ ms}^{-2}$

**Step 2.** Force $= m \times a = 3 \times 0.06$

$= 0.18$ N acting in the direction of motion of the body.

**Q. 10. A body of mass 5 kg is acted upon by two perpendicular forces, 6 N and 8 N. Give the magnitude and direction of acceleration of the body. [NCERT Ex. Q. 5.7, Page 110]**

**Ans.**




Given situation is shown in figure

$m = 5$ kg

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{6^2 + 8^2} = 10 \text{ N}$$

∴ Acceleration of body,

$$a = \frac{F}{m} = \frac{10}{5} = 2 \text{ m/s}^2$$

direction of acceleration

$$\tan\alpha = \frac{F_1}{F_2} = \frac{6}{8} = \frac{3}{4}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{3}{4}\right) \text{ with respect to 8 N of force.}$$

$$\tan\beta = \frac{F_2}{F_1} = \frac{8}{6} = \frac{4}{3}$$

$$\therefore \beta = \tan^{-1}\left(\frac{4}{3}\right) \text{ with respect to 6 N of force.}$$

**Q. 11. The driver of a three wheeler moving with a speed of 36 kmh$^{-1}$ see a child standing in the middle of the road and brings his vehicle to rest in 4 s, just in time to save the child. What is the average retarding force on the vehicle ? The mass of the three wheeler is 400 kg and the mass of the driver is 65 kg. [NCERT Ex. Q. 5.8, Page 110]**

**Ans.** Using, $v = u + at$

When, $u = 36 \text{ kmh}^{-1}$

$= \frac{36 \times 1000}{3600}$ m / s

$= 10 \text{ ms}^{-1}$

$v = 0, t = 4$ s

We get, $0 = 10 + a \times 4$

or $a = -2.5 \text{ ms}^{-1}$

Mass of the driver + three wheeler,

M = 65 + 400

= 465 kg

Retarding force

F = 465 × 2.5 N

= 1162.5 N

**Q. 12. A rocket with a lift-off mass 20,000 kg is blasted upwards with net initial acceleration of 5 ms$^{-2}$, calculate initial thrust (force) of the blast. [NCERT Ex. Q. 5.9, Page 110]**

**Ans.** Here, total acceleration

$a' = (g + a)$

or $a' = (9.8 + 5)$

$= 14.8 \text{ ms}^{-2}$

∴ Thrust of blast of rocket

$= m \times a'$

$= 20000 \times 14.8$

$= 2.96 \times 10^5$ N

**Note**–Clearly the thrust should be such that it overcomes the force of gravity besides it an upward acceleration of 5 ms$^{-2}$.

## Long Answer Type Questions (5 marks each)

**Q. 1. A body of mass 0.40 kg moving initially with a constant speed of 10 m s subject to a constant force of 8.0 N directed towards the south for 30 s. Take the instant the force is applied to be $t = 0$, the position of the body at that time to be predict its position at $t = -5$ s, 25 s, 100 s. [NCERT Ex. Q. 5.10, Page 110]**

**Ans.** Mass of the body, $m = 0.40$ kg

Initial speed of the body, $u = 10$ m/s due north

Force acting on the body, $F = -8.0$ N

Acceleration produced in the body,

$$a = \frac{F}{m} = \frac{-8.0}{0.40} = -20 \text{ m / s}^2$$

**At $t = -5$ s**

Acceleration, $a' = 0$ and $u = 10$ m/s

$$s = ut + \frac{1}{2}a't^2$$

$= 10 \times (-5) = -50$ m

**At $t = 25$ s**

Acceleration, $a'' = -20 \text{ m/s}^2$ and $u = 10$ m/s

$$s' = ut' + \frac{1}{2}a''t^2$$

$$= 10 \times 25 + \frac{1}{2} \times (-20) \times (25)^2$$

$= 250 + 6250 = -6000\ m$

**At $t = 100$ s**

For $0 \le t \le 3s$

$a = -20\ m/s^2$

$u = 10$ m/s

$$s_1 = ut + \frac{1}{2}a''t^2$$

$$= 10 \times 30 + \frac{1}{0} \times (-20) \times (30)^2$$

$= 300 - 9000$

$= -8700$ m

For $30 \text{ s} < t \le 100s$

As per the first equation of motion, for $t = 30$ s, final velocity is given as: $v = u + at$

$= 10 + (-20) \times 30 = -590$ m / s

Velocity of the body after 30 s = –590 m/s

For motion between 30 s to 100 s, i.e., in 70 s:

$$s_2 = vt + \frac{1}{2}a''t^2$$

$= -590 \times 70 = -41300$ m

4 Total distance, $s'' = s_1 + s_2$

$= -8700 - 41300$

$= -50000$ m

**Q. 2. A truck starts from rest and accelerates uniformly at 2.0 ms$^{-2}$. At $t = 10$ s, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the (a) velocity, and (b) acceleration of the stone at $t = 11$ s? (Neglect air resistance.)**

**[NCERT Ex. Q. 5.11, Page 110]**

**Ans. (a)** Initial velocity of the truck, $u = 0$

Acceleration, $a = 2$ m/s$^2$

Time t = 10 s

As per the first equation of motion, final velocity is given as:

$v = u + at$

$= 0 + 2 \times 10 = 20$ m/s

The final velocity of the truck and hence, of the stone is 20 m/s.

At $t = 11$ s, the horizontal component ($v_x$) of velocity, in the absence of air resistance, remains unchanged, i.e.,

$v_x = 20$ m/s

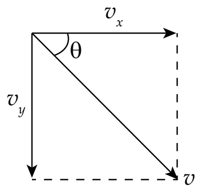
The vertical component ($v_y$)of velocity of the stone is given by the first equation of motion as:

$$v_y = u + a_y \Delta t$$

Where, $\Delta t = 11 - 10 = 1$ s and a = $g = 10$ m/s$^2$

$\therefore$ $v_y = 0 + 10 \times 1 = 10$ m/s

The resultant velocity ($v$) of the stone is given as:




$$v = \sqrt{v_x^2 + v_y^2}$$

$= \sqrt{20^2 + 10^2} = \sqrt{400 + 100}$

$= \sqrt{500} = 22.36$ m / s

Let θ be the angle made by the resultant velocity with the horizontal component of velocity, $v_x$

$$\therefore \tan\theta = \left(\frac{v_y}{v_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{10}{20}\right)$$

$= \tan^{-1}(0.5)$

$= 26.57°$

**(b)** When the stone is dropped from the truck, the horizontal force acting on it becomes zero. However, the stone continues to move under the influence of gravity. Hence, the acceleration of the stone is 10 m/s$^2$ and it acts vertically downward.

**Q. 3. A man of mass 70 kg stands on a weighing scale in a lift which is moving :**

**(a) upward with uniform speed of 10 m/s ?**

**(b) downward with a uniform acceleration of 5 m/s$^2$?**

**(c) upward with uniform acceleration of 5 m/s$^2$. What would be the readings on the scale in each case ?**

**(d) What would be the reading if the lift mechanism failed and it hurtled down freely under gravity ? What is its value ?**

**[NCERT Ex. Q. 5.13, Page 110]**

**Ans.** Given, mass of man ($m$) = 70 kg

In each case the weighing scale will read the reaction R, *i.e.* the apparent weight.

**(a)** As lift is moving upward with a uniform speed, therefore its acceleration $a = 0$

$\therefore$ Normal reaction $w$ = R = $mg$

$= 70 \times 10$ N

$= 700$ N

$w$ acts vertically downward and R acts vertically upwards.

$\therefore$ Reading on weighing scale

$= \frac{700}{10} = 70$ kg

**(b)** Acceleration of the lift $a = 5$ m/s$^2$ (↓)

$\therefore$ Normal reaction, R = $m(g - a)$

$= 70\,(10 - 5)$ N

$= 70 \times 5$ N $= 350$ N

$\therefore$ Reading on weighing scale

$= \frac{350 \text{ N}}{10 \text{ m/s}^2}$

$= 35$ kg.

**(c)** Acceleration of the lift $a = 5$ m/s$^2$ (↑)

$\therefore$ Normal reaction R = $m(g + a)$

$= 70\,(10 + 5)$

$= 1050$ N

$\therefore$ Reading on weighing scale

$= \frac{1050}{10 \text{ m/s}^2} = 105$ kg.

**(d)** Acceleration of the lift when it is falling freely under gravity

$a = g$ (↓)

$\therefore$ Normal reaction R

$= m(g - a)$

$= m(g - a) = 0$

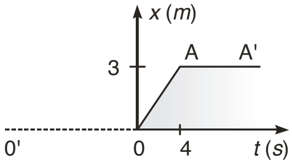
$\therefore$ Reading on weighing scale = 0

This is the state of weightlessness.

**Value :** Overweight gives rise to a number of problems.

**Q. 4. Figure below shows the position-time graph of a particle of mass 4 kg. What is the**

**(a) force on the particle for $t < 0$, $t > 4$ s, $0 < t < 4$ s?**
**(b) impulse at $t = 0$ and $t = 4$ s ? (Consider one dimensional motion only.)**




**[NCERT Ex. Q. 5.14, Page 110]**

**Ans.** Mass of particle = $m$ = 4 kg

**(a)** (1) **For $t < 0$,** the position-time graph is O'O which means displacement of the particles is zero, *i.e.*, the particle is at rest at the origin. Therefore, force on the particle must be zero,

*i.e.*, F = 0

(2) **For $t > 4$ s,** the position-time graph AA' is parallel to time axis. Therefore the particle remains at a distance of 3 m from the origin, *i.e.*, it is at rest. Hence, now force is acting on the particle for this interval, *i.e.*, F = 0.

(3) **For $0 < t < 4$ s,** the particle is changing its position continuously. The position-time graph (OA) during this interval represents uniform motion of the particle, *i.e.*, it moves with a constant speed and thus its acceleration is zero. Therefore

$$F = ma$$
$$= 0 \text{ during this interval}$$

**(b)** (1) **impulse at $t = 0$**

We know that,

$$\text{impulse} = \text{change in momentum}$$
$$= mv - mu$$
$$= m(v - u) \quad ...(i)$$

Hence, $u = 0$ as the particle is at rest before $t = 0$. After $t = 0$, the particle has a constant velocity $v$ where $v$ = slope of line OA = velocity at point O and point A

$$= \frac{3 \text{ m}}{4 \text{ s}} = 0.75 \text{ ms}^{-1} \quad ...(ii)$$

From equation (i) and (ii), we get

$$\text{impulse} = 4\,(0.75 - 0)$$
$$= 4 \times \frac{3}{4} = 3 \text{ kg ms}^{-1}$$

Equation (ii) **Impulse at $t = 4$ s**

Before $t = 4$ s, the particle has a constant velocity

$$u = \frac{3}{4} \text{ ms}^{-1}$$

After $t = 4$ s, the particle is at rest, *i.e.*, $v = 0$

∴ By definition of impulse, we get

$$\text{Impulse} = mv - mu$$
$$= m\,(v - u)$$
$$= 4\left(0 - \frac{3}{4}\right)$$
$$= -3 \text{ kg ms}^{-1}.$$

**Q. 5. Two bodies A and B of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string. A horizontal force F = 600 N is applied to (a) A, (b) B along the direction of string. What is the tension in the string in each case?**

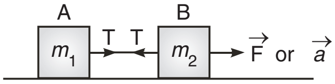
**[NCERT Ex. Q. 5.15, Page 110]**

**Ans.** Given : F = 600 N

Suppose $m_1$ = 10 kg

and $m_2$ = 20 kg

be the masses lying on a frictionless horizontal table.




Suppose T be the tension in the string and '$a$' be the acceleration of the system, in the direction of force applied.

(a) **If force is applied on the heavier mass.**

Then, equations of motion of A and B are

$$m_1 a = T \quad ..(i)$$
$$m_2 a = F - T \quad ...(ii)$$

Dividing equation (ii) by equation (i), we get

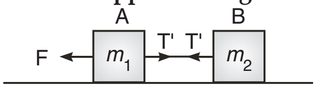
$$\frac{m_2}{m_1} = \frac{F - T}{T} = \frac{F}{T} - 1$$

or $$\frac{20}{10} = \frac{F}{T} - 1$$

or $$\frac{F}{T} = 2 + 1 = 3$$

or $$T = \frac{F}{3} = \frac{600}{3}$$
$$= 200 \text{ N}$$

(b) **If the force is applied on lighter mass :**




Suppose T be the tension in the string in this case

Then, equations of motion of A and B are

$$F - T' = m_1 a \quad ...(iii)$$

and $$T' = m_2 a \quad ...(iv)$$

Equation (iii) and (iv) gives

$$\frac{F - T'}{T'} = \frac{m_1 a}{m_2 a}$$

or $$\frac{F}{T} - 1 = \frac{m_1}{m_2} = \frac{10}{20} = \frac{1}{2}$$

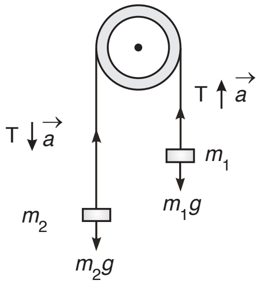
or $$\frac{F}{T'} = 1 + \frac{1}{2} = \frac{2}{3}$$

or $$T' = \frac{2}{3}F = \frac{2}{3} \times 600$$
$$= 400 \text{ N}$$

**Q. 6. Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses and the tension in the string when the masses are released.**

**[NCERT Ex. Q. 5.16, Page 111]**

**Ans.** Suppose $m_1$ and $m_2$ be the masses suspended at the ends of a light inextensible string passing over the pulley.




$\therefore \quad m_1 = 8$ kg, $m_2 = 12$ kg

Suppose, T = tension in the string

$a$ = common acceleration with which $m_1$ moves upward and $m_2$ moves downward

The equations of motion of $m_1$ and $m_2$ are given by

$$T - m_1g = m_1a \quad ..(i)$$

and $$m_2g - T = m_2a \quad ...(ii)$$

Adding equations (i) and (ii),

$$(m_2 - m_1)\,g = (m_1 + m_2)\,a$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} \quad ...(iii)$$

$\therefore$ From equations (i) and (iii),

$$T = m_1\,g + m_1\frac{(m_2 - m_1)g}{m_1 + m_2}$$

or $$T = \frac{m_1g}{m_1 + m_2}(m_1 + m_2 + m_2 - m_1)$$

or $$T = \frac{2m_1m_2}{m_1 + m_2}g \quad ...(iv)$$

Putting $m_1 = 8$ kg and $m_2 = 12$ kg and $g = 10$ ms$^{-2}$, in equation (iii) and (iv), we get

$$a = \frac{(12-8)}{(8+12)} \times 10$$

or $$= \frac{4}{20} \times 10 = 2\text{ms}^{-2}$$

From eq. (i) $\quad T = m_1a + m_1g$

$$= 8 \times 2 + 8 \times 10 = 96 \text{ N}$$

**Q. 7. A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei, the products must be emitted in opposite directions.**

**[NCERT Ex. Q. 5.17, Page 111]**

**Ans.** Suppose $m$ = mass of the nucleus at rest

$u$ = its initial velocity = 0 as it is at rest

Also suppose $m_1$, $m_2$ be the masses of the two smaller nuclei also called product nuclei and $v_1$, $v_2$ be their respective velocities.

When $\vec{p_i}$ and $\vec{p_f}$ be the initial and final momentum of the nucleus and the two nuclei respectively, then

$$\vec{p_i} = m\,\vec{u} = 0 \quad ...(i)$$

and $$\vec{p_f} = m_1\vec{v}_1 + m_2\vec{v}_2 \quad ...(ii)$$

Now according to the law of conservation of linear momentum, we know that

$$\vec{p_i} = \vec{p_f}$$

$$\Rightarrow \quad 0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\Rightarrow \quad m_2\vec{v}_2 = -m_1\vec{v}_1$$

$$\Rightarrow \quad \vec{v}_2 = -\frac{m_1\vec{v}_1}{m_2} \quad ...(iii)$$

The negative sign in equation (iii) show that $\vec{v}_1$ and $\vec{v}_2$ are in opposite directions, *i.e.*, the two smaller nuclei are emitted in opposite direction.

**Q. 8. Two billiard balls each of mass 0.05 kg moving in opposite directions with speed 6 ms$^{-1}$ collide and rebound with the same speed. What is the impulse imparted to each ball due to the other ?**

**[NCERT Ex. Q. 5.18, Page 111]**

**Ans.** Given : mass of each ball, $m$ = 0.05 kg

speed of each ball, $v$ = 6 ms$^{-1}$

$\therefore$ Initial momentum of each ball

$= m\,\vec{v}$

= (0.05) (6) kg ms$^{-1}$

= 0.30 kg ms$^{-1}$

As after collision, the direction of velocity of each ball is reversed on rebounding.

$\therefore$ Final momentum of each ball will be

$\vec{p}_f = m\,(-\,\vec{v}\,)$

= 0.05 (– 6)

= – 0.30 kg ms$^{-1}$

$\therefore$ Impulse imparted to each ball

$= p_b - p_i$

change in momentum of each ball

= – 0.30 – (0.30)

= – 0.60 kg ms$^{-1}$

or magnitude of impulse imparted by one ball due to collision with the other – 0.6 kg ms$^{-1}$. The two impulses are opposite in direction.

**Q. 9. A shell of mass 0.02 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 ms$^{-1}$, what is the recoil speed of the gun ?**

**[NCERT Ex. Q. 5.19, Page 111]**

**Ans. Given :** mass of gun, $m_g$ = 100 kg, mass of shell $m_b$ = 0.02 kg, velocity of shell, $v_b$ = 80 ms$^{-1}$

Total initial momentum = 0

($\because$ the gun and shell have no initial velocity as they are initially at rest)

Total final momentum = momentum of bullet + momentum of gun

$= m_bv_b + m_gv_g$

Applying law of conservation of momentum,

$$0 = m_b v_b + m_g v_g$$

*i.e.,* $m_b v_b = - m_g v_g$

*i.e.,* $0.02 \times 80 = -100 \times v_g$

or $v_g = -\dfrac{0.02 \times 80}{100}$

$= -0.016\ \text{ms}^{-1}$

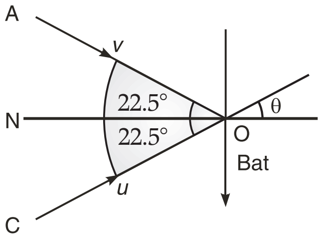
Hence, recoil speed of the gun is

$= 0.016\ \text{ms}^{-1}$.

**Q. 10. A batsman deflects a ball by an angle of 45° without changing its initial speed, which is equal to 54 kmh$^{-1}$. What is the impulse imparted to the ball ? (Mass of the ball is 0.15 kg.)**

**[NCERT Ex. Q. 5.20, Page 111]**

**Ans.** The ball struck by the bat is deflected back such that total angle is 45°.




Now, initial momentum of ball $= mu \cos \theta$

$= \dfrac{0.15 \times 54 \times 1000 \times \cos 22.5}{3600}$

$= 0.15 \times 15 \times 0.9239$ along NO

Final momentum of ball $= mu\cos\theta$ along ON

Impulse = change in momentum

$= mu\cos\theta - (-mu\cos\theta)$

$= 2\,mu\cos\theta$

$= 2 \times 0.15 \times 15 \times 0.9239$

*i.e.,* Impulse $= 4.16$ kg ms$^{-1}$

**Q. 11. There are three forces $F_1$, $F_2$ and $F_3$ acting on a body, all acting on a point P on the body. The body is found to move with uniform speed.**

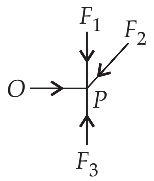
**(a) Show that the forces are coplanar.**

**(b) Show that the torque acting on the body about any point due to these three forces is zero.**

**[NCERT Exemp. Q. 5.34, Page 35]**

**Ans. (a)** As the body is moving with uniform speed after the action of three forces, $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ on a point on body.

Since the body is moving with no acceleration, the sum of the forces is zero $F_1 + F_2 + F_3 = 0$. Let $F_1$, $F_2$, $F_3$ be the three forces passing through a point. Let $F_1$ and $F_2$ be in the plane A (one can always draw a plane having two intersecting lines such that the two lines lie on the plane). Then $F_1 + F_2$ must be in the plane A. Since $F_3 = -(F_1 + F_2)$. $F_3$ is also in the plane A.




**(b)** Consider the torque of the forces about P. Since all the forces pass through P, the torque is zero. Now consider torque about another point O. Then torque about O is

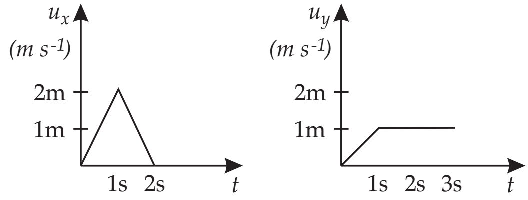
Torque = OP × $(F_1 + F_2 + F_3)$

As a resultant of $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ is zero,

Since $F_1 + F_2 + F_3 = 0$,

Torque $= r \times \vec{F} = 0$

**Q. 12. Figure shows $(v_x, t)$, and $(v_y, t)$ diagrams for a body of unit mass. Find the force as function of time.**




**[NCERT Exemp. Q. 5.36, Page 36]**

**Ans.** $v_x = 2t \qquad 0 < t \le 1$

$a_x = 2(2 - t) \qquad 1 < t < 2$

$=0 \qquad 2 < t$

$F_x = ma = 1 \times 2$

$F_x = 2; \quad 0 < t < 1$

$= -2; \quad 1\text{s} < t < 2\text{s}$

$= 0; \quad 2\text{s} < t$

From figure (b),

$a_y = \dfrac{1}{1} = 1\ \text{m/s}^2 \quad 0 < t < 1$

$F_y = ma = 1 \times 1 = 1$ unit $0 < t < 1$

$a_y = 0$ for $1 < t$

$\vec{F_y} = 1 \times 0 = 0$ units for $1 < t < 2s$

$\vec{F} = \vec{F_x}\hat{i} + \vec{F_y}\hat{j}$

$\vec{F} = 2\hat{i} + 1\hat{j}$ for $0 < t < 1s$

$\vec{F} = -2\hat{i} + 0\hat{j}$ for $1 < t < 2s$

For more than 2 sec $\left.\begin{array}{l} a_y = 0 \\ a_x = 0 \end{array}\right\} \because \vec{F} = 0$

**Q. 13. A cricket bowler releases the ball in two different ways**

**(a) giving it only horizontal velocity , and**

**(b) giving it horizontal velocity and a small downward velocity.**

**The speed $v_s$ at the time of release is the same. Both are released at a height H from the ground. Which one will have greater speed when the ball hits the ground? Neglect air resistance.**

**[NCERT Exemp. Q. 5.39, Page 37]**

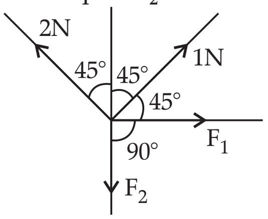
**Ans.** For (a) $\dfrac{1}{2}v_z^2 = gH \qquad v_z = \sqrt{2gH}$

Speed at ground $= \sqrt{v_s^2 + v_z^2} = \sqrt{v_s^2 + 2gH}$

For (b) also $\left[\frac{1}{2}mv_s^2 + mgH\right]$ is the total energy of the ball when it hits the ground.

So the speed would be the same for both (a) and (b).

**Q. 14. There are four forces acting at a point $P$ produced by strings as shown in Fig. 5.11, which is at rest. Find the forces $F_1$ and $F_2$.**




**[NCERT Exemp. Q. 5.40, Page 37]**

**Ans.** Particle is at rest, $a = 0$.

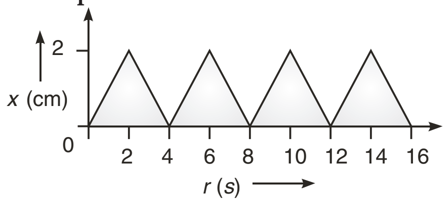
So, resultant force due to all forces will be zero.

$$F_2 = \frac{F_3 + F_4}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}}N$$

$$F_1 + \frac{F_3}{\sqrt{2}} = \frac{F_4}{\sqrt{2}}$$

$$F_1 = \frac{F_4 - F_3}{\sqrt{2}} = \frac{1}{\sqrt{2}}N$$

**Q. 15. The figure given below shows position-time graph of a particle of mass 0.04 kg. Suggest a suitable physical context for the motion. What is the time between two consecutive impulses received by the particle? What is the magnitude of each impulse ?**




**[NCERT Ad. Ex. Q. 5.24, Page 111]**

**Ans. Physical context :** Given graph can be a graph of a ball rebounding between two walls situated at position 0 cm and 2 cm. The ball is rebounding from one wall to the another time and again every 2 s with uniform velocity.

**Note :** Negative sign indicates the universal direction of motion. We can visualize the balls getting rebounded repeatedly on striking against the wall. The linear momentum changes after every collision.

**Impulse :** Here,

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$= \frac{2}{100 \times 2}$$

$$= 0.01 \text{ ms}^{-1}$$

Initial momentum $= mu = 0.04 \times 0.01$

$= 4 \times 10^{-4}$ kg ms$^{-1}$

Final momentum $= mv = 0.04 \times (-0.01)$

$= -4 \times 10^{-4}$ kg ms$^{-1}$

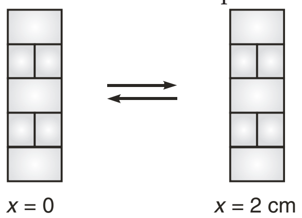
Magnitude of impulse

= change in linear momentum

$= (4 \times 10^{-4}) - (-4 \times 10^{-4})$

$= 8 \times 10^{-4}$ kg ms$^{-1}$

Time between two consecutive impulses is 2 s, *i.e.*, the ball receives an impulse every 2 s.




**Q. 16. A helicopter of mass 1000 kg rises with a vertical acceleration of $15 \text{ ms}^{-2}$. The total mass of the crew and passengers is 300 kg. Give the magnitude and direction of the ($g = 10 \text{ m s}^{-2}$)**

**(a) force on the floor of the helicopter by the crew and passengers.**

**(b) action of the rotor of the helicopter on the surrounding air.**

**(c) force on the helicopter due to the surrounding air.** **[NCERT Ad. Ex. Q. 5.27, Page 112]**

**Ans. (a)** Force on floor of helicopter by crew and passengers = (15 + 10)/300 m/s$^2$ $\times$ 1300 kg

= 32500 N.

**(b)** action of the air on the system is upwards. The action of the rotor on the surrounding air is 32500 N downwards.

**(c)** Force on the helicopter due to the air = 32500 N upwards.

**Q. 17. A stream of water flowing horizontally with a speed of 15 ms$^{-1}$ rushes out of a tube of cross-sectional area 10$^{-2}$ m$^2$ and hits at a vertical wall nearby. What is the force exerted on the wall by the impact of water assuming it does not rebound ? Density of water = 1000 kg m$^{-3}$.**

**[NCERT Ad. Ex. Q. 5.28, Page 112]**

**Ans.** Volume of water striking the wall per second,

$= v \times A$

$= 15 \times 10^{-2}$

$= 0.15 \text{ m}^3 \text{ s}^{-1}$

Mass of water hitting wall per second

$= \rho \times v \times A$

$= 1000 \times 0.15$

$= 150 \text{ kg s}^{-1}$

Initial momentum of water per second

$= 150 \times 15$

$= 2250 \text{ kg ms}^{-1}$

Final momentum of water per second = 0

($\because$ there is no rebounding of water)

Magnitude of force

= Rate of change in momentum = 2250 N

**Q. 18. Ten one-rupee coins are put on top of each other on a table. Each coin has a mass *m*. Give the magnitude and direction of the**

**(a) force on the 7$^{th}$ coin (counted from the bottom) due to all the coins on its top,**

**(b) the force on the 7$^{th}$ coin by the eighth coin,**

**(c) the reaction of the 6th coin on the 7$^{th}$ coin**

**[NCERT Ad. Ex. Q. 5.29, Page 112]**

**Ans.** Force on the seventh coin is exerted by the weight of the three coins on its to p.

Weight of one coin = $mg$

Weight of three coins = $3mg$

Hence, the force exerted on the 7$^{th}$ coin by the three coins on its top is $3mg$. This force acts vertically downward.

Force on the seventh coin by the eighth coin is because of the weight of the eighth coin and the other two coins (ninth and tenth) on its top.

Weight of the eighth coin = $mg$. Weight of the ninth coin = $mg$ Weight of the tenth coin = $mg$

Total weight of these three coins = $3mg$

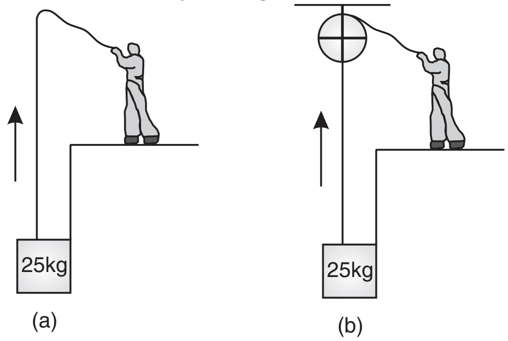
Hence, the force exerted on the 7$^{th}$ coin by the eighth coin is $3mg$. This force acts vertically downward.

The 6$^{th}$ coin experiences a downward force because of the weight of the four coins (7$^{th}$, 8$^{th}$, 9$^{th}$, and 10$^{th}$) on its top.

Therefore, the total downward force experienced by the 6$^{th}$ coin is $4mg$.

As per Newton's third law of motion, the 6$^{th}$ coin will produce an equal reaction force on the 7$^{th}$ coin, but in the opposite direction. Hence, the reaction force of the 6$^{th}$ coin on the 7$^{th}$ coin is of magnitude $4mg$. This force acts in the upward direction.

**Q. 19. A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in figure. What is the action on the floor by the man in the two cases ? If the floor yields a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding ? Find out its value.**




**[NCERT Ad. Ex. Q. 5.32, Page 112]**

**Ans.** Mass of block, $m = 25$ kg

Mass of the man, M = 50 kg

Force required to lift the block

$F$ = weight of the block

$F = mg$

$= 25 \times 10$

$= 250$ N

Weight of the man

$W = Mg$

$= 50 \times 10$

$= 500$ N

**Case (a) :** If the block is raised by the man as shown in Fig. (a) then force is applied by the man is in the upward direction due to which apparent weight of the man increases. Therefore action on the floor by the man

$= F + W$

$= 250 + 500$

$= 750$ N

**Case (b) :** If the block is raised by the man as shown in Fig. (b) then, force is applied by the man in the downward direction due to which apparent weight of the man decreases. Therefore, action on the floor by the man

$= mg - F$

$= 500 - 250 = 250$ N

The floor yields a normal force of 700 N. Action on the floor in case (a) exceeds 700 N and less than 700 N in case (b). Therefore, mode (b) has to be adopted by the man to lift the block.

**Value :** To perform our day to day tasks without any problem, physical fitness is much necessary.

**Q. 20. A monkey of mass 40 kg climbs on a rope (figure) which can stand a maximum tension of 600 N. In which of the following cases will the rope break: the monkey**

**(a) climbs up with an acceleration of 6 ms$^{-2}$**

**(b) climbs down with an acceleration of 4 ms$^{-2}$**

**(c) climbs up with a uniform speed of 5 ms$^{-1}$**

**(d) falls down the rope nearly freely under gravity? (Ignore the mass of the rope).**

**[NCERT Ad. Ex. Q. 5.33, Page 113]**

**Ans. Case (a)**

Mass of the monkey, $m = 40$ kg

Acceleration due to gravity, $g = 10$ m/s$^2$

Maximum tension that the rope can bear,

$T_{max} = 600$ N

Acceleration of the monkey, a = 6 m/s$^2$ upward

Using Newton's second law of motion, we can write the equation of motion as:

$T - mg = ma$

$\therefore\ T = m(g + a)$

$= 40(10 + 6)$

$= 640$ N

Since $T > T_{max}$, the rope will break in this case.

**Case (b)**

Acceleration of the monkey, $a = 4\ m/s^2$ downward

Using Newton's second law of motion, we can write the equation of motion as:

$mg - T = ma$

$\therefore\ T = m(g - a)$

$= 40(10 - 4)$

$= 240\ N$

Since $T < T_{max}$, the rope will not break in this case.

**Case (c)**

The monkey is climbing with a uniform speed of 5 m/s. Therefore, its acceleration is zero, i.e., $a = 0$.

Using Newton's second law of motion, we can write the equation of motion as:

$T - mg = ma$

$T - mg = 0$

$\therefore\ T = mg$

$= 40 \times 10$

$= 400$ N

Since $T < T_{max,}$ the rope will not break in this case.

**Case (d)**

When the monkey falls freely under gravity, its will acceleration become equal to the acceleration due to gravity, i.e., $a = g$

Using Newton's second law of motion, we can write the equation of motion as:

$mg - T = mg$

$\therefore\ T = m(g - g) = 0$

Since $T < T_{max}$, the rope will not break in this case.

**Q. 21. A helicopter of mass 2000 kg rises with the vertical acceleration of 15 ms–2. The total mass of .. and passengers is 500 kg. Give the magnitude and direction of the (g = 10 ms–2)**

**(a) force on the floor of the helicopter by the crew and passengers.**

**(b) action of the rotor of the helicopter on the surrounding air.**

**(c) force on the helicopter due to the surrounding air.**

**[NCERT Exemp. Q. 5.42, Page 37]**

**Ans. (a)** Force on the floor of the helicopter

F= m(g + a) = 500 (10 + 15) = 500 × 25

$= 1.25 \times 10^4$ N

**(b)** Force on the surroundings

R = (M + $a$) ($g$ + $a$) = (2000 + 500) (10 + 15)

$= 2500 \times 25 = 6.25 \times 10^4$ N

**(c)** Force on helicopter due to the air

$F = R = 6.25 \times 10^4$ N

# TOPIC-2
## Friction & Dynamics of Circular Motion

## Quick Review

- **Equilibrium of Concurrent forces :** Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero.

**(i) Conditions of equilibrium of concurrent forces :** If $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, ...... are the concurrent forces acting at the same point, then the point will be in equilibrium if

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ........ = \vec{0}$$

- **Friction :** Friction is an opposing force which comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body. Frictional forces arise due to interlocking of irregularities present on the two surfaces which are in contact. From modern view, the frictional force arises due to strong atomic or molecular force of attraction between the two surfaces at the points of actual contact.

It is of two types :

**(a)** **Internal friction :** It arises on account of relative motion between every two layers of liquid. It is also known as viscosity of liquid.

**(b)** **External friction :** It arises when two bodies in contact with each other try to move or there is an actual relative motion between the two. It is also known as contact friction. Further it is of four types:

**(i)** **Static friction** is an opposing force which comes into play when one body tends to move over the surface of another body, but the actual motion has yet not started. It is a self-adjusting force.

**(ii)** **Limiting friction** is the maximum value of static friction. Limiting friction is the maximum opposing force that comes into play of one body is just at the average of moving over the surface of another body.

**(iii)** **Dynamic or kinetic friction** is the opposing force that acts between two surfaces in contact when one body is actually moving over the surface of another body.

**(iv)** **Rolling friction** is an opposing force that comes into play when one body is actually rolling over the surface of another body.

➢ **Laws of limiting friction :**

**(a)** The magnitude of the force of limiting friction (F) between two bodies in contact is directly proportional to the normal reaction (R) between them, *i.e.*, $F \propto R$.

**(b)** The direction of the force of limiting friction is always opposite to the direction in which one body is at the average of moving over the other.

**(c)** The force of limiting friction is independent of the apparent area of contact as long as normal reaction between the two bodies in contact remains the same.

**(d)** The force of limiting friction between any two bodies in contact depends on the nature of the surfaces in contact.

➢ **Coefficient of limiting friction** between two surfaces in contact ($\mu$) is defined as the ratio of the force of limiting friction (F) and normal reaction (R) between them, *i.e.*,

$$\mu = F/R$$

The value of $\mu$ depends upon the nature of material and state of polish of two surfaces in contact.

Coefficient of kinetic friction, $$\mu_k = \frac{F_k}{R}$$

where, $F_k$ is the force of kinetic friction and R is the normal reaction between the two surfaces in contact.

➢ **Advantages & Disadvantages of friction**

**(a)** **Advantages :**

**(i)** Walking and Brakes of vehicles will not be possible without friction.

**(ii)** No two bodies stick together without friction.

**(iii)** Writing on black board or paper will also be not possible.

**(iv)** Adhesives will lose their purpose.

**(b)** **Disadvantages :**

**(i)** Friction always opposes the relative motion between any two bodies unnecessary expense of energy. It means output is always less than input.

**(ii)** Friction causes wear and tear of the parts of machinery in contact. Thus their life time reduces.

**(iii)** Frictional forces results in the production of heat, which causes damage to the machinery.

➢ **Ways of Reducing Friction.**

**(i)** By polishing

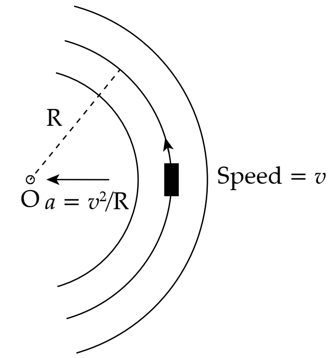
**(ii)** By lubrication

**(iii)** By proper selection of materials

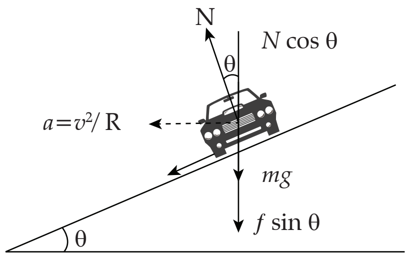
**(iv)** By streamlining

**(v)** By using ball bearings.

- **Motion of car on a road.**




$$V_{max} = \sqrt{\mu_s R_g}$$




$$V_{max} = \sqrt{\left(rg.\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$$

For $\mu_s = 0$ i.e., for a frictionless banked road

$$V_{max} = \sqrt{rg \tan\theta}$$

where, $\mu_s$ is the coefficient of friction and R is the radius of the circle.

## Know the Terms

- **Concurrent forces :** The forces which are acting at a point.
- **Contact forces** arises due to contact with some other object: solid or fluid.
- **Non-contact forces** are forces experienced by an object without actual contact. e.g., force of gravity of earth.
- **Centripetal force** is the force required to move a body uniformly in a circle. This force acts along the radius and towards the centre of the circle.
- **Angle of friction** is the angle in which the direction of resultant of the force of friction and normal reaction makes with the direction of normal reaction. It is represented by θ.

$$\tan \theta = \mu$$

- **Angle of repose** is the maximum angle of inclination of a plane with the horizontal, at which the body placed on the plane is just in limiting equilibrium.
- **Centrifugal force** is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.

## Know the Formulae

- **Angle of friction,** $\tan \theta = \mu.$
- **Angle of repose,** $\mu = \frac{F}{R} = \tan \theta.$
- **Centripetal force** $= \frac{mv^2}{r} = mr\omega^2.$

$= mr(2\pi\nu)^2.$

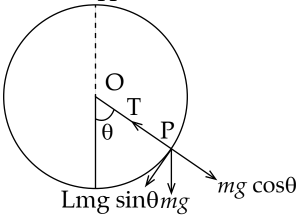
- $\tan \theta = v^2/rg.$

➢ $$\tan\theta = \frac{h}{\sqrt{b^2 - h^2}} = v^2/rg.$$

$h$ = height between outer edge and inner edge
$b$ = breadth of road

➢ $$v_{max} = \left[\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}\right]^{½}$$

➢ **At any position of angular displacement θ along a vertical circle**




$$T = \frac{mv^2}{r} + mg\cos\theta.$$

➢ **At lowest point of vertical circle,** $\theta = 0°$; $T_L = \frac{mv_L{}^2}{r} + mg.$

➢ **At highest point of vertical circle,** $\theta = 180°$

$$T_H = \frac{mv_H{}^2}{r} - mg.$$

➢ **Minimum velocity at highest point, at H** $v_H = \sqrt{gr}$.

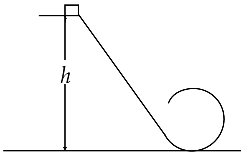
➢ **Minimum velocity at lowest point, at L** $v_L = \sqrt{5gr}$.

➢ **When,** $\theta = 90°$

$$v = \sqrt{3gr}.$$

➢ **Height through which a body should fall for looping the vertical loop**

$h = 5r/2.$




## Know the Links

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☞ *https://byjus.com*
☞ *www.vedantu.com*

## MCQ/Fillups/True or False **(1 mark each)**

### (A) Multiple Choice Questions

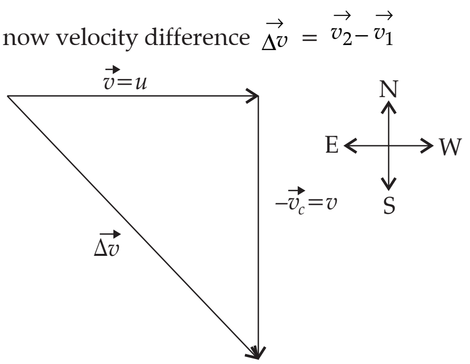
**Q. 1. A hockey player is moving northward and suddenly turns westward with the same speed to avoid an opponent. The force that acts on the player is**

**(a) Frictional force along westward.**
**(b) Muscle force along southward.**
**(c) Frictional force along south-west.**
**(d) Muscle force along south-west.**

**[NCERT Exemp. Q. 5.6, Page 27]**

**Ans. Correct option:** (c)

***Explanation:*** Suppose initial velocity of player $\vec{v_1}$ = $v$ northward and final velocity of player $\vec{v_2}$ = $v$ = westward now velocity difference $\vec{\Delta v} = \vec{v_2} - \vec{v_1}$




Here velocity difference (acceleration) direction is south-west. So Frictional force along south-west.

**Q. 2. In Fig., the co-efficient of friction between the floor and the body B is 0.1. The co-efficient of friction between the bodies B and A is 0.2. A force F is applied as shown on B. The mass of A is m/2 and of B is m. Which of the following statements are true?**

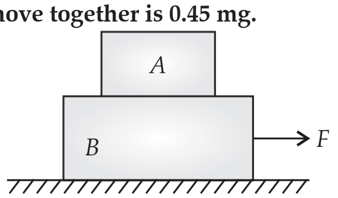
**(a) The bodies will move together if $F$ = 0.25 mg.**

**(b) The body A will slip with respect to B if $F$ = 0.5 mg.**

**(c) The bodies will move together if $F$ = 0.5 mg.**

**(d) The bodies will be at rest if $F$ = 0.1 mg.**

**(e) The maximum value of $F$ for which the two bodies will move together is 0.45 mg.**




**[NCERT Exemp. Q. 5.11, Page 31]**

**Ans. Correct option:** (a), (b), (c) and (d)

***Explanation:***

$$N_1 = \frac{\text{mg}}{2},\ N_2 = N_1 + \text{mg} = \frac{3\text{ mg}}{2}$$

$$F_1 = \mu_1 N = 0.2\left(\frac{\text{mg}}{2}\right) = 0.1\text{ mg}$$

$$F_2 = \mu_2 N_2 = 0.1\left(\frac{3\text{ mg}}{2}\right) = 0.15\text{ mg}$$

**(a)** Both the bodies will move together, if $F = F_1 + F_2$ = 0.1 mg + 0.15 mg = 0.25 mg

**(b)** If, $F$ = 0.5 mg then $F_1 = F - F_2$ = 0.5 mg - 0.15 mg
$F_1$ = 0.35 mg $\because F_1 > 0.1$ mg, the body will slip.

**(c)** When $F$ = 0.5 mg, body A will slip with respect to B. Hence both the bodies cannot move together all the time.

**(d)** Minimum force required to move bodies together $F$ = 0.25 mg

**(e)** The maximum force($F_{max}$) applied on block B to move both blocks together with acceleration a, such that the force transmitted to A is not more than $F_1$.

$\therefore$ for block A, $F_1 = \left(\frac{m}{2}\right)a \Rightarrow 0.1\text{ mg} = \frac{m}{2}a \Rightarrow a = 0.2\text{ g}$

for block B, $F_{max} - F_1 - F_2$ = ma

$\Rightarrow F_{max} - 0.1\text{ mg} - 0.15\text{ mg} = 0.2\text{ mg}$

$\Rightarrow F_{max} = 0.45\text{ mg}$

**Q. 3. Mass $m_1$ moves on a slope making an angle θ with the horizontal and is attached to mass $m_2$ by a string passing over a frictionless pulley as shown in Fig. 5.2. The co-efficient of friction between $m_1$ and the sloping surface is $\mu$.**

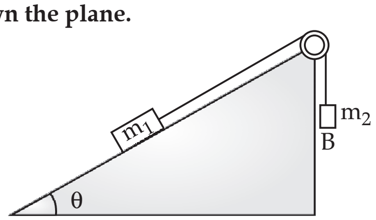
**Which of the following statements are true ?**

**(a) If $m_2 > m_1 \sin\theta$, the body will move up the plane.**

**(b) If $m_2 > m_1 (\sin\theta + \mu\cos\theta)$, the body will move up the plane.**

**(c) If $m_2 < m_1 (\sin\theta + \mu\cos\theta)$, the body will move up the plane.**

**(d) If $m_2 < m_1 (\sin\theta - \mu\cos\theta)$, the body will move down the plane.**




**[NCERT Exemp. Q. 5.12, Page 32]**

**Ans. Correct option:** (b) and (c)

***Explanation:***

Normal by inclined plane on $m_1$,

$N_1 = m_1 g\cos\theta$ and,

$\mu$ = coefficient of friction

Friction force, $F = \mu N_1 \Rightarrow \mu m_1 g\cos\theta$

$\because$ friction opposes the relative $F$ motion, so when block is moving up $F$ will be down along the inclined plane and when it is moving downward $f$ will be up along the plane.

If mass $m_1$ is moving up along the inclined plane, then

$$m_2 g > (m_1 g\sin\theta + \mu m_1 g\cos\theta)$$
$$\Rightarrow m_2 > m_1(\sin\theta + \mu\cos\theta)$$

If mass $m_1$ is moving down along the inclined plane, then,

$$m_1 g\sin\theta > (\mu m_1 g\cos\theta + m_2 g)$$
$$\text{or } m_1\sin\theta > (\mu m_1\cos\theta + m_2)$$
$$\text{or } m_1(\sin\theta - \mu\cos\theta) > m_2$$
$$\text{or } m_2 < m_1(\sin\theta - \mu\cos\theta)$$

**Q. 4. In Fig., a body A of mass m slides on plane inclined at angle $\theta_1$ to the horizontal and $\mu_1$ is the coefficient of friction between A and the plane. A is connected by a light string passing over a frictionless pulley to another body B, also of mass m, sliding on a frictionless plane inclined at angle $\theta_2$ to the horizontal. Which of the following statements are true?**

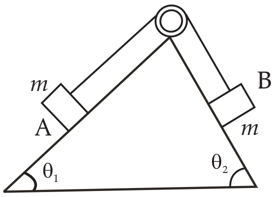



**Fig. 5.3**

**(a) A will never move up the plane.**

**(b) A will just start moving up the plane when $\mu = \dfrac{\sin\theta_2 - \sin\theta_1}{\cos\theta_1}$**

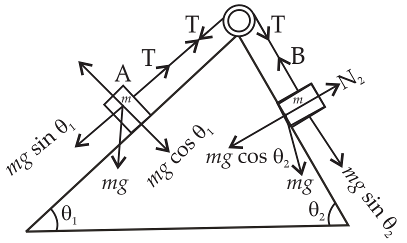
**(c) For A to move up the plane, $\theta_2$ must always be greater than $\theta_1$.**

**(d) B will always slide down with constant speed.**

**[NCERT Exemp. Q. 5.13, Page 32]**

**Ans. Correct option:** (b) and (c)

*Explanation:*




From figure block A can move up the inclined plane depending on values of $\theta_1, \theta_2$ and $\mu$.

Tension in the string

$$T = mg\sin\theta_2, N_1 = mg\cos\theta_1, f = \mu N_1 = \mu mg\cos\theta_1$$

Suppose block A is just move up the inclined plane,

$$T = f + mg\sin\theta_1$$

or $mg\sin\theta_2 = \mu mg\cos\theta_1 + mg\sin\theta_1$

or $\sin\theta_2 - \sin\theta_1 = \mu\cos\theta_1$

or $\mu = \dfrac{\sin\theta_2 - \sin\theta_1}{\cos\theta_1}$

$\because \mu > 0, \sin\theta_2 > \sin\theta_1 \Rightarrow \theta_2 > \theta_1$

$\therefore$ (b) & (c) are correct

**Q. 5. One end of a string of length $l$ is connected to particle of mass $m$ and the other to a small peg on smooth horizontal table. If the particle moves in a circle with speed $v$, the net force on the particle (directed towards the centre) is : (T is the tension in the string)**

**(a)** $T = \dfrac{mv^2}{l}$

**(b)** $T - \dfrac{mv^2}{l}$

**(c)** $T + \dfrac{mv^2}{l}$

**(d)** 0

**[NCERT Ex. Q. 5.4, Page 110]**

**Ans. Correct option:** (a)

*Explanation:*

The net force on the particle is T because tension T provided the necessary centripetal force, *i.e.*,

$$T = \frac{mv^2}{l}$$

## Very Short Answer Type Questions (1 mark each)

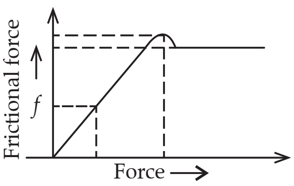
**Q. 1. A block placed on a rough horizontal surface is pulled by a horizontal force $F$. Let $f$ be the force applied by the rough surface on the block. Plot a graph of $f$ versus $F$.**

**[NCERT Exemp. Q. 5.21, Page 34]**

**Ans.** When a small force $F_1$ is applied on a heavier box, it does not move. At this stage, $f$ is equal to $F$.

$f = F$ until the block is stationary.

$f$ remains constant if $F$ increases beyond this point and the block starts moving.




**Q. 2. Why are mountain roads generally made winding upwards rather than going straight up?**

**[NCERT Exemp. Q. 5.25, Page 34]**

**Ans.** On an inclined plane force of friction on a body going upward is $f = \mu R = \mu mg\cos\theta$ is the force of friction, if $\theta$ is angle made by the slope. If $\theta$ is small, force of friction is high and there is less chance of skidding. The road straight up would have a larger slope and smaller value of friction, therefore there are more chances of skidding.

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. A block of mass $M$ is held against a rough vertical wall by pressing it with a finger. If the coefficient of friction between the block and the wall is $\mu$ and the acceleration due to gravity is $g$, calculate the minimum force required to be applied by the finger to hold the block against the wall? [NCERT Exemp. Q. 5.30, Page 35]**

**Ans.** Let $F$ force is applied by finger on a body of mass $M$,

Under Balanced condition,

$F = N$

The normal reaction of the wall on the book. The minimum upward frictional force needed to ensure that the block does not fall is Mg. The frictional force = $\mu N$. Thus, minimum value of

$$F = \frac{Mg}{\mu}.$$

**Q. 2. A bob of mass 0.1 kg hung from the ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 ms$^{-1}$. What is the trajectory of the bob if the string is cut when the bob is :**

**(a) at one of its extreme position.**

**(b) at its mean position. [NCERT Exemp. Q. 5.12, Page 110]**

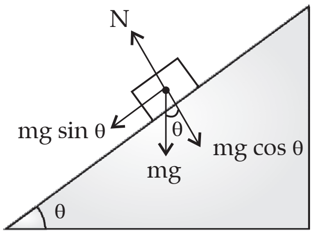
**Ans. (a)** We know that at each extreme position, the instantaneous velocity of the bob is zero. If the string is cut at the extreme position, it is under the action of '$g$' only, hence, the bob will fall vertically downwards.

**(b)** When the bob is at the mean position, it is affected by gravity. At mean position of the bob is having velocity of 1 ms$^{-1}$ along the tangent to the arc which is in the horizontal direction. If the string is cut at the mean position, the bob will behave as a horizontal projectile. Hence, it will follow a parabolic path.

## Long Answer Type Questions (5 marks each)

**Q. 1. When a body slides down from rest along a smooth inclined plane making an angle of 45° with the horizontal, it takes time $T$. When the same body slides down from rest along a rough inclined plane making the same angle and through the same distance, it is seen to take time $pT$, where $p$ is some number greater than 1. Calculate the co-efficient of friction between the body and the rough plane. [NCERT Exemp. Q. 5.35, Page 36]**

**Ans.** For smooth inclined plane




Let $s$ = length of inclined plane

or $mg\sin\theta = ma$

or $a = g\sin\theta$

Applying 2$^{nd}$ kinematic equation of motion

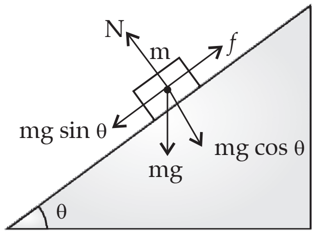
$$s = ut + \frac{1}{2}at^2$$

$$\text{or } s = 0\times T + \frac{1}{2}(g\sin\theta)T^2 \quad (\because t = T)$$

$$\text{or } s = \frac{1}{2}g\sin\theta T^2$$

$$\text{or } s = \frac{1}{2\sqrt{2}}gT^2 \quad (i) \quad (\because \theta = 45^\circ)$$

For rough inclined plane.




$$f = \mu N = \mu mg\cos\theta$$

$$mg\cos\theta - f = ma_1$$

$$\text{or } a_1 = (\sin\theta - \mu\cos\theta)g = \frac{1}{\sqrt{2}}(1-\mu)g$$

Using, $s = ut + \frac{1}{2}at^2$

$$\text{or } s = 0\times(pT) + \frac{1}{2}\left\{\frac{1}{\sqrt{2}}(1-\mu)g\right\}\times p^2T^2$$

$$\text{or } s = \frac{1}{2\sqrt{2}}(1-\mu)gp^2T^2 \quad (ii)$$

∴ by (i) & (ii), we get

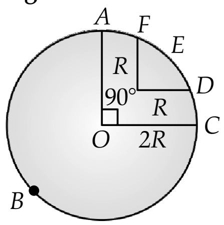
$$\frac{1}{2\sqrt{2}}gT^2 = \frac{1}{2\sqrt{2}}(1-\mu)gp^2T^2$$

$$\text{or } 1 = (1-\mu)p^2$$

$$\text{or } \mu = \left(1 - \frac{1}{p^2}\right)$$

**Q. 2. A racing car travels on a track (without banking) ABCDEFA in fig. ABC is a circular arc of radius 2 R. CD and FA are straight paths of length R and DEF is a circular arc of radius R = 100 m. The co-efficient**

**of friction on the road is $\mu$ = 0.1. The maximum speed of the car is 50 ms$^{-1}$. Find the minimum time for completing one round.**




**[NCERT Exemp. Q. 5.37, Page 36]**

**Ans.** The centripetal force to keep car in circular motion is provided by frictional force (inward to centre).

For DEF

$$\not{m}\frac{v^2}{R} = \not{m}g\mu$$

$$v_{max} = \sqrt{g\mu R} = \sqrt{100} = 10\,\text{m s}^{-1}$$

For ABC

$$\frac{v^2}{R} = g\mu,\ v = \sqrt{200} = 14.14\ \text{m s}^{-1}$$

Time for DEF= $\frac{\pi}{2}\times\frac{100}{10} = 5\pi s$

Time for ABC= $\frac{3\pi}{2}\frac{200}{14.14} = \frac{300\pi}{14.14}s$

For FA and DC= $2\times\frac{100}{50} = 4s$

Total time $= 5\pi + \frac{300\pi}{14.14} + 4 = 86.3s$

**Q. 3. The displacement vector of a particle of mass m is given by $r(t) = \hat{i}A\cos\omega t + \hat{j}B\sin\omega t$.**

**(a) Show that the trajectory is an ellipse.**

**(b) Show that $F = -m\omega^2 r$.**

**[NCERT Exemp. Q. 5.38, Page 36]**

**Ans.** **(a)** $x$ = A cos ω$t$, $y$ = B sin ω$t$

$$\frac{x}{A} = \cos\omega t \qquad (1)$$

$$\frac{y}{B} = \sin\omega t \qquad (2)$$

Squaring & adding (1), (2)

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$$

which is the equation of ellipse

$$\frac{dr}{dt} = v = -\hat{i}\,\omega A\sin\omega t + \hat{j}\,\omega B\cos\omega t$$

$$\frac{dv}{dt} = a = -\omega^2 r; F = m\omega^2 r$$

**Q. 4. A rectangular box lies on a rough inclined surface. The co-efficient of friction between the surface and the box is $\mu$. Let the mass of the box be m.**

**(a) At what angle of inclination θ of the plane to the horizontal will the box just start to slide down the plane?**

**(b) What is the force acting on the box down the plane, if the angle of inclination of the plane is increased to $\alpha > \theta$ ?**

**(c) What is the force needed to be applied upwards along the plane to make the box either remain stationary or just move up with uniform speed?**

**(d) What is the force needed to be applied upwards along the plane to make the box move up the plane with acceleration a ?**

**[NCERT Exemp. Q. 5.41, Page 37]**

**Ans.** **(a)** θ = tan$^{-1}$ ($\mu$)

**(b)** If $\alpha > \theta$, angle of inclination of plane with horizontal, it will slide down.

**(c)** To keep box either stationary or just move it up with uniform velocity

$$F_2 = mg\sin\alpha + f \text{ or } mg\,(\sin\alpha + \mu\cos\alpha)$$

**(d)** Force applied to move box upward with acceleration a,

$$mg(\sin\alpha + \mu\cos\alpha) + ma.$$

**Q. 5. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with speed 40 rev./min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?**

**[NCERT Ex. Q. 5.21, Page 111]**

**Ans.** Frequency of revolution of stone is given by

$$f = 40 \text{ rev/min} = \frac{40}{60} \text{ rev/s}$$

Mass of stone, $m$ = 0.25 kg

Radius of circle, $r$ = 1.5 m

By formula angular speed of the stone,

$$\omega = 2\pi f$$

or $$\omega = 2\pi \times \frac{40}{60}$$

$$= \frac{4\pi}{3} \text{ rad s}^{-1}$$

T = tension in the string = ?

$T_{max}$ = maximum tension in the string = 200 N

*i.e.,* $$T = \frac{mv^2}{r} = mr\omega^2$$

or $$T = 0.25 \times 1.5 \times \left(\frac{4\pi}{3}\right)^2 \text{ N}$$

or $$T = 0.25 \times 1.5 \times \frac{16}{9} \times 9.87$$

$(\because \pi^2 = 9.87)$

or T = 6.58 N ≈ 6.6 N

As the string can withstand a maximum tension of 200 N,

$$\therefore \quad T_{max} = \frac{mv^2_{max}}{r}$$

or $v_{max} = \sqrt{\dfrac{rT_{max}}{m}}$

$= \sqrt{\dfrac{1.5 \times 200}{0.25}}$

or $v_{max} = 34.64 \text{ ms}^{-1}$.

**Q. 6. If in Q. 5 the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks ?**

**(a) The stone jerks radially outwards.**

**(b) The stone flies off tangentially from the instant the string breaks.**

**(c) The stone flies off at the angle with the tangent whose magnitude depends on the speed of particle.**

**[NCERT Ex. Q. 5.22, Page 111]**

**Ans.** **(b)** Correctly describes the trajectory of the stone after the string breaks, *i.e.*, the stone flies off tangentially from the instant the string breaks.

The velocity always acts tangentially to the circle at each point in the circular motion. At the time, the string breaks, the particle continues to move in the tangential direction according to Newton's first law of motion.

**Q. 7. Explain why :**

**(a) a horse cannot pull a cart and run in empty space,**

**(b) passengers are thrown forward from their seats when a speeding bus stops suddenly,**

**(c) it is easier to pull a lawn mover than to push it,**

**(d) a cricketer moves his hands backwards when holding a catch.**

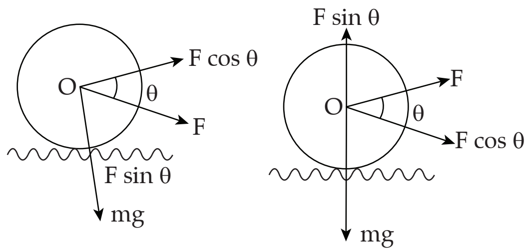
**[NCERT Ex. Q. 5.23, Page 111]**

**Ans. (a)** While trying to pull a cart, a horse pushes the ground backwards with a certain force at an angle. The ground offers an equal reaction in opposite direction; on the feet of the horse. The forward component of this reaction is responsible for the motion of cart.

In empty space, there is no reaction and hence horse cannot pull the cart and run.

**(b)** This is due to inertia of motion possessed by the passengers in a speeding bus.

**(c)** Consider the free body diagram in the two cases, the lawn mover is represented by the point O.




The forces acting on the lawn mover are (i) applied force F, (ii) weight of the mover W downward, (iii) force of friction acting opposite to the direction of motion, (iv) reaction of ground on the mover (R).

In case of pushing the mover, the horizontal component of push $F\cos\theta$ in forward direction and vertical component $F\sin\theta$ in downward direction. The total downward force

$= W + F\sin\theta$.

Therefore, the reaction

$$R = W + F\sin\theta$$

(Newton's third law of motion) and force of friction,

$$f = \mu R$$
$$= \mu(W + F\sin\theta) \qquad (\because W = mg)$$

where, $\mu$ = coefficient of friction.

When P be the net forward force, then

$$P = F\cos\theta - f$$
$$= F\cos\theta - \mu(mg + F\sin\theta) \qquad \text{...(i)}$$

In case of pull, the forward component of pull

$$= F\cos\theta$$

and vertical component $= F\sin\theta$

where R′ be the normal reaction in the case, then

$$F' = W - F\sin\theta$$
$$= mg - F\sin\theta$$

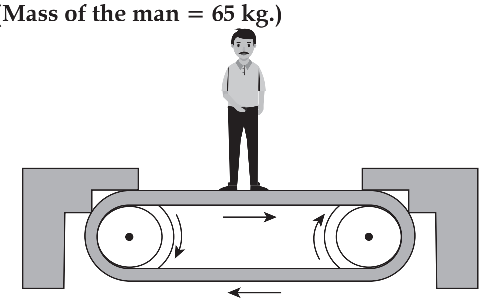
$\therefore$ if $f'$ be the force of friction in this case, then net force

$$P' = F\cos\theta - f' = F\cos\theta - \mu R'$$
$$= F\cos\theta - \mu(mg - F\sin\theta) \qquad \text{...(ii)}$$

From equation (i) and (ii), we see that $P' > P$. Therefore, it is easier to pull than to push the lawn mover.

**(d)** While holding a catch, the impulse received by the hands, $F \times t$ = change in momentum of the ball is constant. By moving his hands backwards, the cricketer increases the time of impact ($t$) to complete the catch. Since $t$ increases, F decreases and as a reaction, the ball hurts hands lesser.

**Q. 8. Figure shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1 $ms^{-2}$. What is the net force on the man? If the coefficient of static friction between the man's shoes and the belt is 0.2, up to what acceleration of the belt can the man continue to be stationary relative to the belt? (Mass of the man = 65 kg.)**



**[NCERT Ad. Ex. Q. 5.25, Page 111]**

**Ans.** Mass of the man, $m = 65$ kg

Acceleration of the belt, $a = 1$ m/s$^2$

Coefficient of static friction, $\mu = 0.2$

The net force $F$, acting on the man is given by Newton's second law of motion as:

$F_{net} = ma = 65 \times 1 = 65$ N

The man will continue to be stationary with respect to the conveyor belt until the net force on the man is less than or equal to the frictional force $f_s$ exerted by the belt, i.e.,

$F'_{net} = f_S$

$ma' = \mu mg$

$\therefore\ a' = 0.2 \times 10 = 2$ m/s$^2$

Therefore, the maximum acceleration of the belt up to which the man can stand stationary is 2 m/s$^2$.

**Q. 9. Stone of mass *m* tied to the end of a string is revolved in a vertical circle of radius R. The net forces at the lowest and highest points of the circle directed vertically downwards are (Choose the correct alternative) :**

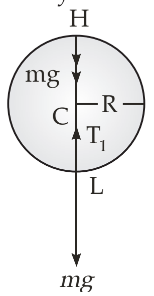
| Lowest point | Highest point |
|---|---|
| (a) $mg - T_1$ | $mg + T_2$ |
| (b) $mg + T_1$ | $mg - T_2$ |
| (c) $mg + T_1 - (mv_1^2)/R$ | $mg - T_2 + (mv_2^2)/R$ |
| (d) $mg - T_1 - (mv_1^2)/R$ | $mg + T_2 + (mv_2^2)/R$ |

**Here $T_1$, $T_2$ and ($v_1$, $v_2$) denote the tension in the string (and the speed of the stone) at the lowest and the highest point, respectively.**

**[NCERT Ad. Ex. Q. 5.26, Page 112]**

**Ans.** In the given figure shown here L and H show the lowest and highest points, respectively.

**At point L :** $T_1$ acts towards the centre of the circle and $mg$ acts vertically downward.




Now, Net force on the stone at the lowest point in the downward direction = $mg - T_1$

**At point H :** Both $T_2$ and $mg$ act vertically downward towards the centre of the vertical circle.

Again, Net force on the stone at the highest point in the downward direction = $T_2 + mg$

Hence, option (a) is the correct alternative.

**Q. 10. An aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at 15°. What is the radius of the loop ?**

**[NCERT Ad. Ex. Q. 5.30, Page 112]**

**Ans. Given :** Speed of aircraft, $v = 720$ kmh$^{-1}$

or $\quad v = 720 \times \dfrac{5}{18}$

or $\quad v = 200$ ms$^{-1}$

Angle of banking, $\theta = 15°$,

Radius of the loop $= r = ?$

We know that for a banked curve,

$$\tan\theta = \frac{v^2}{rg}$$

or $r = \dfrac{v^2}{g\tan\theta} = \dfrac{(200)^2}{9.8 \times \tan 15°}$

or $r = \dfrac{40000}{9.8 \times 0.2679}$

$= 15.24 \times 10^3$ m

or $r = 15.24$ km

**Q. 11. A train rounds an unbanked circular bend of radius 30 m at a speed of 54 kmh$^{-1}$. The mass of the train is $10^6$ kg. What provides the centripetal force required for this purpose ? The engine or the rails ? What is the angle of banking to prevent wearing out of the rail ?**

**[NCERT Ad. Ex. Q. 5.31, Page 112]**

**Ans. Given :** Radius of circular bend,

$r = 30$ m

Speed of train $= v = 54$ kmh$^{-1}$

$= 54 \times \dfrac{5}{18}$

$= 15$ ms$^{-1}$

**Angle of banking** $= \theta = ?$

The centripetal force is provided by the lateral thrust by the outer rail. According to Newton's third law of motion, the train exerts (*i.e.,* causes) an equal and opposite thrust on the outer rail causing its wear and tear.

The angle of banking by formula

$$\tan\theta = \frac{v^2}{rg}$$

$\Rightarrow \quad \tan\theta = \dfrac{(15)^2}{30 \times 9.8}$

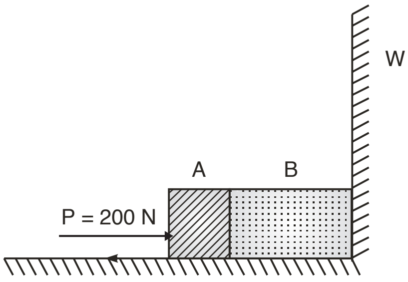
$= \dfrac{225}{30 \times 9.8} = 0.7653$

$\Rightarrow \quad \tan\theta = \tan 37° 25'$

$\therefore \quad \theta = 37°25'$

or $\quad \theta = 37.42°$.

**Q. 12. Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid partition. The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally at A. What are (a) the reaction of the partition, (b) the action-reaction forces between A and B ? What happens when the partition is removed ? Does the answer to (b) change, when the bodies are in motion ? Ignore difference between $\mu_s$ and $\mu_k$.**




**[NCERT Ad. Ex. Q. 5.34, Page 113]**

**Ans.** Given : mass of body A,

$m_1 = 5$ kg

Mass of body B, $m_2 = 10$ kg

Coefficient of friction between the bodies and the table, μ = 0.15. Horizontal force applied on body A, P = 200 N

(a) **Reaction of partition = ?**

Suppose $f$ = force of limiting friction acting to the left as shown in figure, then

$$f = \mu R$$
$$= \mu (m_1 + m_2) g \quad [\because R = (m_1 + m_2)g]$$
$$= 0.15 (5 + 10) \times 9.8$$
$$= 22.05 \text{ N}$$

∴ When F′ = Net force to the right exerted on the partition, then from

$$F' = P - f = 200 - 22.05$$
$$= 177.95 \text{ N}$$

∴ From Newton's 3rd law of motion,

Reaction of the partition = F′

= 177.95 N to the left.

(b) **Action-reaction forces between A and B = ?**

Suppose, $f_1$ = force of limiting friction acting on body A.

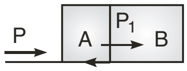
$$\therefore \quad f_1 = \mu R_1 = \mu m_1 g \quad (\because R_1 = m_1 g)$$
$$= 0.15 \times 5 \times 9.8$$
$$= 7.35 \text{ N}$$

When P′ be the net force applied by body A on body B,

then $P' = P - f_1 = 200 - 7.35$

$= 192.65$ N.

*i.e.* action of A on B = P′ = 192.65 N towards right




∴ From Newton's 3rd law of motion,

The force (reaction) of B on A = P′ = 192.65 N towards left.

**If the partition is removed :** If the partition is removed, the kinetic friction comes into play the masses move together as a system of two bodies under the action of net force F′ given by

or $F' = P - f = 200 - 22.05$

or $F' = 177.95$ N.

If $a$ = acceleration produced in the system by F′, then

$$a = \frac{F'}{m_1 + m_2} = \frac{177.95}{5 + 10}$$

$$= \frac{177.95}{15} = 11.83 \text{ ms}^{-2}.$$

**Does the answer to (b) change, if $m_1$ and $m_2$ are in motion ?**

Yes, if the bodies are in motion, then the answer to (b) changes and can be proved as under :

If the bodies are moving, the force exerted by A on B is given by

$F_{BA}$ = P′ – Force required to produce and acceleration of 11.86 $ms^{-2}$ in body A alone

or $F_{BA} = P - f_1 - m_1 a$

or $F_{BA} = 200 - 7.35 - 5 \times 11.86$

or $F_{BA} = 200 - 7.35 - 59.30$

or $F_{BA} = 192.65 - 59.30$

or $F_{BA} = 133.35$ N

*i.e.,* Action of A = 133.35 N

∴ Reaction of B on A when partition is removed = 133.35 N to the left.

**Q. 13. A block of mass 15 kg is placed on a long trolley. The coefficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 $ms^{-2}$ for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by**

**(a) a stationary observer on the ground,**

**(b) an observer moving with the trolley.**

**[NCERT Ad. Ex. Q. 5.35, Page 113]**

**Ans.** Mass of the block, $m$= 15 kg

Coefficient of static friction, $\mu = 0.18$

Acceleration of the trolley, $a = 0.5$ m/s$^2$

As per Newton's second law of motion, the force (*F*) on the block caused by the motion of the trolley is given by the relation:

$F = ma = 15 \times 0.5 = 7.5$ N

This force is acted in the direction of motion of the trolley.

Force of static friction between the block and the trolley:

$f = \mu mg$

$= 0.18 \times 15 \times 10 = 27$ N

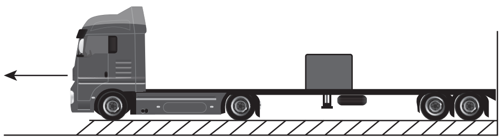
**(a)** The force of static friction between the block and the trolley is greater than the applied external force. Hence, for an observer on the ground, the block will appear to be at rest.

When the trolley moves with uniform velocity there will be no applied external force. Only the force of friction will act on the block in this situation.

**(b)** An observer, moving with the trolley, has some acceleration. This is the case of non-inertial frame of reference. The frictional force, acting on the trolley backward, is opposed by a pseudo force of the same magnitude. However, this force acts in the opposite direction. Thus, the trolley will appear to be at rest for the observer moving with the trolley.

**Q. 14. The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in figure. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from**

**rest and accelerates with 2 ms$^{-2}$. At what distance from the starting point does the box fall off the truck? (Ignore the size of the box).**



**[NCERT Ad. Ex. Q. 5.36, Page 113]**

**Ans.** Mass of the box, $m = 40$ kg

Coefficient of friction, $\mu = 0.15$

Initial velocity, $u = 0$

Acceleration, $a = 2$ m/s$^2$

Distance of the box from the end of the truck, $s' = 5$ m

As per Newton's second law of motion, the force on the box caused by the accelerated motion of the truck is given by: F = $ma$

F = 40 × 2

= 80 m/s$^2$

As per Newton's third law of motion, a reaction force of 80 N is acting on the box in the backward direction. The backward motion of the box is opposed by the force of friction $f$, acting between the box and the floor of the truck. This force is given by:

$f = \mu mg = 0.15 \times 40 \times 10 = 60$ N

Net force acting on the block:

$F_{net} = 80 - 60 = 20$ N backward

The backward acceleration produced in the box is given by:

$$a_{back} = \frac{F_{net}}{m} = \frac{20}{40} = 0.5 m/s^2$$

Using the second equation of motion, time $t$ can be calculated as:

$$s' = ut + \frac{1}{2}a_{back}t^2$$

$$5 = 0 + \frac{1}{2} \times 0.5 \times t^2$$

$$\therefore t = \sqrt{20s}$$

Hence, the box will fall from the truck after $\sqrt{20}$ $s$ from start.

The distance $s$, travelled by the truck in $\sqrt{20}$ $s$ is given by the relation:

$$s = ut + \frac{1}{2}at^2$$

$$0 + \frac{1}{2} \times 2 \times \left(\sqrt{20}\right)^2$$

=20 m

**Q. 15. A disc revolves with a speed of $33\frac{1}{3}$ rev/min, and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the co-efficient of friction between the coins and the record is 0.15, which of the coins will revolve with the record?**

**[NCERT Ad. Ex. Q. 5.37, Page 113]**

**Ans.** Coin placed at 4 cm from the center

Mass of each coin = $m$

Radius of the disc, $r$ = 15 cm = 0.15

$$\text{Frequency of revolution}, v = 33\frac{1}{3} \text{ rev/min}$$

$$= \frac{100}{3 \times 60} = \frac{5}{9}$$

Coefficient of friction, $\mu$= 0.15

In the given situation, the coin having a force of friction greater than or equal to the centripetal force provided by the rotation of the disc will revolve with the disc. If this is not the case, then the coin will slip from the disc.

Coin placed at 4 cm:

Radius of revolution, $r'$ = 4 cm = 0.04 m

$$\text{Angular frequency}, \omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{5}{9} = 3.49\,s^{-1}$$

Frictional force, $f = \mu mg = 0.15 \times m \times 10 = 1.5$ m$N$

Centripetal force on the coin

$Fcent = mr'\omega^2$

$= m \times 0.04 \times (3.49)^2$

= 0.49 m$N$

Since $f > F_{cent}$, the coin will revolve along with the record.

Coin placed at 14 cm:

Radius, $r'' = 14\ cm = 0.14\ m$

Angular frequency, $\omega = 3.49\ s^{-1}$

Frictional force, $f'$ = 1.5mN

Centripetal force is given as:

$Fcent = mr''\omega^2$

$= m \times 0.14 \times (3.49)^2$

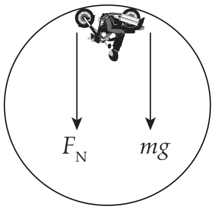
= 1.7$m$ N

Since $f < F_{cent.}$, the coin will slip from the surface of the record.

**Q. 16. You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death-well' (a hollow spherical chamber with holes, so the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?**

**[NCERT Ex. Q. 5.38, Page 113]**

**Ans.** In a death-well, a motorcyclist does not fall at the top point of a vertical loop because both the force of normal reaction and the weight of the motorcyclist act downward and are balanced by the centripetal force. This situation is shown in the following figure.




The net force acting on the motorcyclist is the sum of the normal force ($F_N$) and the force due to gravity ($F_g = mg$).

The equation of motion for the centripetal acceleration $a_c$, can be written as:

$F_{net} = ma_c$.

$F_N + F_g = ma_c$

$$F_N + mg = \frac{mv^2}{r}$$

For, ($v_{min}$), $F_N = 0$

$$mg = \frac{mv^2_{min}}{r}$$

$$\therefore v_{min} = \sqrt{rg}$$

$$= \sqrt{25 \times 10}$$

$$= 15.8 \text{ m/s}$$

**Q. 17. A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed ?**

**[NCERT Ad. Ex. Q. 5.39, Page 113]**

**Ans.** The cylinder being vertical, the normal reaction of the wall on the man acts horizontally and provides the necessary centripetal force which is given below,

$$R = mr\omega^2 \quad ...(i)$$

The frictional force F, acting upwards balances his weight, *i.e.*,

$$F = mg \quad ...(ii)$$

The man will remain stuck to the wall after the floor is removed, *i.e.*, he will continue to rotate with the cylinder without slipping.

When $\mu \geq \frac{F}{R}$

or $F \leq \mu R$

or $mg \leq \mu\, mr\omega^2$

equations (i) and (ii),

or $\omega^2 \geq \frac{g}{\mu r}$

or $\omega \geq \sqrt{\frac{g}{\mu r}}$ ...(iii)

$\therefore$ The minimum angular speed of rotation of the cylindrical drum is given by

$$\omega_{min} = \sqrt{\frac{g}{\mu r}} \quad ...(iv)$$

Here, $\mu = 0.15$, $r = 3$ m, $g = 9.8$ ms$^{-2}$. ...(v)

From equations (iv) and (v),

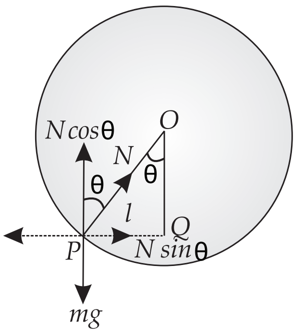
$$\omega_{min} = \sqrt{\frac{9.8}{0.15 \times 13}} \text{ rad s}^{-1}$$

$$= 4.67 \simeq 5 \text{ rad s}^{-1}$$

**Q. 18. A thin circular loop of radius $R$ rotates about its vertical diameter with an angular frequency $\omega$. Show that a small bead on the wire loop remains at its lowermost point for $\omega \leq \sqrt{g/R}$ What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $w = \sqrt{2g/R}$? Neglect friction.**

**[NCERT Ad. Ex. Q. 5.40, Page 113]**

**Ans.** Let the radius vector joining the bead with the center make an angle $\theta$, with the vertical downward direction.




OP = $R$ = Radius of the circle

$N$ = Normal reaction

The respective vertical and horizontal equations of forces can be written as:

$mg = N\cos\theta$ ...(i)

$ml\omega^2 = N\sin\theta$ ...(ii)

In $\Delta OPQ$, we have:

$$\sin\theta = \frac{l}{R}$$

$l = R\sin\theta$ ...(iii)

Substituting equation (iii) in equation (ii), we get:

$m(R\sin\theta)\,\omega^2 = N\sin\theta$

$mR\omega^2 = N$ ...(iv)

Substituting equation (iv) in equation (i), we get:

$mg = mR\omega^2\cos\theta$

$$\cos\theta = \frac{g}{R\omega^2} \quad ...(v)$$

Since $\cos\theta \leq 1$, the bead will remain at its lowermost point for $\frac{g}{R\omega^2} \leq 1$ i.e., $\omega \leq \sqrt{\frac{g}{R}}$ for

For $\omega = \sqrt{\frac{2g}{R}}$ or $\omega^2 = \frac{2g}{R}$ ...(iv)

On equating equations (v) and (vi), we get:

$$\frac{2g}{R} = \frac{g}{R\cos\theta}$$

$$\cos\theta = \frac{1}{2}$$

$$\therefore \theta = \cos^{-1}(0.5)$$

$$= 60°$$

**TIPS... & TRICKS...**

- *Study Newton's laws of motion.*
- *Learn about conservation of Linear momentum.*
- *Understand about Rocket propulsion.*
- *Understand about various type at Friction.*
- *Study and understands about uniform and non-uniform circular motions.*
- *Study about Horizontal circular motion and vertical circular motion.*

## Some Commonly Made Errors

- Students got confused in determining the direction of force.
- Students got confused in objective concept of force in physics with the subjective concept of feeling of force.
- The centripetal force should not be regarded as yet another kind of force.

### EXPERT ADVICE

- *All the three laws of Newton should be will understood and students should be able to relate these laws to general day to day life activities.*
- *Students should practices questions from mechanics involving friction and resolving forces into various components and equalizing them.*

### OSWAAL LEARNING TOOLS

***For*** **Suggested Online Videos**

| | |
|---|---|
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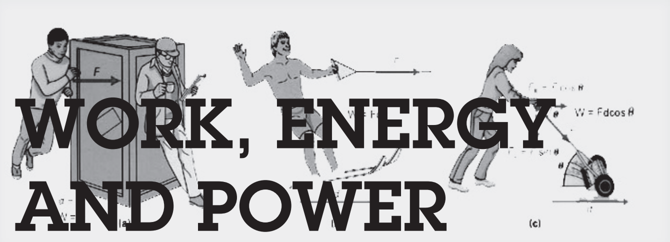
CHAPTER

# 6 WORK, ENERGY AND POWER




## Chapter Objective

**This chapter will help you understand :**

- *Work done by a constant force and a variable force, power.*
- *Kinetic energy, work-energy theorem, Notion of Potential energy, Potential energy of a spring, conservative forces, conservation of Mechanical energy (Kinetic and Potential energy's), Non-conservative forces, Motion in vertical circle, Elastic and inelastic collisions in one and two dimensions.*

## TOPIC-1
## Work and Power



### Quick Review

- **Work :** Work is done when the body is displaced actually through some distance in the direction of applied force.

$$W = \vec{F} \cdot \vec{s}$$

$$W = Fs\cos\theta$$

  - Work done is a scalar quantity. However, work done is positive when θ lies between 0 (zero) and π/2. Work done is negative when θ lies between π/2 and π.
  - S. I. unit of work is joule (J)and the C.G.S unit of work is erg, where 1 joule = $10^7$ erg.
  - Work done by a body does not depend on the time taken to complete the work.
- **Internal work or zero work.** The work in which muscles are strained, but work done is not useful, is called internal work. For example, when a person carrying load keeps on standing at the same place, work done is zero, but he gets tired on account of internal work.

  **Dimensions :** $[ML^2 T^{-2}]$

  **Power :** Power of a body is defined as the time rate of doing work by the body. Thus, in power, time taken by the body to complete the work is significant.

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

  - Here, θ is the angle between force $\vec{F}$ and velocity $\vec{v}$ of the body.
  - **Unit** : 1 W = 1 $Js^{-1}$
  - **Dimensions :** $[ML^2T^{-3}]$
  - Power is a scalar quantity.

### Know the Terms

- **Conservative force** is a force if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between initial and final positions, *e.g.*, gravitational force, electrostatic force between two electric charges, all central forces, etc.

- **Non-conservative force** is a force if work done by or against the force in moving a body from one position to another depends on the path followed between these two positions. *e.g.*, frictional forces, elastic forces, etc.

## Know the Formulae

- Work = Force × Displacement
- $$W = \vec{F}.\vec{s}$$
  $$= Fs\cos\theta$$
- Maximum work
  When $\cos\theta = 0°$
  $W = Fs$
- Minimum work
  When $\cos\theta° = 90°$
  $W = Fs \times \cos 90° = 0$
- Work done by variable force :
  $$W = \int_{x_B}^{x_A} \vec{F}\cdot d\vec{s}$$
- $$\text{Power} = \frac{\text{Work done}}{\text{Time Taken}}$$
- $$P = \vec{F}\cdot\vec{v}$$
- $$P = Fv\cos\theta.$$

## Know the Links

- *www.vedantu.com*
- *www.learncbse.com*
- *www.gradestack.com*

## MCQ/Fillups/True or False (1 mark each)

### (A) Multiple Choice Questions

**Q. 1. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is because,**

**(a) the two magnetic forces are equal and opposite, so they produce no net effect.**
**(b) the magnetic forces do not work on each particle.**
**(c) the magnetic forces do equal and opposite (but non-zero) work on each particle.**
**(d) the magnetic forces are necessarily negligible.**

**[NCERT Exemp. Q. 6.1, Page 38]**

**Ans. Correct option:** (b)

*Explanation:* As, the magnetic field due to motion of electron and proton act in a direction perpendicular to the direction of motion, no work is done by the forces. This is why one ignores the magnetic force of one particle on another.

**Q. 2. A proton is kept at rest. A positively charged particle is released from rest at a distance *d* in its field. Consider two experiments: one in which the charged particle is also a proton and in another, a positron. In the same time *t*, the work done on the two moving charged particles is**

**(a) same as the same force law is involved in the two experiments.**
**(b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.**
**(c) more for the case of a positron , as the positron moves away a larger distance.**
**(d) same as the work done by charged particle on the stationary proton.**

**[NCERT Exemp. Q. 6.2, Page 38]**

**Ans. Correct option:** (c)

*Explanation:* Force between two protons = force between a proton and a positron. Because of having much lighter weight than proton, positron moves away a larger distance compared to proton.

As, work done = force × displacement, therefore in the same time t, work done in case of positron is more than that of proton.

**Q. 3. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is**

**(a) constant and equal to *mg* in magnitude.**

**(b) constant and greater than mg in magnitude.**

**(c) variable but always greater than *mg*.**

**(d) at first greater than mg, and later becomes equal to mg.**

**[NCERT Exemp. Q. 6.3, Page 39]**

**Ans. Correct option:** (d)

*Explanation:* In the whole process, the man exerts a variable force (F) on the ground to set his body in motion. This force is in addition to the force required to support his weight ($mg$). Once the man is in standing position, F become zero.

**Q. 4. A bicyclist comes to a skidding stop in 10m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is**

**(a) + 2000J** **(b) −200J**

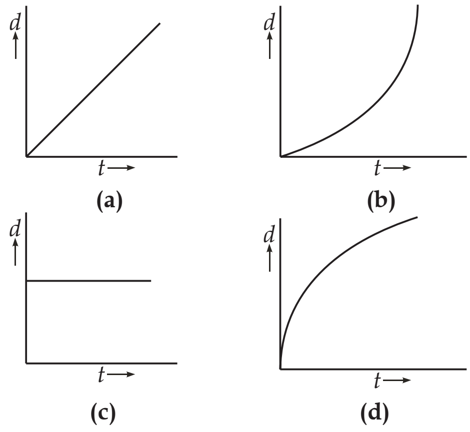
**(c) zero** **(d) −20,000J**

**[NCERT Exemp. Q. 6.4, Page 39]**

**Ans. Correct option:** (c)

*Explanation:* Just because road does not move at all so the work done by the cycle on the road must be zero.

**Q. 5. A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams shown in figure correctly shows the displacement – time curve for its motion?**




**[NCERT Exemp. Q. 6.11, Page 41]**

**Ans. Correct option:** (b)

*Explanation:* For constant power,
displacement $s \propto t^{3/2}$

**Q. 6. A circket ball of mass 150 g moving with a speed of 126 km/h hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.001 s, the force that the batsman had to apply to hold the bat firmly at its place would be**

**(a) 10.5 N**

**(b) 21.0 N**

**(c) $1.05 \times 10^4$ N**

**(d) $2.1 \times 10^4$ N**

**[NCERT Exemp. Q. 6.18, Page 44]**

**Ans. Correct option:** (c)

*Explanation:*

$$F = \frac{\Delta p}{t} = \frac{mv - (-mv)}{t}$$

$$= \frac{2mv}{t}$$

$$= \frac{2 \times 150 \times 10^{-3} \text{kg} \times 35 \text{m/s}}{0.001 \text{s}}$$

$$= 10500 \text{N}$$

$$= 1.05 \times 10^4 \text{N}$$

**Q. 7. A man of mass m, standing at the bottom of the staircase, of height L climbs it and stands at its top.**

**(a) Work done by all forces on man is equal to the rise in potential energy mgL.**

**(b) Work done by all forces on man is zero.**

**(c) Work done by the gravitational force on man is mgL.**

**(d) The reaction force from a step does not do work because the point of application of the force does not move while the force exists.**

**[NCERT Exemp. Q. 6.19, Page 44]**

**Ans. Correct option:** (b) and (d)

*Explanation:*

All forces are internal force, gravitational force and frictional force.

Work done by gravitational force = $-mgL$

Work done by internal force

= −work done against gravitational force = $mgL$

Work done by frictional force = F × 0 = 0

Work done by all forces = $mgL - mgL + 0 = 0$

Work done by reaction force = N × 0 = 0

**Q. 8. A body of mass 0.5kg travels in a straight line with velocity $v = a\, x^{\frac{3}{2}}$ where $a = 5\text{m}^{-1/2}\text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is**

**(a) 1.5 J** **(b) 50 J**

**(c) 10 J** **(d) 100 J**

**[NCERT Exemp. Q. 6.10, Page 41]**

**Ans. Correct option:** (b)

*Explanation:*

Given: $m$=0.5 kg, $v = kx^{3/2}$ where $k = 5\, m^{-1/2}\text{s}^{-1}$
(Here k is used as constant a)

Acceleration, $a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{vdv}{dx}$ ...(i)

Differentiating (i) with respect to $x$,

$$2v\frac{dv}{dx} = 3k^2x^2$$

$\{\because v^2 = k^2x^3\}$

$\therefore$ Acceleration, $a = \frac{3}{2}k^2x^2$

Force, F = Mass × Acceleration $= \frac{3}{2}mk^2x^2$

Work done, w = $\int F dx = \int_0^2 \frac{3}{2}mk^2x^2 dx$

$$w = \frac{3}{2}mk^2\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{3}{6}\times 0.5 \times 5^2 \times [2^3 - 0]$$

$$= 50 \text{ J}$$

**Q. 9. A body is initially at rest. It undergoes one dimensional motion with constant acceleration. The power delivered to it in time *t* is proportional to :**

**(a)** $t^{1/2}$, **(b)** $t$, **(c)** $t^{3/2}$, **(d)** $t^2$

**[NCERT Ex. Q. 6.9, Page 136]**

**Ans. Correct option : (b)**

**Explanation :**

Suppose, $m$ = mass of the body

Given :

$a$ = acceleration produced in the body

$v$ = velocity of the body

$P$ = Power delivered to the body in a time t

Using Newton's second law of motion, the force is given by

F = $ma$

Both $m$ and $a$ are constants.

Hence, force will be a constant

F = $ma$ = constant ...(i)

For velocity $v$, acceleration is given by

$$a = \frac{dv}{dt}$$

$$v = at$$

$$p = Fv$$

or $$P = a^2 mt$$

Since, a and m are constants, therefore

$P$ = constant × $t$

or $P \propto t$

## Very Short Answer Type Questions **(1 mark each)**

**Q. 1. A rough inclined plane is placed on a cart moving with a constant velocity u on horizontal ground. A block of mass M rests on the incline. Is any work done by force of friction between the block and incline? Is there then a dissipation of energy?**

**[NCERT Exemp. Q. 6.22, Page 45]**

**Ans.** No, Force of friction on body is due to the tendency of block M to slide down over inclined plane.

No, as there is no work done so there no dissipation of energy.

**Q. 2. Why is electrical power required at all when the elevator is descending? Why should there be a limit on the number of passengers in this case?**

**[NCERT Exemp. Q. 6.23, Page 45]**

**Ans. (i)** Power is required to increase the velocity due to free fall.

**(ii)** Because limited or specified power can stop the speed of free falling of passenger along with elevator.

**Q. 3. A body is being raised to a height *h* from the surface of earth. What is the sign of work done by**

**(a) applied force**

**(b) gravitational force?**

**[NCERT Exemp. Q. 6.24, Page 45]**

**Ans.** (a) Positive

(b) Negative

(a) Work done by applied force,

$W_A$ = F × displacement × cos θ

or $W_A = mgh\cos 0^o = mgh$ $\{\because \theta = 0^o; F = mg\}$

$W_A \rightarrow$ positive

(b) Work done by gravitational force

$W_g = mg \times \text{displacement} \times \cos\theta$

$= mgh\cos 180^\circ$ $(\because \theta = 180^\circ)$

$= mgh\cos 180^\circ$ $(\because \theta = 180^\circ)$

$= -mgh \Rightarrow W_g \rightarrow$ negative

**Q. 4. Calculated the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2m. [NCERT Exemp. Q. 6.25, Page 45]**

**Ans.** Work done by car against gravity W=

weight of car × displacement× cos θ

= $mg$ × $s$ × cos θ

cos 90° = 0

Hence, $\theta = 90^\circ$, as gravity acting vertically downward

So, W = $mgs$ × 0

W = 0

**Q. 5. A body is moved along a closed loop. Is the work done in moving the body necessarily zero? If not, state the condition under which work done over a closed path is always zero.**

**[NCERT Exemp. Q. 6.27, Page 45]**

**Ans.** No, work done by a body moving along a closed loop is necessarily zero, only if all the forces acting on the system are conservative.

**Q. 6. Calculate the power of a crane in watts, which lifts a mass of 100 kg to a height of 10 m in 20s.**

**[NCERT Exemp. Q. 6.29, Page 45]**

**Ans.** F = $mg$ = 100 × 9.8 N [$m$ = 100 kg, $g$ = 9.8 m/s$^2$]

Power $P = \frac{mgh}{t}$ W

$$= \frac{100 \times 9.8 \times 10}{20} \text{ W}$$
$$= 490 \text{ W}$$

**Q. 7. The average work done by a human heart while it beats once is 0.5 J. Calculate the power used by heart if it beats 72 times in a minute.**

**[NCERT Exemp. Q. 6.30, Page 46]**

**Ans.** Work done in 72 beats by heart = $0.5 \times 72$ J
= 36 J

$$\text{Power} = \frac{\text{Work done}}{\text{Time}}$$
$$= \frac{36 \text{ J}}{60 \text{ s}}$$
$$= 0.6 \text{ Watt}$$
$$= 0.6 \text{W}$$

**Q. 8. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative :**

**(a) Work done by a man in lifting a bucket out of well by means of a rope tied to the bucket.**

**(b) Work done by gravitational force in the above case.**

**(c) Work done by friction on a body sliding down an inclined plane.**

**(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.**

**(e) Work done by the resistive force of air on a vibrating pendulum is bringing it to rest.**

**[NCERT Ex. Q. 6.1, Page 134]**

**Ans.** As, W= Fscos θ

**(a)** $\theta = 0^\circ$, work is positive.

**(b)** $\theta = 180^\circ$, work is negative.

**(c)** As force of friction act opposite to direction of motion, work is negative.

**(d)** As applied force acts in the direction of motion, work is positive.

**(e)** The resistive force of air acts opposite to direction of the motion, hence work is negative.

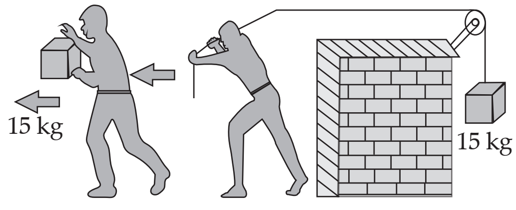
**Q. 9. Answer the following:**

**(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?**

**(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?**

**(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?**

**(d) In Figure (i) the man walks 2 m carrying a mass of 15 kg on his hands. In Figure (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?**




**(i)** **(ii)**

**[NCERT Ex. Q. 6.5, Page 135]**

**Ans. (a)** Rocket

The burning of the casing of a rocket in flight (due to friction) results in the reduction of the mass of the rocket.

According to the conservation of energy:

Total energy = Potential energy + Kinetic energy = $mgh + (1/2)\, mv^2$

The reduction in the rocket's mass causes a drop in the total energy. Therefore, the heat energy required for the burning is obtained from the rocket.

**(b)** Gravitational force is a conservative force. Since the work done by a conservative force over a closed path is zero, the work done by the gravitational force over every complete orbit of a comet is zero.

**(c)** When an artificial satellite, orbiting around earth, moves closer to earth, its potential energy decreases because of the reduction in the height. Since the total energy of the system remains constant, the reduction in P.E. results in an increase in K.E. Hence, the velocity of the satellite increases. However, due to atmospheric friction, the total energy of the satellite decreases by a small amount.

**(d) Work done in Figure (i)**

Mass, $m$= 15 kg

Displacement, $s$ = 2 m

Work done, $W = Fs \cos\theta$

Where, θ = Angle between force and displacement

$W = mgs \cos\theta = 15 \times 2 \times 9.8 \cos 90°$
$= 0$

**Work done in figure (ii)**

Mass, $m$ = 15 kg

Displacement, $s$ = 2 m

Here, the direction of the force applied on the rope and the direction of the displacement of the rope are same.

Therefore, the angle between them, $\theta = 0°$

Since $\cos 0° = 1$

Work done, $W = Fs\cos\theta = mgs$

$= 15 \times 9.8 \times 2 = 294$ J

Hence, more work is done in the Figure (ii).

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. A body constrained to move along Z-axis of a co-ordinate system is subjected to a constant force $\vec{F} = (-\hat{i} + 2\hat{j} + 3\hat{k})$ Newton. Where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the *x*, *y* and *z* axis of the system respectively. What is the work done by this force, in moving the body over a distance of 4 m along the Z-axis ?**

**[NCERT Ex. Q. 6.11, Page 136]**

**Ans.** Here,

$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{s} = 4\hat{k}$

($\because \vec{s} =$ 4 m distance is along Z-axis).

$W = \vec{F}.\vec{s}$

$W = \left(-\hat{i} + 2\hat{j} + 3\hat{k}\right)\left(4\hat{k}\right)$

$= 0 + 0 - 3 \times 4\hat{k}.\hat{k}$

$= 12\hat{k}\cdot\hat{k} = 12J \qquad \left(\because \hat{k}.\hat{k} = 1\right)$

**Q. 2. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m$^3$ in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?**

**[NCERT Ex. Q. 6.15, Page 136]**

**Ans.** Volume of the tank, $V = 30$ m$^3$

Time of operation, $t = 15$ min $= 15 \times 60 = 900$ s

Height of the tank, $h = 40$ m

Efficiency of the pump, $\eta = 30$ %

Density of water, $\rho = 10^3$ kg / m$^3$

Mass of water, $m = \rho V = 30 \times 10^3\ kg$

Output power can be obtained as:

$$P_0 = \frac{\text{Work done}}{\text{Time}} = \frac{mgh}{t}$$

$$= \frac{30 \times 10^3 \times 9.8 \times 40}{900} = 13.067 \times 10^3 \text{ W}$$

For input power $P_i$, efficiency $\eta$, is given by the relation:

$$\eta = \frac{P_0}{P_i} = 30\%$$

$$P_i = \frac{13.067}{30} \times 100 \times 10^3$$

$= 0.436 \times 10^5$ W

$= 43.6$ kW

**Q. 3. A trolley of 300 kg carrying a sand bag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the trolleys floor at the rate of 0.05 kgs$^{-1}$. What is the speed of the trolley after the entire sand bag is empty?**

**[NCERT Ex. Q. 6.19, Page 137]**

**Ans.** As the trolley carrying the sand bag moving uniformly, therefore, external force on system= zero.

When the sand leaks out, it does not lead to application of any external force on the trolley. Hence, the speed the trolley shall not change.

So, speed will remain same

Speed = 27 km/h.

## Long Answer Type Questions (5 marks each)

**Q. 1. A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force ? (b) Fat supplies 3.8 × 10$^7$ J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up ?**

**[NCERT Ex. Q. 6.22, Page 137]**

**Ans. (a)** Mass = 10 kg, $h = 0.5$ m

Number of times the weight is lifted, $n = 1000$

$\therefore$ Work done against gravitational force $= n(mgh)$

$= 1000 \times 10 \times 9.8 \times 0.5 = 49$ kJ

**(b)** Energy equivalent of 1 kg of fat = 3.8 × 107 J

Efficiency rate = 20%

$$\text{Mechanical energy} = \frac{20}{100} \times 3.8 \times 10^7 \text{ J}$$

$= 76 \times 10^5$ J

$$\text{Equivalent mass of fat lost by dieter} = \frac{1}{76 \times 10^5} \times 49 \times 10^3$$

$= 6.45 \times 10^{-3}$ kg

# TOPIC-2
## Energy and Collision

## Quick Review

- **Energy :** Energy of a body is defined as the capacity of the body to do the work. Energy is a scalar quantity. It has the same units and dimensions as those of work. Some practical units of energy and their relation with S.I. unit of energy (joule) are :
  - **(i)** 1 calorie = 4.2 J
  - **(ii)** 1 kiloWatt hour (kWh) = $3.6 \times 10^6$ J
  - **(iii)** 1 electron volt (1 eV) = $1.6 \times 10^{-19}$ J
- **Work-Energy Theorem :** According to this principle, work done by net force in displacing a body is equal to change in kinetic energy of the body and *i.e.*

$$W = K_f - K_i$$

$K_f$ = final K.E.

$K_i$ = initial K.E.

- **Mechanical Energy Conservation :** Total mechanical energy of a system is conserved if the forces, doing work on it are conservative. It is also called principle of conservation of total mechanical energy.

$$K + U = \text{constant}$$

K = Kinetic energy

U = Potential energy

- **Principle of Conservation of Energy :** This law states that energy can neither be created nor be destroyed, within an isolated system, *i.e.*, sum total of all kinds of energy in this universe remains constant. However, energy can be converted from one form to another, such that amount of energy disappearing in one form is always equal to amount of energy appearing in the other form.

  For example, when a body falls freely, its potential energy goes on decreasing and its kinetic energy goes on increasing. The total mechanical energy of falling body remains constant at all points.
- **Collisions :** When a body strikes against another body such that there is exchange of energy and linear momentum take place then the two are said to collide. Collisions are of two types :
  - **(i) Perfectly elastic collision** is that in which there is no change in kinetic energy of the system, *i.e.*,

    Total K.E. before collision = Total K.E. after collision.

    *e.g.*, collisions between atomic and subatomic particles are perfectly elastic collisions.
  - **(ii) Perfectly inelastic collision** is that in which K.E. is not conserved. Here, the bodies stick together after impact.

    Linear momentum is conserved in every collision elastic as well as inelastic, further total energy is also conserved in all such collisions. Kinetic energy alone is not conserved in inelastic collisions.

## Know the Terms

- **Kinetic Energy** is the energy possessed by the body by virtue of its motion. It is always positive.
- **Potential Energy** is the energy possessed by the body by virtue of its position. It can be both negative as well as positive.
- **Gravitational Potential Energy** is the energy possessed by the body by virtue of its position with respect to center of earth or other body.
- **Potential Energy of spring** is the energy associated with the state of compression or expansion of an elastic spring.

- **Heat Energy** is the energy possessed by a body by virtue of random motion of molecules of body.
- **Internal Energy** is the energy possessed by the body by virtue of particular configuration of its molecules.
- **Chemical Energy** comes from the molecules participating in the chemical reaction having different binding energies.
- **Nuclear Energy** is the energy obtained from conversion of nuclear mass into energy in the atomic nucleus. *i.e.,* Nuclear fission, Nuclear fusion.
- **Transformation of Energy** is the process of change of energy from one form to other.
- **Coefficient of Restitution or Coefficient of Resilience** is the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is denoted by '$e$'.

## Know the Formulae

- K.E. $= \frac{1}{2} mv^2$,

  where, $m$ = mass, $v$ = velocity of particles
- $$\text{P.E.} = mgh$$
- $$v = \sqrt{2gh}$$
- $$F = -kx,$$

  where, $k$ = spring constant, $x$ = compression.
- **Mass Energy Equivalence :**

  $$E = mc^2$$

  where, $m$ = mass that disappears

  E = energy that appears

  $c$ = velocity of light
- **Coefficient of Resilience** $(e) = \dfrac{\text{Relative velocity of separation (after collision)}}{\text{Relative velocity of approach (before collision)}}$

  $$e = \frac{v_2 - v_1}{u_1 - u_2}$$

  **for perfectly elastic collision,** $e = 1$

  **for perfectly inelastic collision,** $e = 0$
- **Elastic collision in 1-Dimension :**

  $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

  $$v_2 = \frac{2m_1u_1}{m_1 + m_2} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$
- **Inelastic Collision in 1–Dimension :**

  $$v = \frac{m_1u_1}{m_1 + m_2}$$

  $$\text{Loss in K.E.} = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$$

## Know the Links

- *www.vedantu.com*
- *www.learncbse.com*
- *www.gradestack.com*

# MCQ/Fillups/True or False **(1 mark each)**

## (A) Multiple Choice Questions

**Q. 1. A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall?**

**(a) Kinetic energy.**
**(b) Potential energy.**
**(c) Total mechanical energy.**
**(d) Total linear momentum.**

**[NCERT Exemp. Q. 6.5, Page 39]**

**Ans. Correct option:** (c)

*Explanation:* A body is falling under the gravity then kinetic energy increases and potential energy decreases but total mechanical energy (Kinetic energy + Potential energy) is constant.

**Q. 2. During inelastic collision between two bodies, which of the following quantities always remain conserved?**

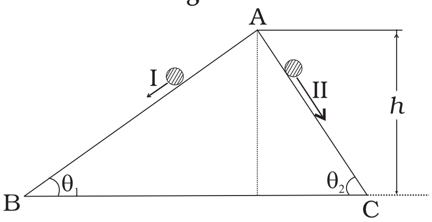
**(a) Total kinetic energy.**
**(b) Total mechanical energy.**
**(c) Total linear momentum.**
**(d) Speed of each body.**

**[NCERT Exemp. Q. 6.6, Page 39]**

**Ans. Correct option:** (c)

*Explanation:* In inelastic collision total linear momentum remains conserved. But total energy does not remain conserved.

**Q. 3. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure.**




**Which of the following statement is correct?**

**(a) Both the stones reach the bottom at the same time but not with the same speed.**
**(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.**
**(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.**
**(d) Both the stones reach the bottom at different times and with different speeds.**

**[NCERT Exemp. Q. 6.7, Page 39]**

**Ans. Correct option:** (c)

*Explanation:*

According to law of conservation of mechanical energy

PE at the top = KE at the bottom

$\therefore \quad mgh = \frac{1}{2}mv_1^2$ ...(i)

and $\quad mgh = \frac{1}{2}mv_2^2$ ...(ii)

by (i) & (ii), we get $v_1 = v_2$

Acceleration of the 2 stones are $a_1 = g \sin \theta_1$ & $a_2 = g \sin \theta_2$ respectively

As $\quad \theta_2 > \theta_1 \therefore a_2 > a_1$

from $\quad v = u + at = 0 + at \Rightarrow t = \frac{v}{a}$

$t \propto \frac{1}{a}$, and $a_2 > a_1$

$\therefore \quad t_1 > t_2$

Hence, stone 2 will take lesser time & reach the bottom earlier than stone 1.

**(c)** According to law of conservation of mechanical energy,

PE at the top = KE at the bottom

and

$\therefore mgh = \frac{1}{2}mv_1^2$ (i)

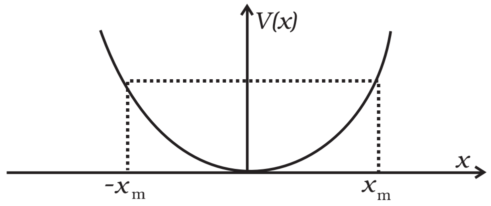
$mgh = \frac{1}{2}mv_2^2$ (ii)

by (i) & (ii), we get $v_1 = v_2$

$\therefore$ both the stones will reach the bottom with the same speed.

**Q. 4. The potential energy function for a particle executing linear SHM is given by V($x$) = $\frac{1}{2}kx^2$ where k is the force constant of the oscillator (Fig.). For k = 0.5N/m. The graph of V($x$) versus $x$ is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$, then which of the following is correct?**

**(a) V = 0, K = E**
**(b) V = E, K = 0**
**(c) V < E, K = 0**
**(d) V = 0, K < E.**




**[NCERT Exemp. Q. 6.8, Page 40]**

**Ans. Correct option:** (b)

*Explanation:*

At any instant of time, the total energy of an oscillator is the sum of kinetic energy and potential energy.

Total energy E = U + $K$

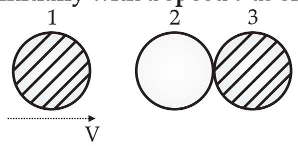
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

At $x = \pm x_m$, the particle turns back

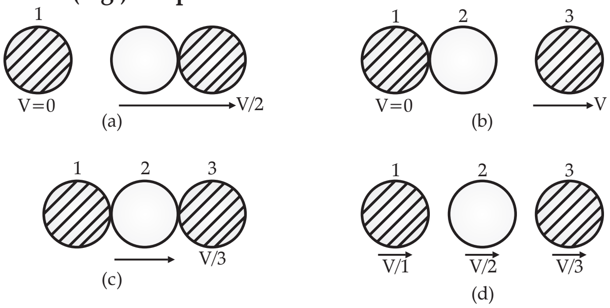
$\therefore$ its velocity at this point is zero, i.e. $v = 0$

$$\therefore\ k=0 \quad \therefore E=\frac{1}{2}kx^2$$

**Q. 5. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed $v$ as shown in Fig.**




**If the collision is elastic, which of the following (Fig.) is a possible result after collision?**




**[NCERT Exemp. Q. 6.9, Page 40]**

**Ans. Correct option:** (b)

***Explanation:***

Let $m$ be the mass of each ball bearing.

Total kinetic energy of the system before collision,

$$=\frac{1}{2}mv^2+0$$
$$=\frac{1}{2}mv^2$$

In (a), kinetic energy of the system after collision,

$$k_1=\frac{1}{2}(2m)\left(\frac{v}{2}\right)^2=\frac{1}{4}mv^2$$

In (b), kinetic energy of the system after collision,

$$k_2=\frac{1}{2}(m)(v)^2=\frac{1}{2}mv^2$$

In (c), kinetic energy of the system after collision,

$$k_3=\frac{1}{2}(3m)\left(\frac{v}{3}\right)^2=\frac{1}{6}mv^2$$

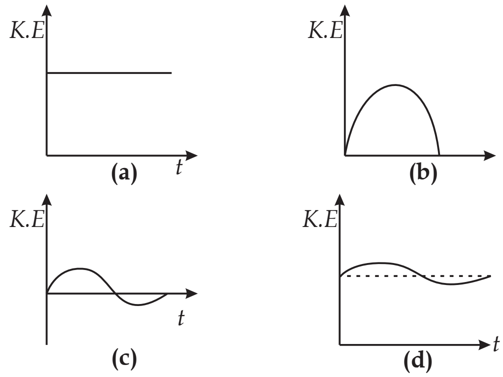
In (d), kinetic energy of the system after collision,

$$k_4=\frac{1}{2}mv^2+\frac{1}{2}m\left(\frac{v}{2}\right)^2+\frac{1}{2}m\left(\frac{v}{3}\right)^2$$
$$=\frac{49}{72}mv^2$$

K is only conserved in (b)

$\therefore$ (b) is the only possibility

**Q. 6. Which of the diagrams shown in Fig. 6.6 most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit?**

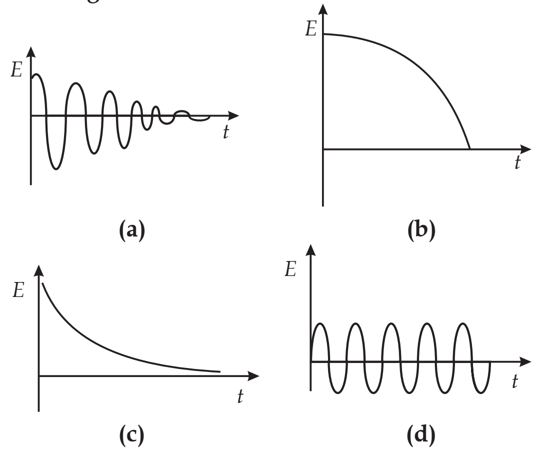



**[NCERT Exemp. Q. 6.12, Page 41]**

**Ans. Correct option:** (d)

***Explanation:*** As Earth moves once around the sun in its elliptical orbit, it kinetic energy is maximum when it is farthest from the sun. As kinetic energy is never zero during its motion, $\therefore$ option (d) is right.

**Q. 7. Which of the diagrams shown in Fig. represents variation of total mechanical energy of a pendulum oscillating in air as function of time?**




**[NCERT Exemp. Q. 6.13, Page 42]**

**Ans. Correct option:** (c)

***Explanation:***

During oscillations of a pendulum in air, its total mechanical energy decreases exponentially with time. $\therefore$option (c) represents correct diagram.

**Q. 8. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be**

**(a)** $250\pi^2$ **(b)** $100\ \pi^2$

**(c)** $5\ \pi^2$ **(d)** 0

**[NCERT Exemp. Q. 6.14, Page 42]**

**Ans. Correct option:** (a)

***Explanation:***

Given mass, $m$ = 5 kg,

Radius, R = 1 m

$$v = 300\text{rpm} = \frac{300}{60}\text{rps} = 5\text{ rps}$$

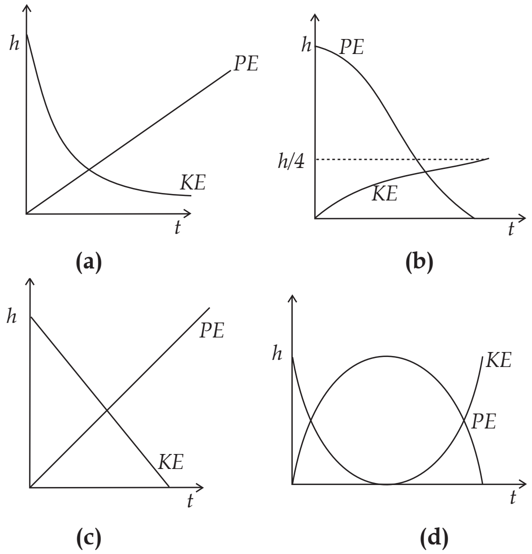
The angular speed,

$\omega = 2\pi v = 2\pi\times 5 = 10\pi$ rad/s

The linear speed is

$$v = \omega R = (10\pi)(\text{rad/s})(1\text{m})$$
$$= 10\pi \text{ m/s}$$
$$\text{K.E.} = k = \frac{1}{2}mv^2$$
$$k = \frac{1}{2} \times (5\text{kg})(10\pi \text{ m/s})^2$$
$$= 250\pi^2 \text{ J}$$

**Q. 9. A raindrop falling from a height *h* above ground, attains a near terminal velocity when it has fallen through a height (3/4)*h*. Which of the diagrams shown in Figure correctly shows the change in kinetic and potential energy of the drop during its fall up to the ground?**




**[NCERT Exemp. Q. 6.15, Page 42]**

**Ans. Correct option:** (b)

***Explanation:***

At $h$, PE of raindrop is maximum and KE=0. As raindrop falls its PE goes on decreasing and KE goes on increasing up to point $\frac{h}{4}$ above the ground. At this stage, rain drop has acquired near terminal velocity (=constant). ∴ At this stage, KE tends to be constant. PE becomes zero when raindrop falls on the ground. Hence (b) is most appropriate.

**Q. 10. In a shot put event an athlete throws the shot put of mass 10 kg with an initial speed of 1 m/s, at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m/s$^2$, the kinetic energy of the shot put when it just reaches the ground will be**

**(a) 2.5 J** **(b) 5.0 J**
**(c) 52.5 J** **(d) 155.0 J**

**[NCERT Exemp. Q. 6.16, Page 42]**

**Ans. Correct option:** (d)

***Explanation:***

$$\text{Initial K.E.} = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(10\text{kg}) \times (1\text{m/s})^2$$
$$= 5 \text{ J}$$

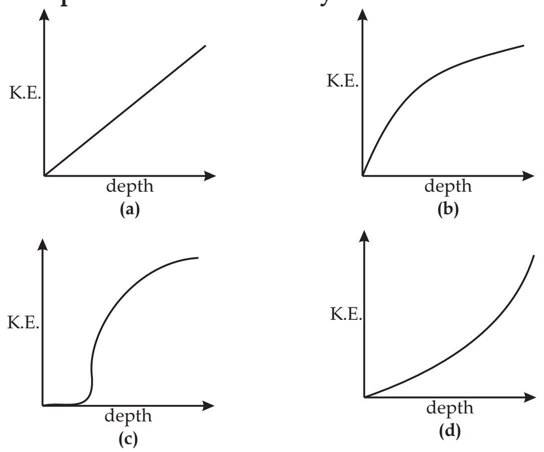
Initial PE at height 1.5m

$$= mgh$$
$$= (10\text{kg})(10\text{m/s}^2)(1.5\text{m})$$
$$= 150 \text{ J}$$

Total initial energy

$$= 155 \text{ J}$$

**Q. 11. Which of the diagrams in Figure correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it a terminal velocity?**




**[NCERT Exemp. Q. 6.19, Page 43]**

**Ans. Correct option:** (b)

***Explanation:***

When iron sphere falls freely in a lake, its motion is accelerated due to gravity and retarted due to viscous force. The effect is increase in velocity and therefore increase in KE till the sphere acquired terminal velocity which is constant. Hence (b) is answer.

**Q. 12. A bullet of mass m fired at 30° to the horizontal leaves the barrel of the gun with a velocity *v*. The bullet hits a soft target at height *h* above the ground while it is moving downward and emerges out with half the kinetic energy it had before hitting the target.**

**Which of the following statements are correct in respect of bullet after it emerges out of the target?**

**(a) The velocity of the bullet will be reduced to half its initial value.**

**(b) The velocity of the bullet will be more than half of its earlier velocity.**

**(c) The bullet will continue to move along the same parabolic path.**

**(d) The bullet will move in a different parabolic path.**

**(e) The bullet will fall vertically downward after hitting the target.**

**(f) The internal energy of the particles of the target will increase.** **[NCERT Exemp. Q. 6.20, Page 44]**

**Ans. Correct option:** (b), (d) and (f)

*Explanation:*

**(a)** $K = \frac{1}{2}mv^2$ or $v = \sqrt{\frac{2K}{m}}$

or $v \propto \sqrt{K}$, at constant mass

$\therefore \frac{v_1}{v_2} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{2K}{K}} = \sqrt{2}$

or $v_2 = \frac{v_1}{\sqrt{2}} = 0.707v_1$

$\therefore$(a) is incorrect

**(b)** $\because v_2 = 0.707v_1 > 0.5v_1$
so (b) is correct

**(c)** velocity of the bullet changed after hitting the target so it follows different parabolic path.
So, (c) is incorrect but (d) is correct.

**(e)** Bullet will follow parabolic path as it has both horizontal as well as vertical velocity after emerging from target, so (e) is incorrect.

**(f)** is correct because loss in KE is used to increase the internal energy of particles of the target.

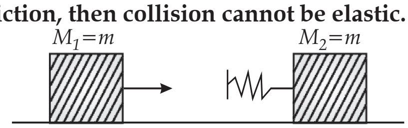
**Q. 13. Two blocks $M_1$ and $M_2$ having equal mass are free to move on a horizontal frictionless surface. $M_2$ is attached to a massless spring as shown Figure. Initially $M_2$ is at rest and $M_1$ is moving toward $M_2$ with speed v and collides head-on with $M_2$.**

**(a) While spring is fully compressed all the KE of $M_1$ is stored as PE of spring.**

**(b) While spring is fully compressed the system momentum is not conserved, though final momentum is equal to initial momentum.**

**(c) If spring is massless, the final state of the $M_1$ is state of rest.**

**(d) If the surface on which blocks are moving has friction, then collision cannot be elastic.**




**[NCERT Exemp. Q. 6.21, Page 44]**

**Ans. Correct option:** (c) and (d)

*Explanation:*

**(c)** The two bodies of equal mass exchange their velocities in a head on elastic collision between them.

**(d)** The collision is inelastic even if the force of friction does not involve.

**Q. 14. A body is moving uni-directionally under the influence of a source of constant power. Its displacement in time t is proportional to :**
**(a) $t^{1/2}$, (b) $t$, (c) $t^{3/2}$, (d) $t^2$**

**[NCERT Ex. Q. 6.10, Page 136]**

**Ans. Correct Option (c)**

***Explanation :***

Suppose constant power acts on the body of the mass $m$ for a time $t$ to give it a velocity $v$

K.E. = work done = power × time

or $\frac{1}{2}mv^2 = Pt$

or $v = \sqrt{\frac{2Pt}{m}}$ ...(i)

We know that $v = \frac{dx}{dt}$

or $dx = vdt$

If x be the displacement of the body, then

$= \int dx = \int vdt = \sqrt{\frac{2P}{m}} \int t^{1/2}dt$

$= \sqrt{\frac{2P}{m}} \left( \frac{t^{1/2+1}}{\frac{1}{2}+1} \right)$

$= \sqrt{\frac{2P}{m}} \frac{t^{3/2}}{3/2} = \frac{2}{3}\sqrt{\frac{2P}{m}} t^{3/2}$

Here, P = constant and $m$ is also constant for a body, so $x$ = constant × $t^{3/2}$ or $x$ is directly proportional to $t^{3/2}$.

## (B) True or False

**Q. 15. State if each of the following statements is true or false. Give reasons for your answer.**

**(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.**

**(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.**

**(c) Work done in the motion of a body over a closed loop is zero for every force in nature.**

**(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.**

**[NCERT Ex. Q. 6.7, Page 135]**

**Ans. (a)** False, it is total momentum and total energy which are conserved.

**(b)** False, force on the system can change total energy of the system.

**(c)** False, in the case of non-conservative forces like friction, the work done in a closed loop is not zero.

**(d)** True, there is at least some loss of kinetic energy in an inelastic collision.

# Very Short Answer Type Questions (1 mark each)

**Q. 1. A body falls towards earth in air. Will its total mechanical energy be conserved during the fall? Justify.**

**[NCERT Exemp. Q. 6.26, Page 45]**

**Ans.** No, the mechanical energy is not conserved, Because resistive force of air also acts on the body which is a non-conservative force, So the gain in KE would be smaller than the loss in PE.

**Q. 2. In an elastic collision of two billiard balls, which of the following quantities remain conserved the short time of collision of the balls (i.e., when they are in contact).**
**(a) Kinetic energy.**
**(b) Total linear momentum?**
**Given reason for your answer in each case.**
**[NCERT Exemp. Ex. Q. 6.28, Page 45]**

**Ans. (b)** Total linear momentum remain conserved.
While balls are in contact, there may be deformation which means elastic potential energy which came from part of KE. Momentum is always conserved.

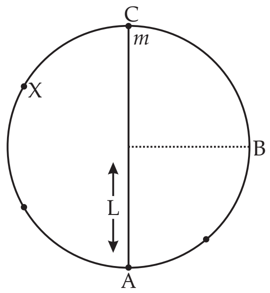
**Q. 3. Give example of a situation in which an applied force does not result in a change in kinetic energy.**
**[NCERT Exemp. Q. 6.31, Page 46]**

**Ans. (i)** Work done during circular motion as force and displacement are perpendicular to each other in circular motion.
**(ii)** A charged particle moving in an uniform magnetic field.

**Q. 4. Two bodies of unequal mass are moving in the same direction with equal kinetic energy. The two bodies are brought to rest by applying retarding force of same magnitude. How would the distance moved by them before coming to rest compare?**
**[NCERT Exemp. Q. 6.32, Page 47]**

**Ans.** By work energy theorem, work done is equal to change in K.E.
Both bodies had same K.E. and hence same amount of work is needed to be done. Since force applied is same, they would come to rest within the same distance.

**Q. 5. A bob of mass m suspended by a light string of length L is whirled into a vertical circle as shown in Figure. What will be the trajectory of the particle if the string is cut at**
**(a) point B?**
**(b) point C?**
**(c) point X?**




**[NCERT Exemp. Q. 6.33, Page 47]**

**Ans. (a)** If the string is cut at point B, the trajectory is straight line vertically downward.
**(b)** At point C, parabolic path with vertex at C
**(c)** At point X, it is parabolic path with vertex higher than C.

**Q. 6. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the**
**(a) Work done by the applied force in 10 s,**
**(b) Work done by friction in 10 s,**
**(c) Work done by the net force on the body in 10 s,**
**(d) Change in kinetic energy of the body in 10 s,**
**And interpret your results.**
**[NCERT Ex. Q. 6.2, Page 134]**

**Ans. Given :** $m = 2$ kg, $u = 0$, F = 7 N, $\mu = 0.1$, $t = 10$ s

Acceleration = $a_1 = \frac{F}{m} = \frac{7}{2} = 3.5 \text{ ms}^{-2}$

Frictional force = $f = \mu R = \mu mg$
$= 0.1 \times 2 \times 9.8$
$= -1.96$ N.

Retardation-
$$a_2 = \frac{-f}{m} = \frac{-1.96}{2} = -0.98 \text{ ms}^{-2}$$

Net acceleration, $a = a_1 + a_2 = 3.5 + (-0.98)$
$= 2.52 \text{ ms}^{-2}$

Distance, in 10 seconds
$$s = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2} \times 2.52 \times (10)^2$$
$= 126$ m

**(a)** Work done (applied force) = F × S = 7 × 126
= 882 J.

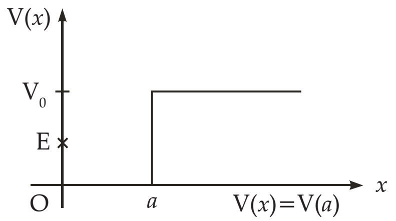
**(b)** Work done (frictional force) = −F × S
= −1.96 × 126
= −246.9 J

**(c)** work done (net force) = Net force × $s$
net force, F = 7 + (–1.96) = 5.04
∴ W = 5.04 × 126 = 635 J.

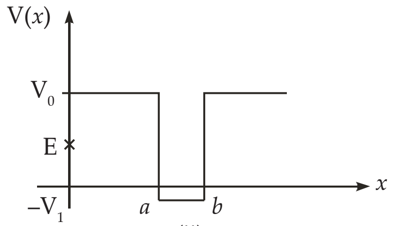
**(d)** Velocity at the end of 10 second.
$v = u + at = 0 + 2.52 \times 10 = 25.2$ m/s.
Initial K.E = 0
$$Final\ KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times (25.2)^2 = 635 J$$

Change in K.E. = 635 −0 = 635J.
∴ K.E.'s change is equal to work done by net force.

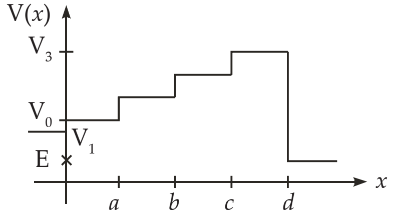
**Q. 7. Given below in figures are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the energy axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.**



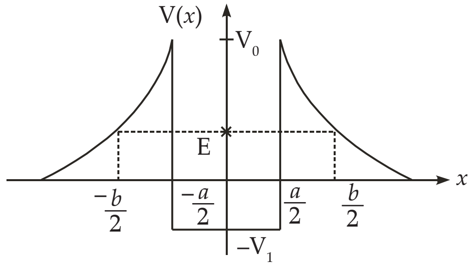

(i)




(ii)




(iii)




(iv)

**[NCERT Ex. Q. 6.3, Page 134]**

**Ans.** It is clear that the total energy of the body is given by

$$E = K.E. + P.E.$$

or $$K.E. = E - P.E. = \frac{1}{2}mv^2$$

Hence, K.E. of the body can never be negative. Thus, P.E. cannot be greater than E.

**(i)** In the region between $x = 0$ and $x = a$, P.E. is zero, so K.E. is positive. In the region $x > a$, the P.E. ($V_0$) has a value greater than E. So, K.E. will be negative in the region. Hence, the particle can't be present in the region $x > a$.

The minimum total energy that the particle can have in this case is zero.

**(ii)** In region $x < a$ and $x > b$, the P.E. is $V_0$ which is greater than the total energy of the particle. So, K.E. will be negative in this region. Thus, the particle cannot be present in the region $x < a$ and $x > b$. In the region between $x > a$ and $x < b$, the P.E. is negative this clears that there is a positive value of K.E. Hence, the particle can be present in the region between $x > a$ and $x < b$.

The minimum total energy that the particle can have in this case is $-V_1$.

**(iii)** Here in all the regions, *i.e.*, $-\infty < x < a, a < x < b, b < x < c, c < x < d$ and $d < x < \infty$, the P.E. is greater than the total energy. Hence, the particle cannot be present in the region $-\infty < x < \infty$.

The minimum total energy that the particle can have in this case is $V_1$.

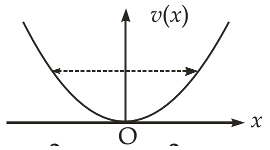
**(iv)** In this case, the P.E. of the particle is more than the total energy (E) in the regions

$$-\frac{b}{2} < x < -\frac{a}{2} \text{ and } \frac{a}{2} < x < \frac{b}{2}.$$

Hence, K.E. of the particle will be negative in these regions, thus it will not be present in the regions $-\frac{b}{2} < x < -\frac{a}{2}$ and $\frac{a}{2} < x < \frac{b}{2}$.

The minimum total energy that the particle can have in this case is $-V_1$.

**Q. 8. The potential energy function for a particle executing linear simple harmonic motion is given by $v(x) = kx^2/2$, where $k$ is the force constant of the oscillator. For $k = 0{\cdot}5\ Nm^{-1}$, the graph of $v(x)$ versus $x$ is shown in figure. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches $x = \pm 2$ m.**




**[NCERT Ex. Q. 6.4, Page 135]**

**Ans.** Total energy,

$$E = K.E. + P.E.$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where, $m$ = mass of particle

$k$ = force constant = 0.5 $Nm^{-1}$

$x$ = displacement, $v$ = velocity of particle

or $$1 = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}x^2$$

or $$x^2 + 2mv^2 = 4$$

Then particle turns back at $x$ when $v = 0$

$\therefore$ $$x^2 + 2m(0)^2 = 4$$

or $$x = \pm 2 \text{ m}.$$

**Q. 9. Underline the correct alternative:**

**(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.**

**(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.**

**(c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.**

**(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.** **[NCERT Ex. Q. 6.6, Page 135]**

**Ans. (a)** Decreases

A conservative force does a positive work on a body when it displaces the body in the direction of force. As a result, the body advances toward the center of force. It decreases the separation between the two, thereby decreasing the potential energy of the body.

**(b)** Kinetic energy

The work done against the direction of friction reduces the velocity of a body. Hence, there is a loss of kinetic energy of the body.

**(c)** External force

Internal forces, irrespective of their direction, cannot produce any change in the total momentum of a body. Hence, the total momentum of a many- particle system is proportional to the external forces acting on the system.

**(d)** Total linear momentum

The total linear momentum always remains conserved whether it is an elastic collision or an inelastic collision.

**Q. 10. Answer carefully, with reasons :**

**(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (*i.e.,* when they are in contact) ?**

**(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?**

**(c) What are the answers to (a) and (b) for an inelastic collision ?**

**(d) Is the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ?**

**(Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy)**

**[NCERT Ex. Q. 6.8, Page 135]**

**Ans. (a)** No, the total kinetic energy is not conserved during the given elastic collision due to a part of the kinetic energy is used in deforming the balls in that short interval for which they are in contact during collision and gets converted into potential energy. In an elastic collision, the K.E. before and after collision is same.

**(b)** Yes, the total linear momentum is conserved during the short time of an elastic collision of two balls.

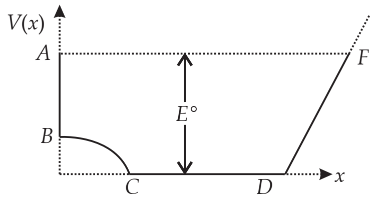
**(c)** In an inelastic collision, total K.E. is not conserved during collision and even after collision. The total linear momentum is however conserved after collision.

**(d)** The collision is elastic because involved forces are conservative.

## Short Answer Type Questions

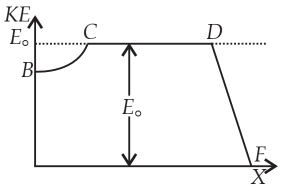
**(2 or 3 marks each)**

**Q. 1. A graph of potential energy V (x) verses x is shown in figure. A particle of energy $E_0$ is executing motion in it. Draw graph of velocity and kinetic energy versus x for one complete cycle AFA.**

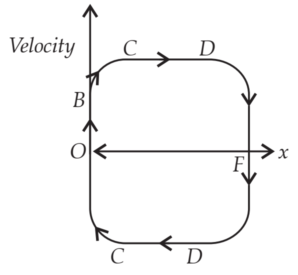



**[NCERT Exemp. Q. 6.34, Page 46]**

**Ans.**




**(a) Kinetic energy versus x.**




**(b) Velocity versus *x*.**

**Q. 2. A ball of mass m, moving with a speed $2v_0$, collides inelastically (e > 0) with an identical ball at rest. Show that**

**(a) For head-on collision, both the balls move forward.**

**(b) For a general collision, the angle between the two velocities of scattered balls is less than 90°.**

**[NCERT Exemp. Q. 6.35, Page 46]**

**Ans. (a)** For head on collision:

Let $V_1$, $V_2$ be the velocities of two balls after collision.

Conservation of momentum

$\Rightarrow \qquad 2mv_0 = mv_1 + mv_2$

or $\qquad 2v_0 = v_1 + v_2$

and $\qquad e = \dfrac{v_2 - v_1}{2v_0} \Rightarrow v_2 = v_1 + 2v_0e$

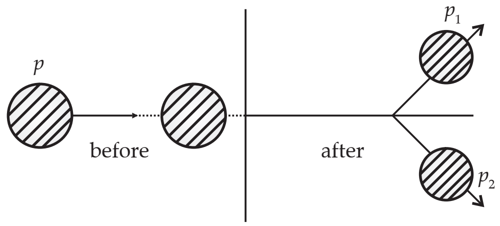
$\therefore \qquad 2v_1 = 2v_0 - 2ev_0$

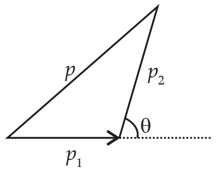
$\therefore \qquad v_1 = v_0(1 - e)$

Since e < 1 ⇒ $v$, has the same sign as $v_0$, therefore the ball moves on after collision.

**(b)** Conservation of momentum ⇒ $p = p_1 + p_2$

But KE is lost ⇒ →




$$KE > KE_1 + KE_2 \Rightarrow \frac{p^2}{2m} > \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$$

$\Rightarrow \therefore p > p_1^2 + p_2^2$

Thus $p$, $p_1$ and $p_2$ are related as shown in the figure.

θ is acute (less than 90°)

($p^2 = p_1^2 + p_2^2$ would give θ = 90°)

**Q. 3. Consider a one-dimensional motion of a particle with total energy E. There are four regions A, B, C and D in which the relation between potential energy V, kinetic energy (k) and total energy E is as given below:**

**Region A : V > E**

**Region B : V < E**

**Region C : K > E**

**Region D : V > K**

**State with reason in each case whether a particle can be found in the given region or not.**

**[NCERT Exemp. Q. 6.36, Page 46]**

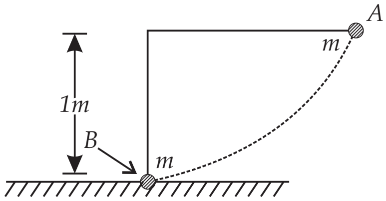
**Ans. Region A :** No, V > E ⇒ E = V + K, K = E – V ⇒ V > E , so K < 0 as KE will become negative.

**Region B :** Yes, V < E, K = E –V, K > 0, total energy can be greater than PE for non zero K.E.

**Region C :** Yes, K > 0, V = E – K, V < 0, KE can be greater than total energy if its PE is negative.

**Region D :** Yes, V > K, K = E – V as PE can be greater than KE.

**Q. 4. The bob A of a pendulum released from horizontal to the vertical hits another bob B of the same mass at rest on a table as shown in figure.**




**If the length of the pendulum is 1 m, calculate**

**(a) The height to which bob A will rise after collision.**

**(b) The speed with which bob B starts moving.**

**(c) Neglect the size of the bobs and assume the collision to be elastic.**

**[NCERT Exemp. Q. 6.37, Page 46]**

**Ans. (a)** When ball A strike to an identical ball at rest then ball A transfer its entire momentum to the ball on the table and does not rise at all.

**(b)** $\frac{1}{2}mv^2 = mgh$

or $\qquad v = \sqrt{2gh}$,

here, $\qquad g = 9.8\ \text{m/s}^2$,

$\qquad h = 1\ \text{m}$

Putting values we get

$v = 4.43$ m/s

**Q. 5. A raindrop of mass 1.00 g falling from a height of 1 km hits the ground with a speed of 50 ms$^{-1}$. Calculate**

**(a) the loss of P.E. of the drop.**

**(b) the gain in K.E. of the drop.**

**(c) Is the gain in K.E. equal to loss of P.E.? If not why.**

**Take g = 10 ms$^{-2}$**

**[NCERT Exemp. Q. 6.38, Page 46]**

**Ans.** Given mass of rain drop, (m)=0.001 kg

$= 1.0 \times 10^{-3}$ kg

Height, $h$=1 km=1000 m

Speed, $v$ = 50 m/s, $u$ = 0.

**(a)** Loss of $PE = mgh = 1 \times 10^{-3} \times 10 \times 10^3 = 10\,J$

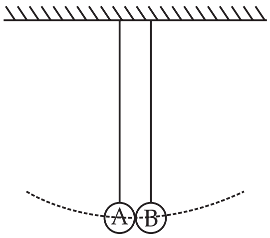
**(b)** Gain in $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 10^{-3} \times 2500 = 1.25\,J$

**(c)** No, because a part of PE is used up in doing work against the viscous drag of air.

**Q. 6. Two pendulums with identical bobs and lengths are suspended from a common support such that in rest position the two bobs are in contact (Fig.). One of the bobs is released after being displaced by 10° so that it collides elastically head-on with the other bob.**

**(a) Describe the motion of two bobs.**

**(b) Draw a graph showing variation in energy of either pendulum with time, for 0 ≤ t ≤ 2T. Where T is the period of each pendulum.**

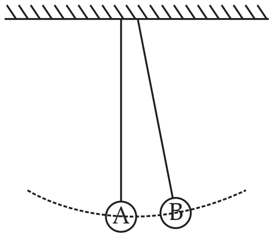



**[NCERT Exemp. Q. 6.39, Page 46]**

**Ans. (a)** At $t = 0$, A is at lowest position and B is at highest position as. K.E. of both bobs are zero while potential energy of B is maximum and A is zero.

Now bob is released.

After collision




At $t = \frac{T}{4}$, B reaches to A and collide elastically as both bobs are identical. The energies are

$KE_B = E, KE_A = 0, PE_A = 0, PE_B = 0$

At $t = \frac{2T}{4}$, A reaches maximum height and B remains at its lowest position.

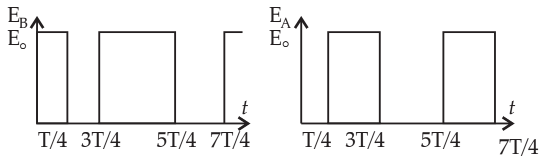
$KE_A = 0, \; KE_B = 0, \; PE_A = E, \; PE_B = 0$

At $t = \frac{3T}{4}$, Bob A hits B which was at rest elastically & A becomes at rest, B moves upward.

$KE_A = 0, \; KE_B = E, \; PE_A = 0, \; PE_B = 0$

$E_A = 0, \; E_B = E$

**(b)**




**Q. 7. Suppose the average mass of raindrops is $3.0 \times 10^{-5}$ kg and their average terminal velocity 9 m $s^{-1}$. Calculate the energy transferred by rain to each square metre of the surface at a place which receives 100 cm of rain in a year.**

**[NCERT Exemp. Q. 6.40, Page 47]**

**Ans. Given :** $m = 3.0 \times 10^{-5}$ kg,

$\rho = 1.0 \times 10^3$ kg/m$^3$,

$v = 9$ m/s

$A = 1m^2$,

$h = 100 \text{ cm} \Rightarrow V = 1m^3$

$M = \rho V = 10^3$ kg,

Energy transferred by rain,

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \times 10^3 \times (9)^3 = 4.05 \times 10^4 \text{ J.}$$

**Q. 8. An engine is attached to a wagon through a shock absorber of length 1.5m. The system with a total mass of 50,000 kg is moving with a speed of 36 km/hr when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, calculate the spring constant.**

**[NCERT Exemp. Q. 6.41, Page 47]**

**Ans.** $m = 50,000$ kg,

$v = 36 \times \frac{5}{18}$ m/s $= 10$ m/s

$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 10^4 \times 10^2 J$

$= 2.5 \times 10^6 J$

90% of KE of wagon lost due to friction and only 10% of this is stored in the spring.

$\frac{1}{2}kx^2 = 2.5 \times 10^4 = 10\% \text{ of } 2.5 \times 10^6 J$

Here, $x = 1$ m

so, $k = \frac{2 \times 2.5 \times 10^4}{(1)^2}$ N/m

$k = 5.0 \times 10^4$ N/m

**Q. 9. An adult weighing 600N raises the centre of gravity of his body by 0.25 m while taking each step of 1 m length in jogging. If he jogs for 6 km, calculate the energy utilised by him in jogging assuming that there is no energy loss due to friction of ground and air. Assuming that the body of the adult is capable of converting 10% of energy intake in the form of food, calculate the energy equivalents of food that would be required to compensate energy utilised for jogging.**

**[NCERT Exemp. Q. 6.42, Page 47]**

**Ans.** $mg = 600$ N, $g = 10$ m/s$^2$, $h = 0.25$m

No. of steps in 6 km $= \frac{6000 \text{ m}}{1 \text{ m / step}} = 6000$ steps

In 6 km there are 6000 steps.

$\therefore$ E = 6000 $(mg)h$

$= 6000 \times 600 \times 0.25$ J

$= 9 \times 10^5$ J

This is 10% of intake energy

$\therefore$ Intake energy $= 10 E = 9 \times 10^6$ J.

**Q. 10. On complete combustion a litre of petrol gives off heat equivalent to $3 \times 10^7$ J. In a test drive a car weighing 1200 kg, including the mass of driver, runs 15 km per litre while moving with a uniform speed on a straight track. Assuming that friction offered by the road surface and air to be uniform, calculate the force of friction acting on the car during the test drive, if the efficiency of the car engine were 0.5. [NCERT Exemp. Q. 6.43, Page 48]**

**Ans.** $\therefore$ Energy given by car in 1 litre petrol

$= 0.5 \times 3 \times 10^7$ J

$= 1.5 \times 10^7$ Joule

With 0.5 efficiency, 1 litre generates $1.5 \times 10^7$ J. which is used for 15 km drive.

$Fd = 1.5 \times 10^7$ J, where $d = 1.5 \times 10^4$ m

Force of friction $F = \frac{1.5 \times 10^7 \text{ J}}{1.5 \times 10^4 \text{ m}}$

$F = 10^3$ N

**Q. 11. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds, (electron mass = $9.11 \times 10^{-31}$ kg, proton mass = $1.67 \times 10^{-27}$ kg, 1 eV = $1.60 \times 10^{-19}$J).**

**[NCERT Ex. Q. 6.12, Page 136]**

**Ans.** Mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg

Mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg

Kinetic energy of the electron, $E_{Ke} = 10$ keV $= 10^4$ eV

$= 10^4 \times 1.60 \times 10^{-19}$

$= 1.60 \times 10^{-15}$ $J$

Kinetic energy of the proton, $E_{Kp} = 100$ $ke$ V $= 10^5$ $eV = 1.60 \times 10^{-14}$ J

For the velocity of an electron $V_e$, its kinetic energy is given by the relation:

$E_{Ke} = (1/2)\, mv^2{}_e$

$\therefore v_e = \sqrt{\frac{2 \times E_{Ke}}{m_e}}$

$= \sqrt{\frac{2 \times 1.60 \times 10^{-15}}{9.11 \times 10^{-31}}} = 5.93 \times 10^7$ m/s

For the velocity of a proton $v_p$, its kinetic energy is given by the relation:

$E_{ke} = (1/2)\, mv^2{}_e$

$v_p = \sqrt{\frac{2 \times E_{kp}}{m_p}} = \sqrt{\frac{2 \times 1.60 \times 10^{-14}}{1.67 \times 10^{-27}}} = 4.38 \times 10^6$ $m/s$

Hence, the electron is moving faster than the proton.

The ratio of their speeds $\frac{v_e}{v_p} = 5.93 \times 10^7 / 4.38 \times 10^6$

$= 13.54 : 1$

Electron is faster; Ratio of speeds is 13.54 : 1

**Q. 12. A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration due to viscous resistance of the air until at half its original height. It attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey ? What is the work done by the resistive force in the entire journey of its speed on reaching the ground is 10 ms$^{-1}$ ?**

**[NCERT Ex. Q. 6.13, Page 136]**

**Ans.** Given, $r = 2$ mm $= 2 \times 10^{-3}$ m

Using formula,

P.E. $= mgh$

where, $m = \frac{4}{3}\pi r^3 \rho$

$m = \frac{4}{3} \times \frac{22}{7} \times (2 \times 10^{-3})^3 \times 10^3$

$= \frac{32 \times 22}{21} \times 10^{-6}$ kg

and distance moved,

$h = \frac{d}{2} = 250$ m

$\therefore$ Work done during each half = P.E.

$= \frac{32 \times 22 \times 10^{-6}}{21} \times 9.8 \times 250$

$= 0.082$ J.

As per the law of conservation of energy, if no resistive force is present, then the total energy of rain drop will remain the same.

$\therefore$ Total energy at the top :

$E_T = mgh + 0$

$= \frac{4}{3} \times 3.14 \times (2 \times 10^{-3})^3 \times 10^3 \times 9.8 \times 500 \times 10^{-5}$

$= 0.164$ J

Due to the presence of a resistive force, the drop hits the ground with a velocity of 10 m/s.

$\therefore$ Total energy at the ground :

$E_G = \frac{1}{2}\, mv^2 + 0$

$= \frac{1}{2} \times \frac{4}{3} \times 3.14 \times (2 \times 10^{-3})^3 \times 10^3 \times 9.8 \times 10^2$

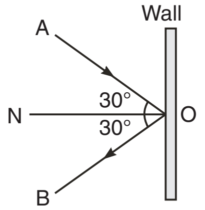
$= 1.675 \times 10^{-3}$ J

$\therefore$ Resistive force $= E_G - E_T$

$= -0.162$ J

**Q. 13. A molecule in a gas container hits a horizontal wall with speed 200 ms$^{-1}$ and angle 30° with normal and rebounds with same speed. Is momentum conserved in the collision ? Is the collision elastic or inelastic ? [NCERT Ex. Q. 6.14, Page 136]**

**Ans.** Since, the wall is too heavy, the recoiling molecule produces no velocity in the wall.




When $m$ is mass of the gas molecule and M is mass of wall, the total K.E. after collision,

$E_2 = \frac{1}{2} m (200)^2 + \frac{1}{2} M (0)^2$

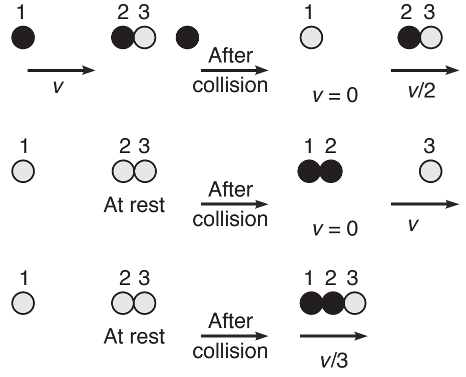
or $E_2 = 2 \times 10^4$ mJ

which is the K.E. of the molecule before collision

$[E_1 = \frac{1}{2} m(200)^2 = 2 \times 10^4 \text{ mJ}]$

Hence, the collision is elastic.

**Q. 14. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed *v*. If the collision is elastic, which of the following figure is a possible result after collision ?**




**[NCERT Ex. Q. 6.16, Page 136]**

**Ans.** Suppose, $m$ = mass of each ball bearing

Before collision, total K.E. of the system

$$= \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2 \quad ...(1)$$

After collision, K.E. of the system is given by

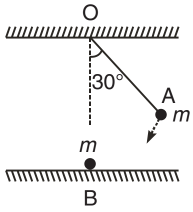
case (i) $$E_1 = \frac{1}{2}2m\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2 \quad ...(2)$$

case (ii) $$E_2 = \frac{1}{2}mv^2 \quad ...(3)$$

case (iii) $$E_3 = \frac{1}{2}(3m)\left(\frac{v}{2}\right)^2 = \frac{1}{6}mv^2 \quad ..(4)$$

Hence from above equations, we see that the K.E. is conserved only in case (ii), so, case (ii) is the only possible result after collision.

**Q. 15. The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in figure given below. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.**




**[NCERT Ex. Q. 6.17, Page 137]**

**Ans.** It is quite clear that in perfectly elastic head on collision, when two equal masses collide with each other, then they exchange their speeds.

In the present case, bob A is moving with certain speed and bob B is at rest. So after collision, bob A comes to rest and the bob B starts moving with the speed of the bob A. The bob A transfers whole of its momentum to bob B and so, bob A, will not rise at all after the collision.

**Note :** The bob shall not rise because when two bodies of same mass undergo an elastic collision, there velocities are interchanged and ball A rests and ball B moves.

## Long Answer Type Questions **(5 marks each)**

**Q. 1. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 30° by a force of 10 N parallel to the inclined surface in figure. The coefficient of friction between block and the in line is 0.1. If the block is pushed up by 10 m along the incline, calculate**
**(a) work done against gravity**
**(b) work done against force of friction**
**(c) increase in potential energy**
**(d) increase in kinetic energy**
**(e) work done by applied force.**

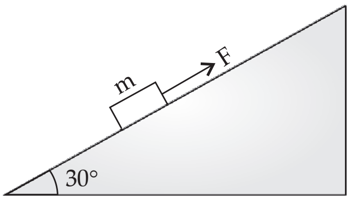



**Fig 6.15**

**[NCERT Exemp. Q. 4.44, Page 48]**

**Ans.** $m = 1\,kg, \theta = 30^\circ, \cos 30^\circ = 0.866, \sin 30^\circ = 0.5$
$F = 10\,N, \mu = 0.1$

Distance $d = 10m$

**(a)** $W_g = mg \sin\theta\, d = 1 \times 10 \times 0.5 \times 10 = 50$ J

**(b)** $W_1 = \mu mg \cos\theta d = 0.1 \times 10 \times 0.866 \times 10 = 8.66$ J

**(c)** $\Delta U = mgh = 1 \times 10 \times 5 = 50$ J

**(d)** $$a = \frac{\{F - (mg\sin 30^\circ + \mu mg\cos 30^\circ)\}}{m}$$

$$a = \frac{[\{10.0 - (5.0 + 0.87)\}]}{1.0} m/s^2$$

$$= 4.13\,m/s^2$$

Apply 3rd Kinematic equation of motion,

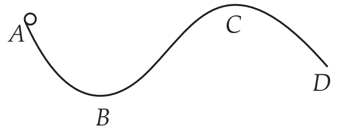
$$v^2 - u^2 = 2ad$$

Change in KE, $\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mad = 41.3J$

**(e)** Work done = Force × displacement
= 10 × 10 J
= 100 J

**Q. 2. A curved surface is shown in figure. The portion BCD is free of friction. There are three spherical balls of identical radii and masses. Balls are**

**released from rest one by one from A which is at a slightly greater height than C.**




**With the surface AB, ball 1 has large enough friction to cause rolling down without slipping; ball 2 has a small friction and ball 3 has a negligible friction.**

**(a) For which balls is total mechanical energy conserved?**

**(b) Which ball (s) can reach D?**

**(c) For balls which do not reach D, which of the balls can reach back A?**

**[NCERT Exemp. Q. 4.45, Page 48]**

**Ans.** **(a)** Force of friction is zero and negligible for ball 1& 3 respectively, so energy is conserved for balls 1 and 3.

**(b)** Ball 1 acquires rotational energy, ball 2 loses energy by friction. They cannot cross at C. Ball 3 can cross over.

**(c)** Ball 3 have negligible friction & crosses C so ball can not reach at A.

Ball 1, 2 turn back before reaching C. Because of loss energy, ball 2 cannot reach back to A. Ball 1 has a rotational motion in "wrong" sense when it reaches B. It cannot roll back to A, because of kinetic friction.

**Q. 3. A rocket accelerates straight up by ejecting gas downwards. In a small time interval $\Delta t$, it ejects a gas of mass $\Delta m$ at a relative speed $u$. Calculate KE of the entire system at $t + \Delta t$ and $t$ and show that the device that ejects gas work = $\left(\frac{1}{2}\right)\Delta mu^2$ in this time interval (neglect gravity).**

**[NCERT Exemp. Q. 4.46, Page 49]**

**Ans.** Let mass of rocket at any time $t$ = M

Velocity of rocket at any time $t = v$

$\Delta m$ = mass of gas ejected in time interval $\Delta t$ .

$$(KE)_{t+\Delta t} = \frac{1}{2}(M-\Delta m)(v+\Delta v)^2 + \frac{1}{2}\Delta m(v-u)^2$$

For rocket For gas

$$= \frac{1}{2}Mv^2 + Mv\Delta v - \Delta mvu + \frac{1}{2}\Delta mu^2$$

Initial $(KE)_i = \frac{1}{2}Mv^2$

$$(KE)_{t+\Delta t} - (KE)t = (M\Delta v - \Delta mu)v + \frac{1}{2}\Delta mu^2$$

By Newton's third law, Reaction force on rocket (upward)=Action force by burnt gases (downward)

$$\frac{Mdv}{dt} = \frac{dm}{dt}|u| \qquad (\because \; F = mu)$$

or $M\Delta v = \Delta mu \Rightarrow M\Delta v - u\Delta m = 0$

substitute this value in (i)

$$K = \frac{1}{2}\Delta mu^2$$

**Q. 4. Two identical steel cubes (masses 50g, side 1 cm) collide head-on face to face with a speed of 10 cm/s each. Find the maximum compression of each. Young's modulus for steel = Y = 2 × $10^{11}$ N/$m^2$. [NCERT Exemp. Q. 4.47, Page 49]**

**Ans.** Mass of cube, $m$ = 50 g = $5.0 \times 10^{-2}$ kg

Speed of cube, $v$ = 10 cm/s = $1.0 \times 10^{-1}$ m/s

Young's modulus Y = $2.0 \times 10^{11}$ N/$m^2$

Side of cube(L) = 1 cm = $1.0 \times 10^{-2}$ m

Apply Hooke's Law,

$$\text{Young modulus } Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$\text{So, } \frac{F}{\Delta L} = \frac{YA}{L} \qquad ...(i)$$

The force acting on each time

$$\text{Or, } \frac{F}{\Delta L} = k \qquad ...(ii)$$

From eqn (i) &eqn (ii)

$$k = \frac{YA}{L} = \frac{YL^2}{L}, \qquad (\text{Here } A = L^2)$$

$$K = YL \qquad ...(iii)$$

$$\text{Initial KE} = 2 \times \frac{1}{2} mv^2 = 5.0 \times 10^{-4} \text{ J}$$

$$\text{Final PE} = 2 \times \frac{1}{2} k \Delta l^2$$

$$= k\Delta l^2 = YL\Delta l^2$$

Apply Law of conservation of energy

$YL\Delta l^2 = 5.0 \times 10^{-4}$

$$\text{or, } \Delta L = \sqrt{\frac{5.0 \times 10^{-4}}{YL}}$$

$$= \sqrt{\frac{5.0 \times 10^{-4}}{(2.0 \times 10^{11} \times 10^{-2})}} \text{ m}$$

$\Delta l = 5.0 \times 10^{-7}$ m

**Q. 5. A balloon filled with helium rises against gravity increasing its potential energy. The speed of the balloon also increases as it rises. How do you reconcile this with the law of conservation of mechanical energy? You can neglect viscous drag of air and assume that density of air is constant.**

**[NCERT Exemp. Q. 4.48, Page 49]**

**Ans.** As dragging viscous force of air on balloon is neglected so there is Net Buoyant Force= V$\rho$g

Let m, V, $\rho_{He}$, denote respectively the mass, volume and density of helium balloon and $\rho_{air}$ density of air

Volume V of balloon displaces volume V of air.

So,

$V(\rho_{air} - \rho_{He})\, g = ma$, or,

$$V(\rho_{air} - \rho_{He})\, g = m\frac{dv}{dt} \quad ...(1)$$

Integrating equation (1) with respect to $t$, we have

$$V(\rho_{air} - \rho_{He})\, gt = mv$$

$$\frac{1}{2}mv^2 = \frac{1}{2}m\frac{V^2}{m^2}(\rho_{air} - \rho_{He})^2 g^2 t^2$$

$$= \frac{1}{2m}V^2(\rho_{air} - \rho_{He})^2 g^2 t^2 \quad ...(2)$$

If the balloon rises to a height h, from $h = ut + \frac{1}{2}at^2$

We get,

$$h = \frac{1}{2}at^2 = \frac{1}{2}\frac{V(\rho_{air} - \rho_{He})}{m}gt^2 \qquad (\because u = 0) \;...(3)$$

From Eqs. (3) and (2)

$$\frac{1}{2}mv^2 = \left[V(\rho_{air} - \rho_{He})g\right]\left[\frac{1}{2m}V(\rho_{air} - \rho_{He})gt^2\right]$$

$$= V(\rho_{air} - \rho_{He})gh$$

Rearranging the terms.

$$\Rightarrow \frac{1}{2}mv^2 + V\rho_{He}gh = V\rho_{air}\, gh$$

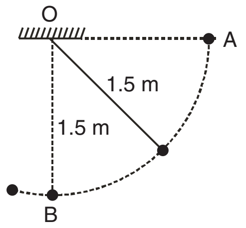
or $KE_{balloon} + PE_{balloon}$ = change in PE of air.

So, as the balloon goes up, and equal volume of air comes down, increase in PE and KE of the balloon is at cost of PE of air [which comes down].

**Q. 6. The bob of a pendulum is released from a horizontal position A as shown in figure. If the length of the pendulum is 1·5 m, what is the speed with which the bob arrives at the lower most point B, given that it dissipates 5% of its initial energy against air resistance.**

**[NCERT Ex. Q. 6.18, Page 137]**

**Ans. Given :** $h$ = 1·5 m, $v$ = ?, Energy dissipated = 5%.
Taking B as the lowest position of the bob, its potential energy at B is zero. At the horizontal position A, total potential energy of the bob is *mgh*. In going from A to B, P.E. of the bob is converted into K.E.




Energy converted = 95% (*mgh*)

When $v$ is velocity acquired at B, then

$$K.E. = \frac{95}{100}mgh$$

or $$K.E. = \frac{1}{2}mv^2$$

or $$= \frac{1}{2}mv^2 = \frac{95}{100}mgh$$

$$v = \sqrt{\frac{95}{100}\times 2gh}$$

or $$v = \sqrt{\frac{19}{20}\times 2\times 9.8\times 1.5}$$

or $$v = 5.285\ ms^{-1}$$

$$\approx 5.3\ ms^{-1}$$

**Q. 7. The blades of a windmill sweep out a circle of area *A*. (a) If the wind flows at a velocity V perpendicular to the circle, what is the mass of the air passing through it in time *t* ?(b) What is the kinetic energy of the air? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that *A*= 30 m³, *v*= 36 km/h and the density of air is 1.2 *kg* m⁻³. What is the electrical power produced?**

**[NCERT Ex. Q. 6.21, Page 137]**

**Ans.** Area of the circle swept by the windmill $= A$
Velocity of the wind $= v$
Density of air $= \rho$

**(a)** Volume of the wind flowing through the windmill per sec $= Av$
Mass of the wind flowing through the windmill per sec $= \rho Av$
Mass $m$, of the wind flowing through the windmill in time $t = \rho Avt$

**(b)** Kinetic energy of air $= (1/2)mv^2$
$= (1/2)(\rho Avt)v^2 = (1/2)\rho Av^3 t$

**(c)** Area of the circle swept by the windmill $= A = 30\ m^2$
Velocity of the wind $= v = 36$ km / h
Density of air, $\rho = 1.2\ kg\ m^{-3}$
Electric energy produced = 25% of the wind energy
$= (25/100) \times$ Kinetic energy of air
$= (1/8)\rho Av^3 t$
Electrical power = Electrical energy / Time
$= (1/8)\rho Av^3 t / t$
$= (1/8)\rho Av^3$
$= (1/8) \times 1.2 \times 30 \times (10)^3$
$= 4.5\ kW$

**Q. 8. A large family uses 8 kW of power.**

**(a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square metre. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW ?**

**(b) Compare the area of that of the roof of a typical house.**

**[NCERT Ex. Q. 6.23, Page 137]**

**Ans.** Power used by a family (P) = 8 kW

**(a)** Solar energy incident on horizontal surface per square metre = 200 W
Electrical energy obtained from solar energy per unit area

$= 200 \times \frac{20}{100}$ W

$= 40$ W

$\therefore$ Area needed to supply 8 kW.

$= \frac{8\,kW}{40\,W} = \frac{8000}{40} = 200\text{ m}^2.$

**(b)** This area is comparable to the roof of a large house of 250 $m^2$.

**Value :** Family values should be encouraged in children.

**Q. 9. A bullet of mass 0.012 kg and horizontal speed 70 $ms^{-1}$ strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.**

**[NCERT Ad. Ex. Q. 6.24, Page 137]**

**Ans.** Mass of the bullet, $m = 0.012$ kg

Initial speed of the bullet, $u_b = 70$ $m/s$

Mass of the wooden block, $M = 0.4$ kg

Initial speed of the wooden block, $u_B = 0$

Final speed of the system of the bullet and the block = $v$

Applying the law of conservation of momentum:

$mu_b + Mu_B = (m+M)v$

$0.012 \times 70 + 0.4 \times 0 = (0.012 + 0.4)$ v

$\therefore$ $v = 0.84 / 0.412 = 2.04$ m / s

For the system of the bullet and the wooden block:

Mass of the system, $m' = 0.412$ kg

Velocity of the system = 2.04 m/ s

Height up to which the system rises = $h$

Applying the law of conservation of energy to this system:

Potential energy at the highest point = Kinetic energy at the lowest point

$m'gh = (1/2)\, m' v^2$

$\therefore \quad h = (1/2)(v^2 / g)$

$= (1/2) \times (2.04)^2 / 9.8$

$= 0.2123$ m

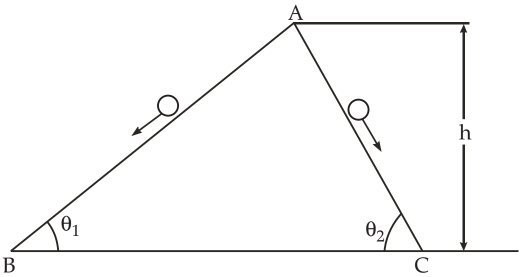
The wooden block will rise to a height of 0.2123 m.

Heat produced = Kinetic energy of the bullet – Kinetic energy of the system

$= (1/2)mu^2 - (1/2)m'v^2 = (1/2) \times 0.012 \times (70)^2 - (1/2) \times 0.412 \times (2.04)^2$

$= 29.4 - 0.857 = 28.54\ J$

**Q. 10. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (in figure). Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given $\theta_1 = 30°$, $\theta_2 = 60°$ and $h = 10$ m, what are the speeds and times taken by two stones?**

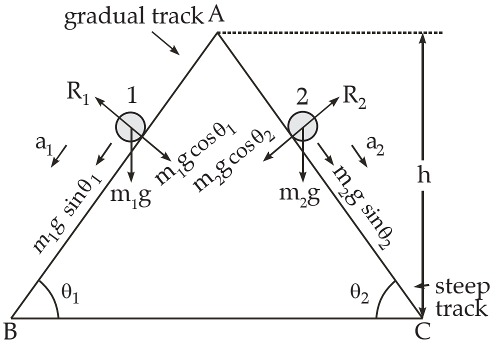



**[NCERT Ad. Ex. Q. 6.25, Page 137]**

**Ans.** AB and AC are two smooth planes or tracks inclined at $\theta_1$ and $\theta_2$ respectively.

No, two stones do not reach the bottom at the same time.

**Explanation:** Supposes $m_1g$ and $m_2g$ be the weights of the two stones on these planes respectively. The rectangular components of $m_1g$ and $m_2g$ are shown in given figure below.




When $a_1$ and $a_2$ be the accelerations produced in the stones 1 and 2 respectively, then clearly

$$m_1a_1 = m_1g\sin\theta_1$$

or $\quad a_1 = g\sin\theta_1$

Similarly $a_2 = g\sin\theta_2$

Also as $\theta_2 > \theta_1$

$a_2 > a_1$

it means $a_1 = g\sin 30° = \frac{g}{2}$

and $\quad a_2 = g\sin 60° = \frac{g\sqrt{3}}{2}$

From the equation $\quad v = u + at,$

$\therefore \quad v = at$

or $\quad t = v/a \quad$ ...(i)

Here $u = 0$ as the two stones are initially at rest

or $\quad t \propto \frac{1}{a}$

$\therefore \quad t_1 \propto \frac{1}{a_1}$

and $\quad t_2 = \frac{1}{a_2}$ or $\frac{t_2}{t_1} = \frac{a_1}{a_2} \quad$ ...(ii)

Now

as $a_2 > a_2$ or $\frac{a_1}{a_2} < 1 \quad$ ...(iii)

From the equations (ii) and (iii),

$$\therefore \quad \frac{t_2}{t_1} < 1$$

$$\text{or} \quad t_2 < t_1$$

It means second stone will take lesser time and reach the bottom earlier than the first stone, i.e., stone on the steep plane reaches the bottom earlier

Yes, the two stones reach the bottom with the same speed as follows:

Explanation : Let h = height of the plane at Point A= 10 m

When $v_1$ and $v_2$ be the speeds of the two stones with which they reach at the bottom, then according to the law of conservation of energy

Loss in P.E. at top = Gain in K.E. at bottom

or $$m_1gh = \frac{1}{2}m_1v_1^2$$

and $$m_2gh = \frac{1}{2}m_2v_2^2$$

or $$v_1 = \sqrt{2gh}$$

and $$v_2 = \sqrt{2gh}$$

or $$v_1 = v_2$$

$$= \sqrt{2 \times 9.8 \times 10}$$

$$= \sqrt{196}$$

$$= 14\, ms^{-1}$$

$\therefore$ from equation (i), $t = \frac{v}{a}$

$$\therefore \quad t_1 = \frac{v_1}{a_1} = \frac{v_1}{g}\sin\theta_1$$

$$t_1 = \frac{14}{9.8 \times \sin 30°} = \frac{14}{9.8 \times \frac{1}{2}}$$

or $$t_1 = \frac{20}{7}s = 2.86s$$

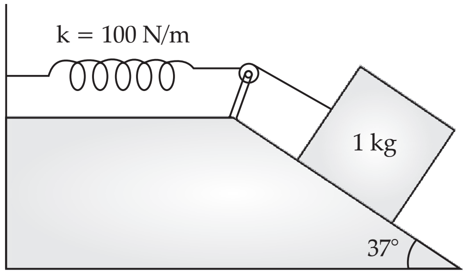
or $$t_2 = \frac{v_2}{a_2} = \frac{14}{g \sin 60^{\circ}}$$

$$= \frac{14}{9.8} \times \frac{2}{\sqrt{3}}$$

So, $$t_2 = \frac{2}{0.7\sqrt{3}}s$$

$$= 1.655$$

**Q. 11. A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m$^{-1}$ as shown in Figure. The block is released from the rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.**




**[NCERT Ad. Ex. Q. 6.26, Page 138]**

**Ans.** Net force on block in downward direction

$= mg \sin\theta - f$

$= mg \sin\theta - \mu mg \cos\theta$

$= mg (\sin\theta - \mu \cos\theta)$

Distance, x = 10cm = 0.1m

In equilibrium condition,

Work done = potential energy of spring

$$mg(\sin\theta - \mu\cos\theta)x = \frac{1}{2}kx^2$$

or $2 \times 1 \times 10 (\sin 37° - \mu \cos 37°) = 100° \; 0.1$

or $20 (0.601 - \mu \times 0.798) = 10$

or $\mu = 0.126$

**Q. 12. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 m/s. It hits the floor of the elevator (length of the elevator = 3 m) and does not rebound. What is the heat produced by the impact ? Would your answer be different, if the elevator were stationary ?**

**[NCERT Ad. Ex. Q. 6.27, Page 138]**

**Ans.** Mass of the bolt ($m$) = 0.3 kg

Length of the elevator ($h$) = 3 m

As the bolt does not rebound, therefore its total P.E. is converted into heat.

$\therefore$ Heat produced = P.E. of bolt

$= mgh$

$= 0.3 \times 9.8 \times 3$

$= 8.82$ J

The answer will not change even, if the elevator is stationary or moving, because the value of acceleration due to gravity remains same in all inertial systems.

**Q. 13. A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m/s relative to the trolley in a direction opposite to the trolley's motion and jumps out of the trolley. What is the final speed of the trolley ? How much has the trolley moved from the time the child begins to run ? [NCERT Ad. Ex. Q. 6.28, Page 138]**

**Ans.** Mass of the trolley ($m_1$) = 200 kg

Speed of the trolley ($v$) = 36 km/h

$$= 36 \times \frac{5}{18} \text{ m/s} \left(\because 1 \text{ km/h} = \frac{5}{18} \text{ m/s}\right)$$

$$= 10 \text{ m/s}$$

Mass of the child ($m_2$) = 20 kg

Initial momentum of the trolley and child before the child starts running,

$$p_i = (m_1 + m_2)v$$
$$= (200 + 20) \times 10 \text{ kg m/s}$$
$$= 2200 \text{ kg-m/s}$$

Speed of the child relative to the trolley = 4 m/s

Let $v'$ be the final speed of the trolley with respect to the earth.

∴ Speed of the child *w.r.t.* the earth

$$= (v' - 4) \text{ m/s}$$

Final momentum of the system when child is running,

$$P_f = m_1 v' + m_2(v' - 4)$$
$$= 200v' + 20(v' - 4)$$
$$= 220\, v' - 80$$

As no external force is acting on the system, therefore according to the law of conservation of momentum.

Initial momentum = Final momentum

$$p_i = p_f$$
$$2200 = 200v' - 80$$

or
$$v' = \frac{2200 + 80}{220} = \frac{2280}{220}$$
$$= 10.36 \text{ m/s}$$

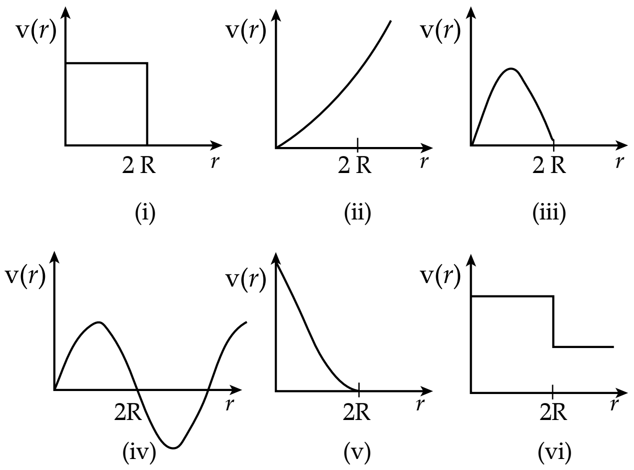
Time taken by the child to run 10 m over the trolley.

$$t = \frac{s}{v} = \frac{10}{4}$$
$$= 2.5 \text{ s}$$

Distance travelled by the trolley in this time interval,

$$S = v' \times t$$
$$= 10.36 \times 2.5$$
$$= 25.9 \text{ m}$$

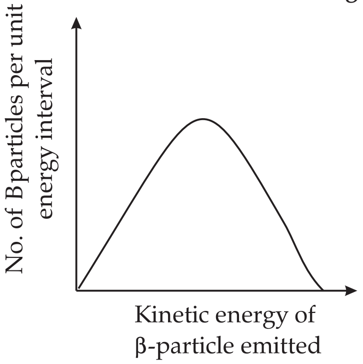
**Q. 14. Which of the following potential energy curves in figure cannot possibly describe the elastic collision of two billiard balls? Here r is the distance between centres of the balls.**




**[NCERT Ad. Ex. Q. 6.29, Page 138]**

**Ans.** The potential energy of a system of two masses is inversely proportional to the separation between them. In the given case, the potential energy of the system of the two balls will decrease as they come closer to each other. It will become *zero* (*i.e.*, V($r$) = 0) when the two balls touch each other, i.e., at $r$ = 2R, where R is the radius of each billiard ball. The potential energy curves given in figures (i), (ii), (iii), (iv) and (vi) do not satisfy these two conditions. Hence, they do not describe the elastic collisions between them.

**Q. 15. Show that the two body decay of this type must necessarily give an electron of fixed energy and therefore, cannot account for the observed continuous energy distribution in the β-decay of a neutron or a nucleus as shown in fig.**




**[Note : The simple result of this exercise was one among the several arguments advanced by W. Pauli to predict the existence of a third particle in the decay products of β-decay. This particle is known as neutrino. We now know that it is a particle of intrinsic spin ½ (like $e^-$.*p* or *n*), but is neutral, and either massless or having an extremely small mass (compared to the mass of electron) and which interacts very weakly with matter. The correct decay process of neutron is : $n \rightarrow p + e + \nu$]. [NCERT Ad. Ex. Q. 6.30, Page 138]**

**Ans.** The decay process of free neutron at rest is given as :

$n \rightarrow p + e^-$

From Einstein's mass – energy relation, we have the energy of electron as $\Delta mc^2$.

where,

$\Delta m$ = Mass defect = Mass of neutron – (Mass of proton + Mass of electron)

$c$ = Speed of light

$\Delta m$ and $c$ are constants. Hence, the given two-body decay is unable to explain the continuous energy distribution in the β-decay of a neutron or a nucleus. The presence of neutrino ν on the LHS of the decay correctly explains the continuous energy distribution.

### TIPS... & TRICKS...

- *Work and Power both are scalar quantities.*
- *Understand about work and power.*
- *Total energy of the system is conserved but it changes one form to another.*
- *Energy is equal to work done if energy loss is zero.*
- *In collision the energy remains conserved.*
- *In elastic collision energy and momentum are conserved but in in elastic collision energy not conserved.*

## Some Commonly Made Errors

- *Students got confused with negative or positive sign of work done.*
- *Students cannot properly understand the concept of WE theorem.*
- *Students do not practice the concept of collision and potential energy of spring which is an important topic.*

### EXPERT ADVICE

- *Work done is a scalar quantity unlike mass and kinetic energy.*
- *The WE theorem is not independent of Newton's second law. It may be viewed as a scalar form of second law.*
- *Every force encountered in mechanics does not have an associated potential energy.*
- *Remember the work done by the friction or viscous force on a moving body is negative.*

### OSWAAL LEARNING TOOLS

***For* Suggested Online Videos**

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|---|---|
| **Visit :** *https://youtu.be/21uVupYLxrs* **Or Scan the Code** | **Visit :** *https://youtu.be/lfJrpR2utWI* **Or Scan the Code** |
| **Visit :** *https://youtu.be/hce4pncZlI4* **Or Scan the Code** | **Visit :** *https://youtu.be/92AKYU5NphU* **Or Scan the Code** |
| **Visit :** *https://goo.gl/iokZK2* **Or Scan the Code** | **Visit :** *https://goo.gl/ag1cVJ* **Or Scan the Code** |

CHAPTER 7

# SYSTEM OF PARTICLES AND ROTATIONAL MOTION

## Chapter Objective

**This chapter will help you understand :**

- *Centre of Mass of a two particle system, Momentum conservation and center of mass. Centre of mass of a rigid body. Centre of mass of a uniform Rod. Moment of force, torque, angular momentum, law of conservation of angular momentum and its applications. Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion. Comparison of linear and rotational motions.*
- *Moment of Inertia, radius of gyration, values of moments of Inertia for simple geometrical objects (no derivation). Statement of parallel and perpendicular axes theorem and their applications.*

# TOPIC-1
## Centre of Mass and Motion of Rotational Particles



## Quick Review

- **Kinds of Motion of Rigid Body**

  **(i)Pure Translational Motion :** All the particles of body are moving together with same velocity at particular instant of time. eg. A car moving is a straight line.

  **(ii)Pure Rotational Motion :** A rigid body rotates about a fixed axis. Every particle of the body moves in a circle which lies in a plane perpendicular to axis and has its center on the axis. eg. A Potter's wheel.

  **(iii)Combination of Translational and Rotational Motion :** The motion of rigid body, which is not pivoted or fixed in some way is either pure translation motion or a combination of translation and rotation. eg. A vehicle's wheel.

- **Center of Mass of a two particle system :** Position vector of centre of mass of a two particle system is such that the product of total mass of the system and position vector of centre of mass is equal to sum of the products of masses of two particles and their respective position vectors.

- **Momentum Conservation :** Total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.

  $\vec{p} = M\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2 + .........+ m_n\vec{v}_n$

  Differentiating it,

  $$\frac{d\vec{p}}{dt} = M\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{ext}.$$

  This is Newton's II law.

  For isolated system, $\vec{F}_{ext} = \vec{0}$.

  $\therefore \quad \frac{d\vec{p}}{dt} = F_{ext} = 0 \text{ or } \vec{p} = \text{constant}$

  $\therefore \quad M\vec{v} = \text{Constant.}$

- **Right handed Screw Rule :** It states that if right handed screw is placed with its axis perpendicular to plane containing two vectors is rotated from direction of $\vec{A}$ to direction $\vec{B}$ through smaller angle, then sense of advancement of tip of screw gives direction of $\left(\vec{A} \times \vec{B}\right)$ or $\vec{C}$.

- **Moment of Force or Torque : Torque** due to a force is moment of force and measures the turning effect to the force about the axis of rotation. The general expression for torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- **Angular Momentum and its Conservation : Angular momentum** of a particle about a given axis is the moment of linear momentum of the particle about the axis. It is equal to the product of linear momentum of the particle and the perpendicular distance of the line of action of linear momentum from the axis of rotation. It is the product of linear momentum and the perpendicular distance of its line of action from the axis of rotation.

$$\vec{L} = \vec{r} \times \vec{p} = rp \sin \phi$$
$$= \vec{p} \times \vec{d}$$

where, $d = r\sin \phi$ = perpendicular distance of line of action of $\vec{p}$ from the axis. Angular momentum is a vector quantity, whose direction is given by **right handed screw rule.**

- $\vec{L} \perp \vec{r}$ and $\vec{L} \perp \vec{p}$
- **Rate of change of angular momentum is torque**, *i.e.*, $\vec{\tau} = d\vec{L}/dt$.

As $\quad \vec{\tau} = \dfrac{d\vec{L}}{dt} = \vec{\tau}_{ext}.$

for isolated system $\vec{\tau}_{ext} = \vec{0}.$

$\therefore \quad \vec{\tau}_{ext} = \dfrac{d\vec{L}}{dt} = \vec{0}.$

So, $\quad \vec{L}$ = constant.

- **Equilibrium of Rigid Bodies :**

  **1st Condition :** A rigid body is said to be in translational equilibrium, if it remain at rest or moving with a constant velocity in a particular direction. For this, the net external force or the vector sum of all external forces acting on the body must be zero *i.e.*, $\quad \Sigma \vec{F}_i = \vec{0}.$

  **Translational Static Equilibrium is of 3 types :**

  (i) Stable Equilibrium
  (ii) Unstable Equilibrium
  (iii) Neutral Equilibrium

  **2nd Condition :** A rigid body is said to be in rotational equilibrium, if the body does not rotate or rotates with constant angular velocity. For this, the net external torque or the vector sum of all the torques acting on the body must be zero *i.e.*, $\quad \Sigma \vec{F}_i = \vec{0}.$

- **Principle of Moments**

  **(a)** According to principle of moments, body will be in rotational equilibrium if algebraic sum of the moments of all forces acting on the body, about a fixed point is zero.

## Know the Terms

- **System** is formed when a collection of any number of particles interact with one another.
- **Internal forces** are all the forces exerted by various particles of the system on one another.
- **External forces** are those forces exerted on a given system by the agencies outside the system.
- **A rigid body** is defined as a system of particles, in which distance between any two particles does not change under the influence of external forces, where, size and shape of the body will remain unaffected under the effect of external forces.
- **The centre of mass** of a body is a point where the whole mass of the body is supposed to be concentrated. If all the forces acting on the body were applied on the centre of mass, the nature of motion of the body shall remain unaffected.

- **Angular velocity** of a particle is defined as the time rate of change of its angular displacement.
- **Angular acceleration** of an object in circular motion is defined as the time rate of change of its angular velocity.
- **Centre of gravity** of a body is a point where the weight of the body acts and total gravitational torque on the body is zero.

## Know the Formulae

- **Position vector of centre of mass of *n* particles system**

$$\vec{r} = \frac{\sum_{i=1}^{n} m_i \vec{r_i}}{M}, \text{ where, M is the total mass to the system } i.e., M = \sum_{i=1}^{n} m_i$$

- **For two particle system**

$$\vec{r} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$$

- **Coordinates of centre of mass will be**

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$z = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

- **If centre of mass lies at origin *i.e.*, $x = y = 0$,**

$\therefore$ $$m_1 x_1 + m_2 x_2 = 0$$

$$x_2 = \frac{-m_1 x_1}{m_2}, \; y_2 = \frac{-m_1 y_1}{m_2}$$

- **Velocity of C.M. of a system of two particles is**

$$\vec{v}_{CM} = \frac{m_1 \vec{v_1} + m_2 \vec{v_2}}{m_1 + m_2}$$

**Note :** For units.
1. All masses in kg
2. All distances in metre
3. All velocity in m/s

- **Cross Product of two Vectors :**

**(i)** $$\vec{A} \times \vec{B} = \vec{C} = AB \sin\theta \, \hat{C}$$

$$\theta = \text{Angle b/w } \vec{A} \,\&\, \vec{B}$$

**(ii)** $$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$

**(iii) Unit Vector**

$$\vec{C} = \frac{\vec{A} \times \vec{B}}{AB\sin\theta} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

**(iv)** $$|\vec{A} \times \vec{B}| = \text{Area of parallelogram.}$$

- **Angular Momentum**

$$\vec{L} = m\vec{v} \times \vec{r}$$

Vector form $$\vec{L} = \vec{r} \times \vec{p} = rp\sin\phi$$

$m$ = kg, $v = m/s$ , $p$ = kg $ms^{-1}$ , L = $kgm^2/s$

➢ **Equations of Rotational Motion :**

(a) $\omega_2 = \omega_1 + \alpha t$

(b) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

(c) $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

(d) $v = r\omega$

$\omega = 2\pi\nu = \frac{2\pi}{T}$

(e) $a = r\alpha$

(f) Centripetal acceleration $= \frac{v^2}{r} = r\omega^2$

where symbols have their usual meanings

➢ **Torque :** $\vec{\tau} = \vec{r} \times \vec{F}$

**(i) Work done by torque** $dW = \tau(d\theta)$

**(ii) Power of torque** $P = \frac{dW}{dt} = \tau\left(\frac{d\theta}{dt}\right)$

$P = \tau\omega$

## Know the Links

- *www.vedantu.com*
- *www.learncbse.in*
- *www.physics.stackexchange.com*
- *www.tiwaryacademy.com*

## MCQ/Fillups/True or False **(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. For which of the following does the centre of mass lie outside the body?**

**(a) A pencil** **(b) A shot put**

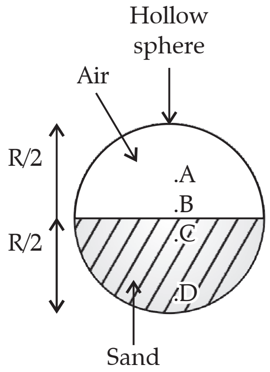
**(c) A dice** **(d) A bangle**

**[NCERT Exemp. Q. 7.1, Page 50]**

**Ans. Correct option:** (d)

*Explanation:* Centre of mass of A bangle at centre.

**Q. 2. Which of the following point is the likely position of the centre of mass of the system shown in figure?**




**(a) A** **(b) B**

**(c) C** **(d) D**

**[NCERT Exemp. Q. 7.2, Page 50]**

**Ans. Correct option:** (c)

*Explanation:* In this figure likely upper half sphere disc is hollow and lower half is solid than correct possible position of centre of mass is option (c).

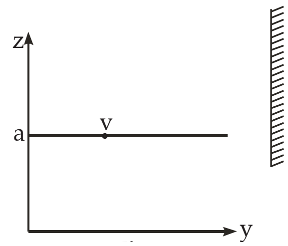
**Q. 3. A particle of mass m is moving in yz-plane with a uniform velocity v with its trajectory running parallel to +ve y-axis and intersecting z-axis at z = a(as shown in fig.). The change in its angular momentum about the origin as it bounces elastically from a wall at y=constant is:**

**(a)** $mva\hat{e}_x$

**(b)** $2mva\hat{e}_x$

**(c)** $ymv\hat{e}_x$

**(d)** $2ymv\hat{e}_x$




**[NCERT Exemp. Q. 7.3, Page 50]**

**Ans. Correct option:** (b)

***Explanation:***

Initial velocity, $v_i = v\hat{e}_y$

After reflection, final velocity $v_f = v\hat{e}_y$,

The trajectory is at constant distance a on z-axis and as particle moves along y-axis, its y component changes

Now, position vector, $\vec{r} = y\hat{e}_y + a\hat{e}_z$

Hence, change in angular momentum-

$\vec{r} \times m(v_f - v_i) = 2mva\hat{e}_x$

**Q. 4. When a disc rotates with uniform angular velocity, which of the following is not true?**

**(a) The sense of rotation remains same.**

**(b) The orientation of the axis of rotation remains same.**

**(c) The speed of rotation is non-zero and remains same.**

**(d) The angular acceleration is non-zero and remains same.**

**[NCERT Exemp. Q. 7.4, Page 51]**

**Ans. Correct option:** (d)

*Explanation:* If disc rotates with constant angular velocity. Then angular acceleration of the disc zero. *i.e.*,

$$\alpha = \frac{\Delta\omega}{\Delta t} = 0$$

**Q. 5. The density of a non- uniform rod of length 1 m is given by $\rho(x) = a(1+bx^2)$ where a and b are constants and $0 \le x \le 1$. The centre of mass of the rod will be at**

**(a)** $\frac{3(2+b)}{4(3+b)}$

**(b)** $\frac{4(2+b)}{3(3+b)}$

**(c)** $\frac{3(3+b)}{4(2+b)}$

**(d)** $\frac{4(3+b)}{3(2+b)}$

**[NCERT Exemp. Q. 7.7, Page 51]**

**Ans. Correct option:** (a)

*Explanation:* Density, $\rho(x) = a(1+bx^2)$, at b = 0, $\rho(x)$= a= constant

centre of mass, at x=0.5 m

**(a)** $\frac{3}{4}\times\frac{2}{3} = \frac{1}{2} = 0.5\text{m}$

**(b)** $\frac{4}{3}\times\frac{2}{3} \neq 0.5\text{m}$

**(c)** $\frac{3}{4}\times\frac{3}{2} \neq 0.5\text{m}$

**(d)** $\frac{4}{3}\times\frac{3}{2} \neq 0.5\text{m}$

**Q. 6. Choose the correct alternatives:**

**(a) For a general rotational motion, angular momentum *L* and angular velocity ω need not be parallel.**

**(b) For a rotational motion about and fixed axis, angular momentum *L* angular velocity ω are always parallel.**

**(c) For a general translational motion, momentum *p* and velocity *v* are always parallel.**

**(d) For a general translational motion, acceleration *a* and velocity *v* are always parallel.**

**[NCERT Exemp. Q. 7.9, Page 51]**

**Ans. Correct option:** (a) and (c)

*Explanation:*

(a) For a general rotational motion where axis of rotation is not symmetric. Angular momentum z and angular velocity ω need not to be parallel.

(c) $\vec{p} = m\vec{v}$ so $\vec{p} \,||\, \vec{v}$

**Q. 7. Figure shows two identical particles 1 and 2, each of mass m, moving in opposite directions with same speed v along parallel lines. At a particular instant, $r_1$ and $r_2$ are their respective position vectors drawn from point A which is in the plane of the parallel lines. Choose the correct options:**

**(a) Angular momentum $l_1$ of particle 1 about A is $l_1 = mvd_1$ ⊙**

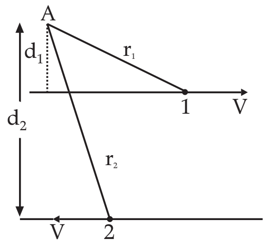
**(b) Angular momentum $l_2$ of particle 2 about A is $l_2 = mvr_2$ ⊙**

**(c) Total angular momentum of the system about A is $l = mv(r_1+r_2)$ ⊙**

**(d) Total angular momentum of the system about A is $l = mv(d_2-d_1)$ ⊗**

**⊙ represents a unit vector coming out of the page.**

**⊗ represents a unit vector going into the page.**




**[NCERT Exemp. Q. 7.10, Page 52]**

**Ans. Correct option:** (a) and (d)

*Explanation:*

(a) $I_1 = mvr_1 \sin\theta = mvd_1$ $\quad (\because d_1 = r_1 \sin\theta)$

and (b) $I = I_2 - I_1 = mvr_2 \sin\theta - m_1 vr_1 \sin\theta$

$= mv(d_2 - d_1)$ $\quad \left(\begin{array}{l}\because r_2 \sin\theta = d_2 \\ r_1 \sin\theta = d_1\end{array}\right)$

**Q. 8. The net external torque on a system of particles about an axis is zero. Which of the following are compatible with it?**

**(a) The forces may be acting radially from a point on the axis.**

**(b) The forces may be acting on the axis of rotation.**

**(c) The forces may be acting parallel to the axis of rotation.**

**(d) The torque caused by some forces may be equal and opposite to that caused by other forces.**

**[NCERT Exemp. Q. 7.11, Page 52]**

**Ans. Correct option:** (a), (b), (c) and (d)

*Explanation:*

**(a)** $\tau = Fr \sin\theta = 0 \because \vec{F} \,||\, \vec{r}$

**(b)** $\tau = Fr \sin\theta = 0 \because \vec{\tau} \,||\, \vec{F}$

**(c)** $\tau = Fr \sin\theta = 0 \because \vec{\tau} \,||\, \vec{F}$

**(d)** $\tau = Fr \sin\theta = 0 \times r \sin\theta = 0$

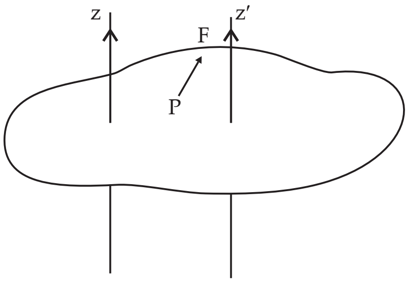
**Q. 9. Figure shows a lamina in x-y plane. Two axes z and z′ pass perpendicular to its plane. A force F acts in the plane of lamina at point P as shown. Which of the following are true? ( The point P is closer to z′ -axis than the z-axis.)**

**(a) Torque $\tau$ caused by F about z axis is along $-\vec{k}$ .**

**(b) Torque $\tau'$ caused by F about z′ axis is along $-\vec{k}$ .**

**(c) Torque $\tau$ caused by F about z axis is greater in magnitude than that about z axis.**

**(d) Total torque is given be $\tau = \tau + \tau'$ .**




**[NCERT Exemp. Q. 7.12, Page 53]**

**Ans. Correct option:** (b) and (c)

*Explanation:*

(a) $\vec{r}' < r, \vec{\tau}_{z'} = \vec{r}' \times \vec{F} = rF\sin\theta'(+k')$

As, $\vec{r}$ and $\vec{F}$ are x-y plane

Therefore, $\vec{r} \times \vec{F}$ will be $+\hat{k}$ direction by Right Hand Thumb Rule.

(b) $\vec{\tau_z} = \vec{r} \times \vec{F}$

$\vec{\tau_z} = rF\sin\theta(-\hat{k})$

[same reason as in (a) verifies option (b)]

(c) $r > r', \theta > \theta'$

$\sin\theta > \sin\theta'$

$rF\sin\theta > r'F\sin\theta'$

$\tau_z > \tau_z'$

[This verifies the option c]

(d) If $\tau_z$ and $\tau_z'$ , are along same axis, $\tau = \tau_z + \tau_z'$ will be true but here axis are different z and $z'$

So, $\tau \neq \tau_z + \tau_z'$

This rejects the option.

**(B) True and False**

**Q. 10. Read each statement below carefully, and state, with reasons, if it is true or false;**

**(a) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.**

**(b) The instantaneous speed of the point of contact during rolling is zero.**

**(c) The instantaneous acceleration of the point of contact during rolling is zero.**

**(d) For perfect rolling motion, work done against friction is zero.**

**(e) A wheel moving down a perfectly frictionless inclined plane will undergo sli ping (not rolling) motion.**

**[NCERT Ex. Q. 7.32, Page 181]**

**Ans. (a)** False

Frictional force acts opposite to the direction of motion of the center of mass of a body. In the case of rolling, the direction of motion of the center of mass is backward. Hence, frictional force acts in the forward direction.

**(b)** True

Rolling can be considered as the rotation of a body about an axis passing through the point of contact of the body with the ground. Hence, its instantaneous speed is zero.

**(c)** False

When a body is rolling, its instantaneous acceleration is not equal to zero. It has some value.

**(d)** True

When perfect rolling begins, the frictional force acting at the lowermost point becomes zero. Hence, the work done against friction is also zero.

**(e)** True

The rolling of a body occurs when a frictional force acts between the body and the surface. This frictional force provides the torque necessary for rolling. In the absence of a frictional force, the body slips from the inclined plane under the effect of its own weight.

## Very Short Answer Type Questions **(1 mark each)**

**Q. 1. The centre of gravity of a body on the earth coincides with its centre of mass for a 'small' object whereas for an 'extended' object it may not. What is the qualitative meaning of 'small' and 'extended' in this regard?**

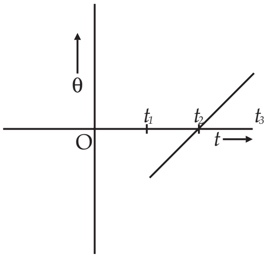
**For which of the following the two coincides? A building, a pond, a lake, a mountain?**

**[NCERT Exemp. Q. 7.14, Page 53]**

**Ans.** Centre of gravity is centre of it's geometry or given structure but centre of mass is a point where whole mass of the body can be considered. When vertical height or geometric centre of object is very small and very near to earth's surface in comparison to earth's radius. So, these objects are small, if it is larger, they are extended objects.

**(a)** Buildings (High) and ponds are small objects.
**(b)** Mountains and lakes are extended objects.

**Q. 2. The variation of angular position θ, of a point on a rotating rigid body, with time t is shown in figure. Is the body rotating clock –wise or anti-clockwise?**

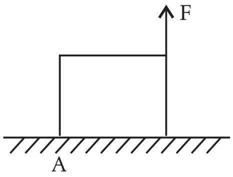



**[NCERT Exemp. Q. 7.17, Page 54]**

**Ans.** In graph, slope of $\theta - t$ is positive, which indicates anti-clockwise rotation.

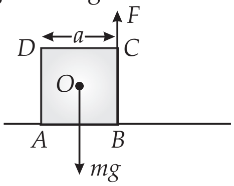
**Q. 3. A uniform cube of mass m and side a is placed on a frictionless horizontal surface. A vertical force F is applied to the edge as shown in figure. Match the following (most appropriate choice):**

| Column-I | Column-II |
|---|---|
| **(a) mg/4<F<mg/2** | **(i) Cube will move up.** |
| **(b) F>mg/2** | **(ii) Cube will not exhibit motion.** |
| **(c) F>mg** | **(iii) Cube will begin to rotate and slip at A.** |
| **(d) F=mg/4** | **(iv) Normal reaction effectively at a/3 from A, no motion.** |




**[NCERT Exemp. Q. 7.17, Page 54]**

**Ans.** Consider the given diagram-




Moment of force due to F at A (anticlockwise)

$$\tau_1 = \vec{AB} \times \vec{F} = a \times \vec{F}$$

Moment of force due to mg at A (Clockwise),

$$\tau_2 = mg \times \frac{a}{2}$$

**(i)** Cube will not move (exhibit motion), if $\tau_1 = \tau_2$

$$a \times \vec{F} = mg\frac{a}{2} \text{ or } \vec{F} = \frac{mg}{2} \quad \text{so, (a)} \rightarrow \text{(ii)}$$

**(ii)** Cube will rotate, if $\tau_1 > \tau_2$

$$Fa > mg\frac{a}{2} \text{ or } F > \frac{mg}{2} \quad \text{So, (b)} \rightarrow \text{(iii)}$$

**(iii)** Let Normal reaction is acting at a/3 from point A,

$$mg \times \frac{a}{3} = F \times a \text{ or } F = \frac{mg}{3} > \frac{mg}{4} \quad \text{So, (c)} \rightarrow \text{(i)}$$

**(iv)** $\frac{mg}{4} < \frac{mg}{3}$ if, so due to F, block will not move,

so, (d)→ (iv)

∴ (a) → (ii), (b) → (iii), (c) → (i), (d) → (iv).

## Short Answer Type Questions **(2 or 3 marks each)**

**Q. 1. The vector sum of a system of non-collinear forces acting on a rigid body is given to be non-zero. If the vector sum of all the torques due to the system of forces about a certain point is found to be zero, does this mean that it is necessarily zero about any arbitrary point?**

**[NCERT Exemp. Q. 7.19, Page 54]**

**Ans.** No, it is not necessarily zero.

Given: $\sum_i \vec{F}_i = 0$

Sum of torques about a certain point A,

$$\sum_i \vec{r}_i \times \vec{F}_i = 0$$

Sum of torques about any other point B,

$$\sum_i (\vec{r}_i - a) \times \vec{F}_i = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i - a\sum_{i=1}^{n} \vec{F}_i$$

So, as a and $\sum_{i=1}^{n} \vec{F}_i$ are not zero.

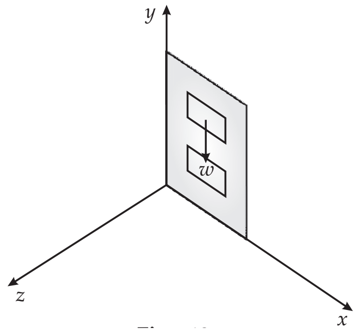
∴ torque is not necessarily zero about any other point.

**Q. 2. A wheel in uniform motion about an axis passing through its centre and perpendicular to its plane is considered to be in mechanical (translational plus rotational) equilibrium because no net external force or torque is required to sustain its motion. However, the particles that constitute the wheel do experience a centripetal acceleration directed towards the centre. How do you reconcile this fact with the wheel being in equilibrium?**
**How would you set a half-wheel into uniform motion about an axis passing through the centre of mass of the wheel and perpendicular to its plane? Will you require external forces to sustain the motion? [NCERT Exemp. Q. 7.20, Page 54]**

**Ans.** Wheel is a rigid elastic body which is in uniform motion about axis passing through its centre and perpendicular to the plane of wheel. The particles that constitute the wheel do experience a centripetal acceleration directed towards the centre. This centripetal acceleration arises due to internal elastic forces which cancel out in pairs.

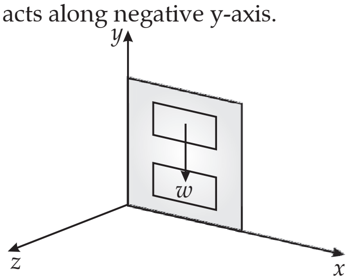
In a half wheel the distribution of mass about its centre of mass (axis of rotation) is not symmetrical. Therefore, the direction of angular momentum does not coincide with the direction of angular velocity and hence an external torque is required to maintain rotation.

**Q. 3. A door is hinged at one end and is free to rotate about a vertical axis (as shown in figure). Does its weight cause any torque about this axis? Give reason for your answer.**




**[NCERT Exemp. Q. 7.21, Page 54]**

**Ans.** Let us consider the given diagram, where weight of the door acts along negative y-axis.




A force can produce torque only along a direction normal to itself as $\tau = r \times f$. So, when the door is in the xy-plane, the torque produced by gravity can only be ± z direction, never about an axis passing through y direction.

Hence, the weight will not produce any torque about y-axis.

**Q. 4. (n−1) equal point masses each of mass m are placed at the vertices of a regular n-polygon. The vacant vertex has a position vector *a* with respect to the centre of the polygon. Find the position vector of centre of mass.**

**[NCERT Exemp. Q. 7.22, Page 55]**

**Ans.** The centre of mass of a regular n-polygon lies at its geometric centre.

Let $\vec{b}$ is the position vector of the centre of mass of regular n- polygon.

From question, $(n-1)$ equal point masses each of mass m are placed at the $(n-1)$ vertices of a regular $n$-polygon,

Then, $$r_{cm} = \frac{(n-1)mb + ma}{(n-1)m + m}$$

Now, mass m is placed at nth remaining vertex, then $r_{cm} = 0$

$$\frac{(n-1)mb + ma}{(n-1)m + m} = 0$$

or $$\vec{b} = \frac{-m\vec{a}}{(n-1)m} = \frac{-\vec{a}}{(n-1)}$$

Negative sign indicates that centre of mass lies other side from nth vertex geometrical centre of n-polygon.

**Q. 5. Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?**

**[NCERT Ex. Q. 7.1, Page 178]**

**Ans.** In all four cases (i),(ii),(iii),(iv), centre of mass is allocated at their respective geometrical centres.

No, it is not necessary that centre of mass of a body lie inside it because in some cases i.e. ring, hollow sphere, a hollow cylinder, etc. centre of mass lies outside.

**Q. 6. In the HCl molecule, the separation between the nuclei of the two atoms is about 1·27 Å (1 Å = $10^{-10}$ m). Find the approximate location of the C.M. of the molecule, given that a chlorine atom is about 35·5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus ?**

**[NCERT Ex. Q. 7.2, Page 178]**

**Ans.** Let us choose the nucleus of the hydrogen atom as the origin for measuring distance.

Mass of hydrogen atom,

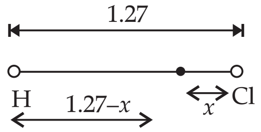
$$m_1 = 1 \text{ unit (say)}$$

Mass of chlorine atom,

$$m_2 = 35{\cdot}5 \text{ unit (say)}$$

Now, $x_1 = 0$

and $x_2 = 1{\cdot}27$ Å $= 1{\cdot}27 \times 10^{-10}$ m




Distance of C.M. of HCl molecule from the origin is given by

$$\frac{m(1.27 - x) + 35.5mx}{m + 35.5m} = 0$$

$$m(1.27 - x) + 35.5\,mx = 0$$

$$1.27 - x = -35.5\,x$$

$$\therefore \quad x = \frac{-1.27}{35.5 - 1} = -0.037 \text{ Å}$$

Here, the negative sign indicates that the centre of mass lies at the left of the molecule.

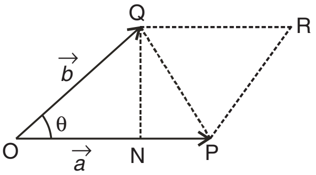
**Q. 7. A child sits stationary at one end of a long trolley moving uniformly with a speed $v$ on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of C.M. (Centre of Mass) of the (trolley + child) system ? [NCERT Ex. Q. 7.3, Page 178]**

**Ans.** Trolley and the child make the system, speed of the C.M. of this system remains same irrespective of the movement of the child inside the trolley in any manner because C.M. is affected by external forces and not by the internal forces.

**Q. 8. Show that the area of the triangle contained between the vectors $\vec{a}$ and $\vec{b}$ is one-half of the magnitude of $\vec{a}\times\vec{b}$.**

**[NCERT Ex. Q. 7.4, Page 178]**

**Ans.** Let $\vec{a}$ be represented by $\vec{OP}$ and $\vec{b}$ be represented by $\vec{OQ}$. Let $\angle POQ = \theta$




Complete the || gm OPRQ. Join PQ draw QN ⊥ OP.

In ΔQNO, $\sin\theta = \frac{QN}{OQ} = \frac{QN}{b}$

$$QN = b\sin\theta$$

Now, by definition,

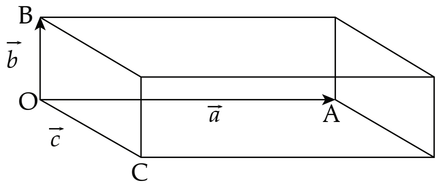
$$|\vec{a}\times\vec{b}| = ab\sin\theta$$
$$= (OP)(QN)$$
$$= \frac{2(OP)(QN)}{2}$$
$$= 2 \times \text{area of } \Delta OPQ$$

$\therefore$ Area of $\Delta OPQ = \frac{1}{2}|\vec{a}\times\vec{b}|$ **Proved.**

**Q. 9. Show that $a.(b \times c)$ is equal in magnitude to the volume of the parallelepiped formed on the three vectors, $a$, $b$ and $c$.**

**[NCERT Ex. Q. 7.5, Page 178]**

**Ans.** A parallelepiped with origin O and sides $a$, $b$, and $c$ is shown in the following figure.




Volume of the given parallelepiped = $abc$

$$\vec{OC} = \vec{a}$$
$$\vec{OB} = \vec{b}$$
$$\vec{OC} = \vec{c}$$

Let $\hat{n}$ be a unit vector perpendicular to both $b$ and $c$. Hence, $n$ and $a$ have the same direction.

$$\therefore \vec{b}\times\vec{c} = bc\sin\theta\,\hat{n}$$
$$= bc\sin 90^\circ n = bc\hat{n}$$
$$\vec{a}.(\vec{b}\times\vec{c}) = a.(bc\hat{n})$$
$$= abc\cos\theta\,\hat{n}$$
$$= abc\cos 0^\circ$$
$$= abc$$
$$= \text{Volume of the parallelepiped}$$

**Q. 10. Find the components along the $x$, $y$, $z$ axes of the angular momentum of a particle, whose position vector is r with components $x, y, z$ and momentum is $p$ with components $p_x$, $p_y$, $p_z$. Show that if the particle moves only in the $x$-$y$ plane the angular momentum has only a $z$-component.**

**[NCERT Ex. Q. 7.6, Page 178]**

**Ans.** Angular momentum $l$ of a particle-

$$\vec{l} = \vec{r}\times\vec{p} \qquad \begin{bmatrix}\vec{r} = \text{position vector}\\ \vec{p} = \text{momentum}\end{bmatrix}$$

But $\vec{r} = \left[x\hat{i} + y\hat{j} + z\hat{k}\right]$

Where x, y, z are components of $\vec{r}$ and

$$\vec{p} = \left[p_x\hat{i} + p_y\hat{j} + p_z\hat{k}\right]$$

$$\therefore \vec{l} = \vec{r}\times\vec{p}$$
$$= \left[x\hat{i} + y\hat{j} + z\hat{k}\right]\times\left[p_x\hat{i} + p_y\hat{j} + p_z\hat{k}\right]$$

$$\left(l_x\hat{i} + l_y\hat{j} + l_z\hat{k}\right) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ p_x & p_y & p_z\end{vmatrix}$$

$$= \left(yp_z - zp_y\right)\hat{i} + \left(zp_x - xp_z\right)\hat{j} + \left(xp_y - yp_x\right)\hat{k}$$

We conclude that-

$$l_x = yp_z - zp_y,\ l_y = zp_x - xp_z,\ l_z = xp_y - yp_x$$

If given particle moves only in x-y plane,

Then $z=0$, $p_z=0$,

hence, $l_x=0$, $l_y=0$ $l_z = \left(xp_y - yp_x\right)\hat{k}$

(which is only z-component of $l$)

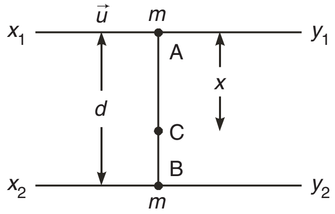
Therefore, particle moves only in $x$-$y$ plane, the angular momentum has only $z$-component.

**Q. 11. Two particles, each of mass $m$ and speed $v$, travel in opposite directions along parallel lines separated by a distance $d$. Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.**

**[NCERT Ex. Q. 7.7, Page 178]**

**Ans.** It is clear from figure, vector angular momentum of the two particles system about any point A on $x_1y_1$.

$L_A = mv \times d + mv \times 0 = m\vec{v}d$




Similarly, vector angular momentum of the two particle system about any point B on $x_2y_2$.

$$\vec{L}_B = m\vec{v} \times d + m\vec{v} \times 0 = m\vec{v}d$$

Let us consider any other point C on AB, where AC = $x$.

$\therefore$ Vector angular momentum of the two particle system about C is

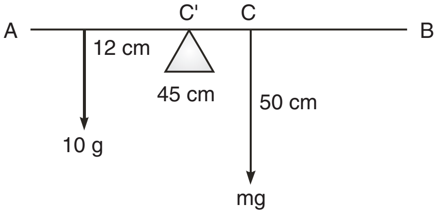
$$\vec{L}_C = m\vec{v}(x) + m\vec{v}(d - x) = m\vec{v}d$$

Clearly, $\vec{L}_A = \vec{L}_B = \vec{L}_C$ **Proved**

**Q. 12. A meter stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12·0 cm mark, the stick is found to be balanced at 45·0 cm. What is the mass of meter stick ?**

**[NCERT Ex. Q. 7.17, Page 179]**

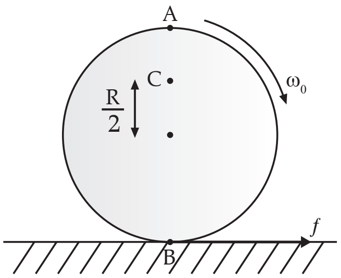
**Ans.** Let $m$ be the mass of the stick concentrated at C, the 50 cm mark




For equilibrium

$$10\,g(45 - 12) = mg(50 - 45)$$
$$10g \times 33 = mg \times 5$$
$$m = \frac{10 \times 33}{5}$$
$$= 66\,g$$

**Q. 13. A disc rotating about its axis with angular speed $\omega_0$ is placed lightly (without any linear push) on a perfectly frictionless table. The radius of disc is R. What are the linear velocities of the points A, B and C on the disc shown in figure. Will the disc roll?**



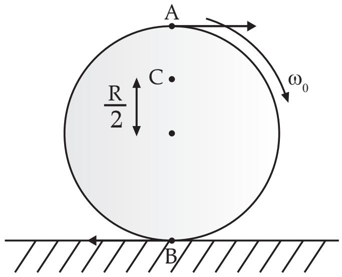

**[NCERT Ad. Ex. Q. 7.28, Page 180]**

**Ans.** For A, $v_A = R\omega_0$ in forward direction

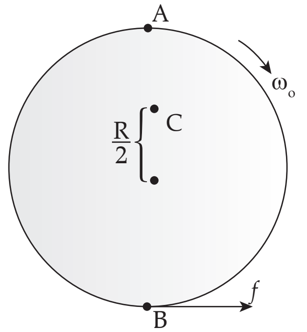
For B, $v_B = R\omega_0$ in backward direction

For C, $v_C = \frac{R}{2}\omega_0$ in forward direction

Since the disc is placed on a frictionless table, it will not roll. This is because the presence of friction is essential for the rolling of a body.




**Q. 14. Explain why friction is necessary to make the disc in figure given roll in the direction indicated. (a) Give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins. (b) What is the force of friction after perfect rolling begins?**




**[NCERT Ad. Ex. Q. 7.29, Page 181]**

**Ans.** A torque is required to roll the given disc. As per the definition of torque, the rotating force should be tangential to the disc. Since the frictional force at point B is along the tangential force at point A, a frictional force is required for making the disc roll.

**(a)** Force of friction acts opposite to the direction of velocity at point B. The direction of linear velocity at point B is tangentially leftward. Hence, frictional force will act tangentially rightward. The sense of frictional torque before the start of perfect rolling is perpendicular to the plane of the disc in the outward direction.

**(b)** Since frictional force acts opposite to the direction of velocity at point B, perfect rolling will begin when the velocity at that point becomes equal to zero. This will make the frictional force acting on the disc zero.

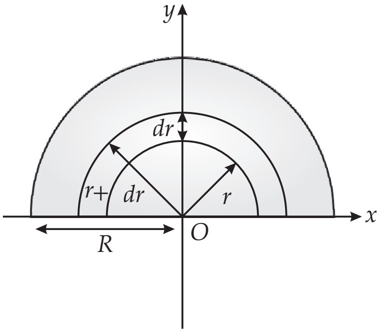
# Long Answer Type Questions (5 marks each)

**Q. 1. Find the centre of mass of a uniform (a) half-disc, (b) quarter-disc. [NCERT Exemp. Q. 7.23, Page 55]**

**Ans.** Let the mass of half disc be M.

Area of half disc $= \frac{\pi R^2}{2}$

Mass per unit area, $m = \frac{2M}{\pi R^2}$




**(a)** The half disc can be divided into a large number of semi-circular strips having mass $dm$, thickness $dr$, and radii varies from $0 \to R$

Surface area of semicircular strip of radius $r$ and thickness

$$dr = \frac{\pi}{2}\left[(r+dr)^2 - r^2\right]$$

$$= \frac{\pi}{2}\left[r^2 + dr^2 + 2rdr - r^2\right]$$

If, $(dr^2)$ is very small,

$$\therefore \quad = \frac{\pi}{2}(2r \times dr)$$

$$= \pi r dr$$

$\therefore$ Mass of the strip,

$$dm = \pi r dr \times \frac{2M}{\pi R^2}$$

$$= \frac{2M}{R^2}.rdr$$

Let $(x, y)$ be the coordinates of c. m. of this strip,

So, $(x, y) = \left(0, \frac{2r}{\pi}\right)$

Thus, $x = 0, y = \frac{2r}{\pi}$

Let $x$ cm and $y$ cm be the co-ordinates of the centre of mass of semicircular strip. Then

$$x = x_{cm} = \frac{1}{M}\int_0^R x dm = \int_0^R 0 \, dm = 0$$

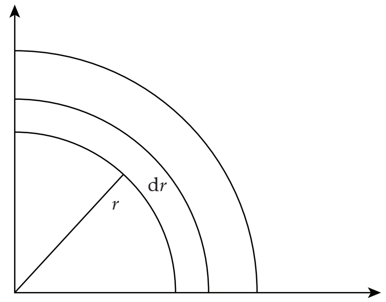
$$y = y_{cm} = \frac{1}{M}\int_0^R y dm = \frac{1}{M}\int_0^R \frac{2r}{\pi} \times \frac{2M}{R^2}.rdr = \frac{1}{M}.\frac{4M}{\pi R^2}\int_0^R r^2 dr$$

$$= \frac{4}{\pi R^2}\left[\frac{r^3}{3}\right]_0^R = \frac{4}{3\pi R^2}.R^3$$

$$\therefore \quad y_{cm} = \frac{4R}{3\pi}$$

So, centre of mass of uniform half disc$= \left(0, \frac{4R}{3\pi}\right)$

**(b)**




Mass per unit area of quarter disc $= \frac{M}{\frac{\pi R^2}{4}} = \frac{4M}{\pi R^2}$

Using symmetry,

For half disc, $y_{cm} = \frac{4R}{3\pi}$

Similarly, for half disc, along x-axis centre of mass, at $x = \frac{4R}{3\pi}$

So, the centre of mass of quarter disc is$= \left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$

**Q. 2. Two cylindrical hollow drums of radii R and 2R, and of a common height $h$, are rotating with angular velocities ω (anti-clockwise) and ω (clockwise), respectively. Their axes, fixed are parallel and in a horizontal plane separated by $(3R+\delta)$. They are now brought in contact $(\delta \to 0)$.**

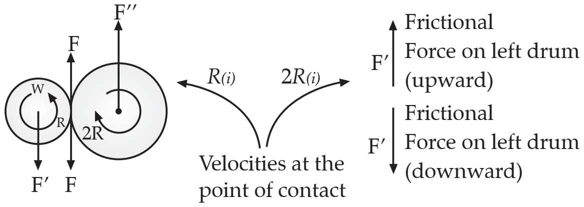
**(a) Show the frictional forces just after contact.**

**(b) Identify forces and torques external to the system just after contact.**

**(c) What would be the ratio of final angular velocities when friction ceases?**

**[NCERT Exemp. Q. 7.26, Page 55]**

**Ans. (a)**




Let us consider the following figure, which shows frictional forces.

The direction of $v_1$ and $v_2$ at point of contact are tangentially upward.

Then

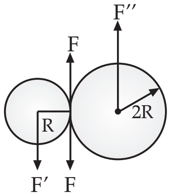
$$\Rightarrow \quad v_1 = \omega R, v_2 = \omega 2R,$$

**(b)** External forces acting on the system are equal and opposite, so net force is zero.

$F' = F = F''$ where F′ and F″ are external forces through support

$F_{net} = 0$

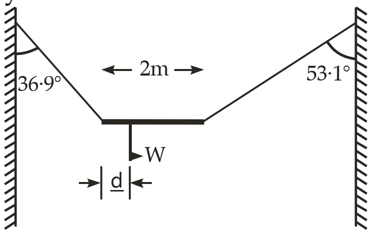
External torque= F × 3R, anticlockwise.
So, as velocity of drum 2 is double, *i.e.* $v_2 = 2v_1$




**(c)** Let $\omega_1$ and $\omega_2$ be final angular velocities (anticlockwise and clockwise respectively) of smaller drum 1 and drum 2 respectively. Finally, when their velocities become equal there will be no friction., due to no slipping at this stage.

Hence, $R\omega_1 = 2\,R\omega_2$ or $\dfrac{\omega_1}{\omega_2} = \dfrac{2}{1}$

**Q. 3. A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in figure. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.**




**[NCERT Ex. Q. 7.8, Page 178]**

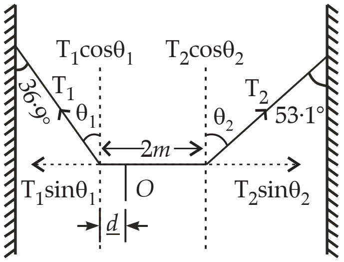
**Ans.** Given: $\theta_1 = 36.9$, $\theta_2 = 53.1$

Let $T_1$& $T_2$ be tensions in two strings. For equilibrium condition,

$T_1 \sin\theta_1 = T_2 \sin\theta_2$

$$\frac{T_1}{T_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 53.1^\circ}{\sin 36.9^\circ}$$

$$\frac{T_1}{T_2} = \frac{0.7407}{0.5477} = 1.3523 \qquad (1)$$




Let O be the position of centre of gravity of rod from left & at distance *d*.

For rotational equilibrium of rod about O, the moment of vertical forces must be equal and opposite :

$$T_1\cos\theta_1 \times d = T_2\cos\theta_2(2-d)$$

or $\dfrac{T_1}{T_2} \times \dfrac{\cos\theta_1}{\cos\theta_2} = \dfrac{2-d}{d}$

From (1)

$$1.3523 \times \frac{\cos 36.9^\circ}{\cos 53.1^\circ} = \frac{2-d}{d}$$

$$1.3523 \times \frac{0.8366}{0.6718} = \frac{2}{d} - 1$$

$$\frac{2}{d} = \frac{1.3523 \times 0.8366}{0.6718} + 1$$

$d = 0.745$ m

**Q. 4. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its center of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.**

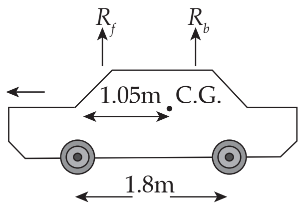
**[NCERT Ex. Q. 7.9, Page 178]**

**Ans.** Mass of the car, $m$ = 1800 kg

Distance between the front and back axles, $d$ = 1.8 m

Distance between the C.G. (center of gravity) and the back axle = 1.05 m

The various forces acting on the car are shown in the following figure.




$R_f$ *and* $R_b$ are the forces exerted by the level ground on the front and back wheels respectively.

At translational equilibrium:

$R_f + R_b = mg$

$= 1800 \times 9.8$

$= 17640$ N …(i)

For rotational equilibrium, on taking the torque about the C.G., we have:

$R_f\,(1.05) = R_b\,(1.8 - 1.05)$

$R_f \times (1.05) = R_b \times 0.75$

$$\frac{R_f}{R_b} = \frac{0.75}{1.05} = \frac{5}{7}$$

$$\frac{R_b}{R_f} = \frac{7}{5}$$

$R_b = 1.4R_f$ …*(ii)*

Solving equations *(i)* and *(ii)*, we get

$1.4R_f + R_f = 17640$

$$R_f = \frac{17640}{2.4} = 7350N$$

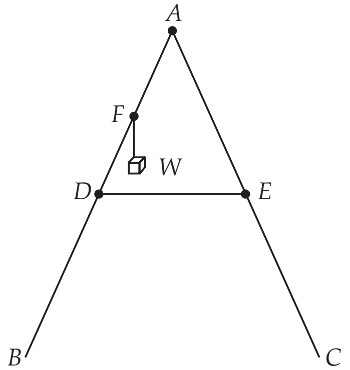
$\therefore R_b = 17640 - 7350 = 10290\,N$

Therefore, the force exerted on each front wheel = $\dfrac{7350}{2}$ = 36757*N*, and

The force exerted on each back wheel = $\dfrac{10290}{2}$ = 5145 N

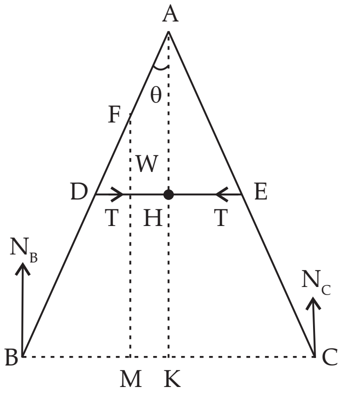
**Q. 5. As shown in Fig., the two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied half way up. A weight 40 kg is**

**suspended from point F. 1.2 m from B along the ladder BA. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. (Take g =9.8 m/s²)**
**(Hint : Consider the equilibrium of each side of the ladder separately).**




**[NCERT Ad. Ex. Q. 7.22, Page 179]**

**Ans.** Given: m=40 kg ⇒ weight = 40 × 9.8 N
= 392 N




$$AB = AC = 1.6m,$$
$$BD = \frac{1}{2} \times 1.6m = 0.8m$$
$$BF = 1.2m, \ DE = 0.5m$$

Now, ΔADE and ΔABC are similar,

$$\therefore \quad BC = DE \times \frac{AB}{AD} = \frac{0.5 \times 1.6}{0.8} = 1.0m$$

Now consider equilibrium at point B, $\Sigma\tau = 0$

$$\therefore W \times (MB) = N_C \times (CB) \quad (1)$$

$$\text{But } MB = \frac{KB \times BF}{BA} = \frac{0.5 \times 1.2}{1.6} = 0.375m$$

Substituting in equation (1), we get

$$N_C = \frac{W \times MB}{CB} = \frac{392 \times 0.375}{1} = 147N$$

Now considering equilibrium at point C,

$$\frac{W \times (MC)}{BC} = N_B \times (BC)$$

$$N_B = \frac{W \times (MC)}{BC}$$

$$= \frac{W \times (BC - BM)}{BC}$$

$$N_B = \frac{393 \times (1 - 0.375)}{1} = 245N$$

Hence, it can be easily shown that tension in string $T = M_B - M_C$
$= 245 - 147$
$= 98$ N

**Q. 6. Separation of motion of a system of particles into motion of the centre of mass and motion about the centre of mass:**

**(a) Show $p = p_i' + m_i v$**

**Where $p_i$ is the momentum of the iᵗʰ particle (of mass $m_i$) and $p_i' = m_i v_i'$. Note $v_i'$ is the velocity of the $i^{th}$ particle relative to the center of mass.**

**Also, prove using the definition of the centre of mass**

$$\sum_i p_i' = 0$$

**(b) Show $K = K' + \frac{1}{2}Mv^2$**

**Where K is the total kinetic energy of the system of particles, K′ is the total kinetic energy of the system when the particle velocities are taken with respect to the center of mass and $Mv^2/2$ is the kinetic energy of the translation of the system as a whole (i.e. of the centre of mass motion of the system). The result has been used in Sec. 7.14.**

**(c) Show L = L′ + R×Mv**

**Where $L' = \sum r_i' \times p_i'$ is the angular momentum of the system about the center of mass with velocities taken relative to the center of mass. Remember $r_i' = r_i - R$; rest of the notation is the standard notation used in the chapter. Note L′ and MR×$v$ can be said to be angular momenta, respectively, about and of the center of the system of particles.**

**(d) Show**

$$\frac{dL'}{dt} = \sum_i r_i' \times \frac{d}{dt}(p_i')$$

**Further, show that**

$$\frac{dL}{dt} = \tau'_{ext}$$

**Where $\tau'_{ext}$ is the sum of all external torques acting on the system about the center of mass.**

**(Hint: Use the definition of center of mass and Newton's Third Law. Assume the internal forces between any two particles act along the line joining the particles.)**

**[NCERT Ad. Ex. Q. 7.33, Page 181]**

**Ans. (a)** Take a system of $i$ moving particles.

Mass of the $i^{th}$ particle = $m_i$

Velocity of the $i^{th}$ particle = $v_i$

Hence, momentum of the $i^{th}$ particle, $p_i = m_i v_i$

Velocity of the center of mass = V

The velocity of the $i^{th}$ particle with respect to the center of mass of the system is given as:

$v_i' = v_i - v$ ... (1)

Multiplying $m_i$ throughout equation (1), we get:

$m_i v_i' = m_i v_i - m_i v$

$\therefore p_i' = p_i - m_i v$

Where,

$p_i' = m_i v_i'$ = Momentum of the $i^{th}$ particle with respect to the center of mass of the system

$p = p_i' + m_i v$

We have the relation: $p_i' = m_i v_i'$

Taking the summation of momentum of all the particles with respect to the center of mass of the system, we get:

$$\sum_i p_i' = \sum_i m_i v_i' = \sum_i m_i \frac{dr_i'}{dt}$$

Where

$r_i'$ = Position vector of $i^{th}$ particle with respect to the center of mass $v_i' = \frac{dr_i'}{dt}$

As per the definition of the centre of mass, we have:

$$\sum_i m_i r_i' = 0$$

$$\therefore \sum_i m_i \frac{dr_i'}{dt} = 0$$

$$\sum_i p_i' = 0$$

**(b)** We have the relation for velocity of the $i^{th}$ particle as:

$v_i = v_i' + v$

$$\sum_i m_i v_i = \sum_i m_i v_i' + \sum_i m_i v \quad ...(2)$$

Taking the dot product of equation (2) with itself, we get:

$$\sum_i m_i v_i \sum_i m_i v_i' = \sum_i m_i (v_i' + v) \sum_i m_i (v_i' + v)$$

$$M^2 \sum_i v_i^2 = M^2 \sum_i v_i^2 + M^2 \sum_i v_i v_i' + M^2 \sum_i v_i' v_i + M^2 v^2$$

Here, for the center of mass of the system of particles

$$\sum_i v_i' v_i = -\sum_i v_i' v_i$$

$$M^2 \sum_i v_i^2 = M^2 \sum_i v_i^2 + M^2 v^2$$

$$\frac{1}{2} M \sum_i v_i^2 = \frac{1}{2} M \sum_i v_i'^2 + \frac{1}{2} M v^2$$

$$K = K' + \frac{1}{2} M v^2$$

where,

$\boldsymbol{K} = \frac{1}{2} M \sum_i v^2{}_1$ = kinetic energy of the system of particles

$\boldsymbol{K'} = \frac{1}{2} M \sum_i v_1'^2$ = Total kinetic energy of the system of particles with respect to the center of mass

$\frac{1}{2} MV^2$ = Kinetic energy of the translation of the system as a whole.

**(c)** Position vector of the $i^{th}$ particle with respect to origin $= r_i$

position vector of the $i^{th}$ particle with respect to the center of mass $= r_i'$

Position vector of the center of mass w.r.t. origin $= R$

It is given that :

$r_i' = r_i - R$

$r_i = r_i' + R$

We have from part (a)

$p_i = p'_i + m_i v$

Taking the cross product of this relation by $r_i$, we get

$$\sum_i r_i \times p_i = \sum_i r_i \times p_i' = \sum_i r_i \times m_i v$$

$$L = \sum_i (r_i \times R) \times p_i' + \sum_i (r_i' \times R) \times m_i v$$

$$= \sum_i r_i \times p_i' + \sum_i R \times p_i' + \sum_i r_i \times m_i v + \sum_i R \times m_i v$$

$$= L' + \sum_i R_i \times p_i' + \sum_i r_i' \times m_i v + \sum_i R \times m_i v$$

where

$$R \times \sum_i p_i' = 0 \text{ and}$$

$$\left(\sum_i r_i'\right) \times Mv = 0$$

$$\sum_i m_i = M$$

$$\therefore L = L' + R \times Mv$$

**(d)** We have the relation:

$$L' = \sum_i r_i' \times p_i'$$

$$\frac{dL'}{dt} = \frac{d}{dt}\left(\sum_i r_i' \times p_i'\right)$$

$$= \frac{d}{dt}\left(\sum_i r_i'\right) \times p_i' + \sum_i r_i' \times \frac{d}{dt} \times (p_i')$$

$$= \frac{d}{dt}\left(\sum_i m_i r_i'\right) \times v_i' + \sum_i r_i' \times \frac{d}{dt}(p_i')$$

Where $r_i'$ is the position vector with respect to the center of mass system of particles

$$\therefore \sum_i m_i r_i' = 0_i$$

$$\frac{dl}{dt} = \sum_i m_i r_i' \times \frac{dl}{dt}(p_i')$$

We have the relation:

$$\frac{dL'}{dt}\sum_i r_i' \times \frac{d}{dt}(p_i')$$

$$= \sum_i r_i' \times m_i \frac{d}{dt}(v_i')$$

Where, $\frac{d}{dt}(v_i')$ is the rate of change of velocity of the $i^{th}$ particle with respect to the center of mass of the system

Therefore, according to Newton's third law of motion, we can write: $m_i \frac{d}{dt}(v_i')$ =External force acting on the $i^{th}$ particle

$$\sum_i (\tau_i')_{ext}$$

*i.e.* $\sum_i r_i' \times m_i \frac{d}{dt}(v_i') = \tau'_{ext}$ = External torque acting on the system as a whole

$$\therefore \frac{dL'}{dt} = \tau'_{ext}$$

# TOPIC-2
## Moment of Inertia and Radius of Gyration

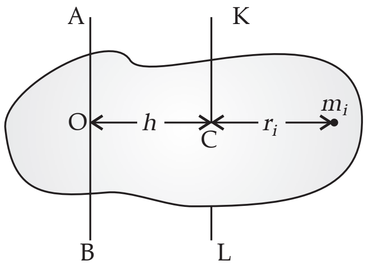
## Quick Review

➢ **Principle of Conservation of Angular Momentum :** According to this principle, when no external torque acts on a system of particles, then the total angular momentum of system always remains a constant. *i.e.*,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + ....... + \vec{L}_n = \text{constant.}$$

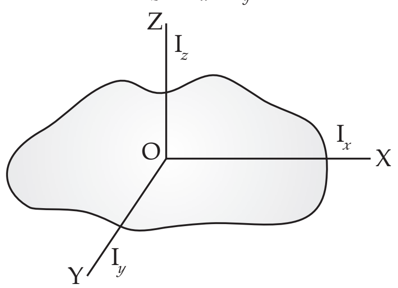
➢ **Theorem of Parallel Axes :** According to this theorem, moment of inertia of a rigid body about any axis AB is equal to moment of inertia of the body about another axis KL passing through center of mass C of the body in a direction parallel to AB, plus the product of total mass M of the body and square of the perpendicular distance between the two parallel axes. *i.e.*,

$$I_{AB} = I_{KL} + Mh^2$$




➢ **Theorem of Perpendicular Axes :** According to this theorem, the M.I. of plane lamina about any axis OZ perpendicular to the plane of the lamina is equal to the sum of moments of inertia of lamina about any two mutually perpendicular axes OX and OY in the plane of lamina, meeting at a point where the given axis OZ passes through the lamina. *i.e.*,

$$I_z = I_x + I_y$$




➢ **Laws of Rotational Motion**

**I Law :** A body continues to be in a state of rest or in a state of uniform rotation about a given axis unless an external torque is applied on the body.

**II Law :** The rate of change of angular momentum of a body about a given axis is directly proportional to external torque applied on the body.

**III Law :** When a rigid body A exerts a torque on another rigid body B in contact with it, then the body B would exert an equal and opposite torque on the body A.

## Know the Terms

- **Moment of inertia** of a body about a given axis is the property by virtue of which, the body opposes any change in its state of rest or state of uniform rotation about that axis. For a single particle, moment of inertia (I) is equal to product of mass ($m$) of the particle and square of perpendicular distance ($r$) of the particle from the axis of rotation.,

  *i.e.,* $I = mr^2$.

  Moment of inertia is a scalar quantity, whose unit is $kgm^2$. It plays the same role in rotational motion as is played by the mass in linear motion.
- **Radius of gyration** of a body about a given axis is the distance (K) of a point from the given axis, where if whole mass of the body is concentrated, the body would have the same moment of inertia, as it has with the actual distribution of mass.
- **Kinetic energy of rotation of a body is the energy possessed by body on account of its rotation about a given axis.**

## Know the Formulae

- **Moment of Inertia—**

  Unit—$kg.m^2$, Dimension—$[ML^2T^0]$

  **(i) Circular ring—** $I = MR^2$

  **(ii) Circular disc—** $I = \frac{1}{2}MR^2$

  **(iii) Angular disc—** $I = \frac{1}{2}(R^2 - r^2)$

  (R = outer radius, r = inner radius)

  **(iv) Thin rod—** $I = \frac{Ml^2}{12}$, Where $l$ is the length of the rod.

  **axis ⊥ to its length.**

  **(v) Solid cylinder—** $I = \frac{1}{2}MR^2$

  **(vi) Hollow cylinder—** $I = MR^2$
  about its long axis of symmetry.

  **(vii) Solid sphere—** $I = \frac{2}{5}MR^2$

  about its diameter.

  **(viii) Hollow sphere—** $I = \frac{2}{3}MR^2$

  Axis = Diameter

  **(ix) Solid cylinder—** $I = M\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$

  about an axis passing through C.G. & ⊥ to its own axis.

  **(x) Hollow cylinder—** $I = M\left(\frac{l^2}{12} + \frac{R^2}{2}\right)$

  about an axis passing through C.G. & ⊥ to its own axis

  **(xi) Uniform rectangular lamina—**

  $$I = M\left(\frac{l^2 + b^2}{12}\right)$$

about an axis passing through C.G. & ⊥ to its plane.

$l$ = length

$b$ = breadth

**(xii) Elliptical disc—**

$$I = \frac{M}{4}\left(a^2 + b^2\right)$$

about an axis passing through its C.G. & ⊥ to its plane.
($a$ = semi-major axis, $b$ = semi-minor axis.)

**(xiii) Uniform cone—** $$I = \frac{3}{10}MR^2$$

about an axis joining the vertex to centre of its base.

**(xiv) Triangular lamina—**

$$I_1 = M \times h^2/6 \text{ (about the base as axis)}$$

$$I_2 = \frac{b^2}{6}.M \text{ (about the height as axis)}$$

$$I_3 = \frac{mb^2h^2}{6\left(b^2 + h^2\right)} \text{ (about the hypotenuse as axis)}$$

➢ **Kinetic energy of rotation—**

**Unit—Joule.** $$K.E. = \frac{1}{2}I\omega^2,$$

➢ **Angular Momentum—** $L = I\omega$

➢ **Relation between Angular Momentum & Torque—**

$$\tau = I\alpha,\ \tau = dL/dt$$

$$L = kgm^2s^{-1},\ \tau = N\text{–}m.$$

➢ **From Principle of Conservation of Angular momentum—**

$$\frac{I_1}{I_2} = \frac{\tau_1}{\tau_2}$$

➢ **Radius of Gyration—**

$$K = \sqrt{\frac{r_1^2 + r_2^2 + .....r_n^2}{n}}$$

Here $r_1, r_2 ........r_n$ = perpendicular distance of particles from axis of rotation
$n$ = total no. of particles.

## Know the Links

☞ *www.learcbse.com*
☞ *www.physics.stackexchange.com*
☞ *www.vedantu.com*

## MCQ/Fillups/True or False **(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind (as shown in figure). The moment of inertia about the z-axis is then**

**(a) increased**
**(b) decreased**
**(c) the same**
**(d) changed in unpredicted manner.**

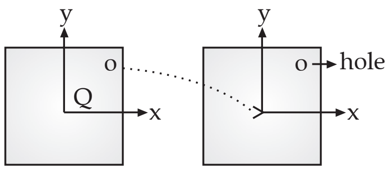



**Fig 7.3**

**[NCERT Exemp. Q. 7.5, Page 51]**

**Ans. Correct option:** (b)

*Explanation:* Moment of Inertia I = $mr^2$

Here distance of small piece from $z$-axis is decreases.

So moment of Inertia about $z$-axis is decreases.

**Q. 2. In problem 1, the CM of the plate is now in the following quadrant of x-y plane,**

**(a) I** **(b) II**

**(c) III** **(d) IV**

**[NCERT Exemp. Q. 7.6, Page 51]**

**Ans. Correct option:** (c)

*Explanation:* Here mass decreases from I quadrant, so c.m. displaced to III quadrant.

**Q. 3. A Merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round).**

**The speed of the round afterwards is**

**(a)** 2ω **(b)** ω

**(c)** $\frac{\omega}{2}$ **(d) 0**

**[NCERT Exemp. Q. 7.8, Page 52]**

**Ans. Correct option:** (b)

*Explanation:*

As no torque is exerted by the person jumping, radially away from the centre of the round, let the total moment of inertia of the system is 2I and the round is revolving with angular speed ω. Since the angular momentum of the person when it jumps off the round is Iω the actual momentum of round seen from ground is 2Iω – Iω = Iω

So, we conclude that the angular speed remains same *i.e.*, ω.

**Q. 4. With reference to figure of a cube of edge a and mass m, state whether the following are true or false.**

**(O is the centre of the cube.)**

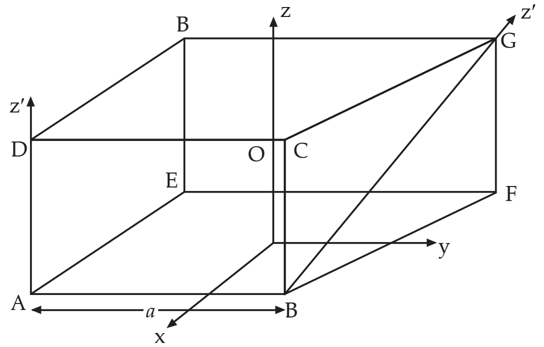



**Fig 7.6**

**(a) The moment of inertia of cube about z-axis is** $I_z = I_x + I_y$

**(b) The moment of inertia of cube about** $z'$ **is** $I'_z = I_z + \frac{ma^2}{2}$

**(c) The moment of inertia of cube about** $z''$ **is** $= I_z + \frac{ma^2}{2}$

**(d)** $I_x = I_y$

**[NCERT Exemp. Q. 7.13, Page 53]**

**Ans. (a) False :** Perpendicular axis theorem is applicable only to a lamina.

**(b) True :** (Parallel axis theorem)

**(c) False :** $z \neq z'$

**(d) True :** (according to symmetry $I_x = I_y$)

## Very Short Answer Type Questions **(1 mark each)**

**Q. 1. Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry? [NCERT Exemp. Q. 7.15, Page 53]**

**Ans.** As, moment of inertia of a given body, $I = \sum m_i r_i^2$

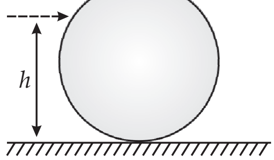
(sum of M.I. of each constituent particles)

All mass in a cylinder lies at distance R from axis of symmetry but most of the mass of a solid sphere lies at a smaller distance than R.

That's why solid sphere have smaller moment of inertia than a hollow cylinder.

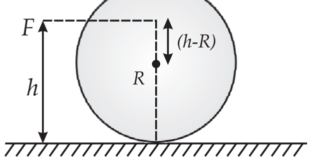
**Q. 2. A uniform sphere of mass m and radius R is placed on a rough horizontal surface (Fig. 7.9). The sphere is struck horizontally at a height h from the floor. Match the following:**

| Column-I | Column-II |
|---|---|
| (a) h=R/2 | (i) Sphere rolls without slipping with a constant velocity and no loss of energy |
| (b) h=R | (ii) Sphere spins clockwise, loses energy by friction. |
| (c) h=3R/2 | (iii) Sphere spins anti-clockwise, loses energy by friction. |
| (d) h=7R/5 | (iv) Sphere has only a translational motion, looses energy by friction. |




**[NCERT Exemp. Q. 7.18, Page 54]**

**Ans.** Let us consider the given figure,



**(i)** The sphere will roll without slipping, when

$$\omega = \frac{v}{r}$$

($v$ = linear velocity, $\omega$ = angular velocity).

By applying, law of conservation of angular momentum just before and after struck-

$$mv(h - R) = I\omega = \left(\frac{2}{5}MR^2\right).\left(\frac{v}{R}\right)$$

$$\text{or } mv(h - R) = \frac{2}{5}mvR$$

$$\text{or } (h - R) = \frac{2}{5}R \text{ or } h = \frac{7}{5}R$$

So, (d) → (i)

**(ii)** Torque (due to applied force),

$$\tau = F(h - R) \text{ (clockwise)}$$

$$\text{If } h = R, \tau = 0$$

So, (b) → (iv)

**(iii)** Sphere will spin clockwise,

if $\tau > 0$

$(h - R) \times F > 0$ or $h > R$

So, (c) → (ii)

**(iv)** Sphere will spin anticlockwise,

if $\tau < 0$

$(h - R)F < 0$

$h < R \quad (\because F \not< 0)$

So, (a) → (iii)

Therefore, (a) → (iii),(b) → (iv), (c) → (ii), (d) → (i)

## Short Answer Type Questions **(2 or 3 marks each)**

**Q. 1. A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7·6 kg-m². (a) What is his new angular speed ? (Neglect friction) (b) Is K.E. conserved in the process ? If not from where does the change come out ? [NCERT Ad. Ex. Q. 7.23, Page 180]**

**Ans. (a)** Initial moment of inertia

$$I_1 = 7{\cdot}6 \times 2 \times 5 \times (0{\cdot}9)^2$$
$$= 15{\cdot}7 \text{ kg m}^2$$
$$\omega_1 = 30 \text{ rpm}$$

Final moment of inertia

$$I_2 = 7{\cdot}6 + 2 \times 5(0{\cdot}2)^2$$
$$= 8{\cdot}0 \text{ kg m}^2$$
$$\omega_2 = ?$$

According to the principle of conservation of angular momentum,

$$I_2\omega_2 = I_1\omega_1$$

$$\omega_2 = \frac{I_1}{I_2}\,\omega_1 = \frac{15 \cdot 7 \times 30}{8 \cdot 0}$$

$$= 58{\cdot}88 \text{ rpm}$$

**(b)** No, kinetic energy is not conserved in the process. In fact, as moment of inertia decreases, K.E. of rotation increases. This change comes about as work is done by the man in bringing his arms closer to his body.

**Q. 2. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1·0 m wide and weights 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embedded into it.**

**[Hint : The moment of inertia of the door about the vertical axis at one end is $\frac{ML^2}{3}$ ]**

**[NCERT Ad. Ex. Q. 7.24, Page 180]**

**Ans.** Angular momentum imparted by the bullet

$$L = mv \times r$$
$$= (10 \times 10^{-3}) \times 500 = 2{\cdot}5 \text{ kg-m}^2$$

Also, $I = 4 \text{ kg-m}^2$

As $L = I\omega$

$$\therefore \quad \omega = \frac{L}{I} = \frac{2.5}{4} = 0.625 \text{ rad/sec.}$$

**Q. 3. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius the cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time ?**

**[NCERT Ex. Q. 7.11, Page 179]**

**Ans.** Moment of inertia of the cylinder

$$I_1 = MR^2$$

and Moment of inertia of the sphere

$$I_2 = \frac{2}{5}MR^2$$

Angular acceleration of cylinder

$$\alpha_1 = \frac{\tau}{I_1} = \frac{\tau}{MR_2}$$

Angular acceleration of sphere

$$\alpha_2 = \frac{\tau}{I_2} = \frac{\tau}{\frac{2}{5}MR^2}$$

$$= 2{\cdot}5\,\frac{\tau}{MR_2}$$

$$= 2{\cdot}5\,\alpha_1$$

**Note :** From $\omega = \omega_0 + \omega t$, we find that for given $\omega_0$ and $t$, $\omega_2 > \omega_1$ *i.e.*, angular speed of solid sphere will be greater than angular speed of hollow cylinder.

**Q. 4. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s$^{-1}$. The radius of the cylinder is 0·25. What is the kinetic energy associated with the rotation of the cylinder ? What is the magnitude of angular momentum of the cylinder about its axis ?**

**[NCERT Ex. Q. 7.12, Page 179]**

**Ans.** As $(K.E.)_{rot} = \frac{1}{2} I\omega^2$

Here, $I = \frac{1}{2} MR^2 = \frac{20}{2} \times (0{\cdot}25)^2$

$= 0{\cdot}625$ kg-m$^2$

$\omega = 100$ rad $s^{-1}$,

we get $(K.E.)_{rot} = \frac{1}{2} \times 0{\cdot}625 \times (100)^2$

$= 3125$ J

Angular momentum,

$L = I\omega$

$= 0{\cdot}625 \times 100$

$= 62{\cdot}5$ kgm$^2$s$^{-1}$.

**Q. 5. (a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.**

**(b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?**

**[NCERT Ex. Q. 7.13, Page 179]**

**Ans.** Given: $\omega_1 = 40$ rev/min., $I_2 = \frac{2}{5} I_1$

**(a)** As no external torque is acting on system

So, L=constant= $I\omega$

$I_1\omega_1 = I_2\omega_2$

or $\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{5}{2} \times 40 = 100\,\text{rpm}$

**(b)** $$\frac{K.E_f}{K.E_i} = \frac{\frac{1}{2} I_2\omega_2^2}{\frac{1}{2} I_1\omega_1^2} = \left(\frac{I_2}{I_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2$$

$$= \frac{2}{5} \times \left(\frac{100}{40}\right)^2$$

$$= 2.5$$

Therefore, $K.E_{final} = 2.5 K.E_{initial}$

This increase in K.E. is because of fact that child spends his internal energy in folding his hands which is converting into its kinetic energy.

**Q. 6. A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N ? What is the linear acceleration of the rope ? Assume that there is no slipping?**

**[NCERT Ex. Q. 7.14, Page 179]**

**Ans.** Here, M = 3 kg,

R = 40 cm = 0·4 m.

Moment of inertia of the hollow cylinder about its axis

$I = MR^2$

$= 3 \times (0{\cdot}4)^2$ kgm$^2$

$= 0{\cdot}48$ kg-m$^2$

When force of 30 N is applied over the rope wound on the cylinder, the torque will act on the cylinder. It is given by

$\tau = FR = 30 \times 0{\cdot}4 = 12$ Nm.

If $\alpha$ is angular acceleration produced, then,

$\tau = I\alpha$

or $\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25$ rad $s^{-2}$.

The linear acceleration of the rope

$a = R\alpha = 0{\cdot}4 \times 25 = 10$ ms$^{-2}$

**Q. 7. To maintain a rotor at a uniform angular speed of 200 rad s$^{-1}$, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine ? Assume that the engine is 100% efficient.**

**[NCERT Ex. Q. 7.15, Page 179]**

**Ans.** Here, $\tau = 180$ Nm, $\omega = 200$ rads$^{-1}$.

As $P = \tau\omega$

$\therefore$ $P = 180 \times 200$

$= 36000$ W $= 36$ kW

Hence, the power required by the engine is 36 kW.

# Long Answer Type Questions

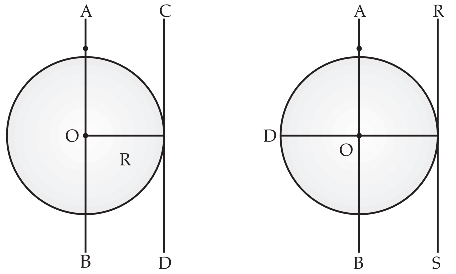
**(5 marks each)**

**Q. 1. (a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.**

**(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.**

**[NCERT Ex. Q. 7.10, Page 178]**

**Ans. (a)**




Given:

$I_{AB} = \frac{2}{5}MR^2$

$I_{CD} = ?$

From parallel axes theorem,

$I_{CD} = I_{AB} + MR^2$

$\frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

**(b)** In fig AB & CD are two diameters symmetrical to each other.

$I_{AB} = I_{CD} = \frac{1}{4}MR^2 (Given)$

⇒ From perpendicular axes theorem,

M.I. of disc about an axis passing through its centre and normal to its plane

$I_{XY} = I_{AB} + I_{CD} = 2 \times \frac{1}{4}MR^2 = \frac{1}{2}MR^2$

⇒ From parallel axes theorem,

MI. of the disc about on axis passing through its edge and perpendicular to its plane.

$I_{RS} = I_{XY} + MR^2$

$= \frac{1}{2}MR^2 + MR^2$

$I_{RS} = \frac{3}{2}MR^2$

**Q. 2. From a uniform disk of radius R, a circular section of radius R/2 is cut out, the centre of the hole is at R/2, from the centre of the original disc. Locate the centre of mass of the resulting flat body.**

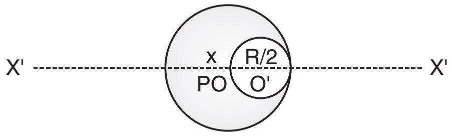
**[NCERT Ex. Q. 7.16, Page 179]**

**Ans.** Suppose mass per unit area of the disc = $m$

∴ Mass of portion removed from the disc,

$$M' = \pi (R/2)^2 \times m$$

$$= \frac{\pi R^2}{4} m = \frac{M}{4}$$




In figure, mass M is concentrated at O and mass M′ is concentrated at O′, where OO′ = R/2.

After the circular disc of mass M′ is removed, the remaining portion can be considered as a system of two masses M at O and – M′ = – $\frac{M}{4}$ at O′.

If $x$ is the distance of centre of mass (P) of the remaining part,

then, $$x = \frac{M \times 0 - M' \times R/2}{M + M'}$$

$$= \frac{\frac{-M}{4} \times \frac{R}{2}}{M - \frac{M}{4}}$$

$$= \frac{-MR}{8} \times \frac{4}{3M}$$

$$= \frac{-R}{6}$$

Negative sign shows that P is to the left of O.

**Q. 3. A solid sphere rolls down two different inclined planes of the same height but different angles of inclination.**

**(a) Will it reach the bottom with the same speed in each case ?**

**(b) Will it take longer to roll down one plane than the other ? Is if so, which one and why ?**

**[NCERT Ex. Q. 7.18, Page 179]**

**Ans.** Let $v$ be the speed of the solid sphere at the bottom of the inclined plane Applying principle of conservation of energy, we get

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

As $$I = \frac{2}{5}mR^2$$

$$\frac{1}{2}mv + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = mgh$$

$$R\omega = v$$

$$\frac{1}{2}mv^2 + \frac{1}{5}mv^2 = mgh$$

$$v = \sqrt{\frac{10}{7}gh}$$

Since $h$ is same for both the inclined planes

∴ $v$ is the same

**(b)** Consider two inclined planes with inclinations $\theta_1$ and $\theta_2$ where $\theta_1 < \theta_2$

The acceleration produced in the sphere when it rolls down the inclined plane at $\theta_1$ is g $\sin\theta_1$

Similarly, acceleration at plane of the inclined plane of angle $\theta_2$ is g $\sin\theta_2$.

$\because \theta_2 > \theta_1$

$\therefore$ g $\sin\theta_2$ > g $\sin\theta_1$

$\therefore a_2 > a_1$ ...(ii)

Initial velocity, u = 0

Final velocity, $v$ = constant.

Using first equation of motion,

$v$ = u+at

$$t \propto \frac{1}{a}$$

$$\left.\begin{array}{l}\text{for inclination } \theta_1 : t_1 \propto \dfrac{1}{a_1} \\ \text{for inclination } \theta_2 : t_2 \propto \dfrac{1}{a_2}\end{array}\right\} \quad .....(iii)$$

from equations (ii) and (iii)

$t_2 < t_1$

Hence, the sphere roll take a longer time-to reach the bottom of the inclined plane having smaller inclination.

**Q. 4. A hoop is of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it ?**

**[NCERT Ex. Q. 7.19, Page 179]**

**Ans.** R = 2 m, M = 100 kg

$v$ = 20 cm/s = 0·2 m/s

$$\text{Total energy of the hoop} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$(\because v = R\omega)$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\,\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$$

Work required to stop the hoop total energy of the hoop

$$W = Mv^2$$
$$= 100\,(0{\cdot}2)^2$$
$$= 4 \text{ Joule.}$$

**Q. 5. The oxygen molecule has a mass of $5.30 \times 10^{-26}$ kg and a moment of inertia of $1.94\times 10^{-46}$ kg m$^2$ about an axis through its center perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.**

**[NCERT Ex. Q. 7.20, Page 179]**

**Ans.** Mass of an oxygen molecule, $m = 5.30 \times 10^{-26}$ kg

Moment of inertia, I= $1.94 \times 10^{-46}$ kg m$^2$

Velocity of the oxygen molecule, $v$ = 500 m/s

The separation between the two atoms of the oxygen molecule = $2r$

Mass of each oxygen atom = $\dfrac{m}{2}$

Hence, moment of inertia $I$, is calculated as:

$$\left(\frac{m}{2}\right)r^2 + \left(\frac{m}{2}\right)r^2 = mr^2$$

$$r = \sqrt{\frac{I}{m}}$$

$$r = \sqrt{\frac{1.94\times10^{-46}}{5.36\times10^{-26}}} = 0.60\times10^{-10}m$$

It is given that:

$$KE_{rot} = \frac{2}{3}KE_{trans}$$

$$\frac{1}{2}I\omega^2 = \frac{2}{3}\times\frac{1}{2}\times mv^2$$

$$mr^2\omega^2 = \frac{2}{3}mv^2$$

$$\omega = \sqrt{\frac{2}{3}}\frac{v}{r}$$

$$= \sqrt{\frac{2}{3}}\times\frac{500}{0.6\times10^{-10}}$$

$$= 6.80\times10^{12}\text{ rad/s}$$

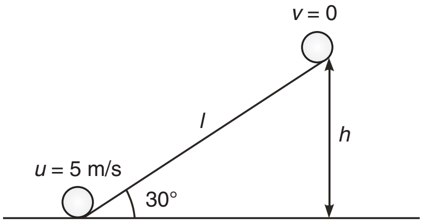
**Q. 6. A solid cylinder rolls up an inclined plane of an angle of inclination 30°. At the bottom of the inclined plane of the centre of mass of the cylinder has a speed of 5 m/s.**

**(a) How far will the cylinder go up the plane ?**

**(b) How long will it take to return to the bottom ?**

**[NCERT Ex. Q. 7.21, Page 179]**

**Ans.**




If '$a$' is the acceleration (*i.e.*, deceleration) in the present case

$$a = \frac{g\sin\theta}{1+\dfrac{K^2}{r^2}}$$

In case of a solid cylinder

$$K = \frac{1}{\sqrt{2}}r$$

$$a = \frac{g\sin 30^\circ}{1+\dfrac{\frac{1}{2}r^2}{r^2}}$$

$$= \frac{g\dfrac{1}{2}}{1+\dfrac{1}{2}} = \frac{g}{3}$$

From $\quad v^2 = u^2 + 2as$

$$0 = (5)^2 + 2\left(\frac{-g}{3}\right)l$$

$$0 = 25 - \frac{2g}{3}l$$

$$l = \frac{25\times3}{2g} = \frac{75}{2\times9.8}$$

$$= 3{\cdot}8 \text{ m}$$

**(b)** Now time for going up and returning must be equal.

From $s = ut + \frac{1}{2}at^2$,

we get, $l = 0\times t + \frac{1}{2}\left(\frac{-g}{3}\right)t^2$

$$t^2 = \frac{6l}{g} = \frac{6\times3.8}{9.8}$$

$$t = \sqrt{\frac{6\times3.8}{9.8}}$$

$$= 1{\cdot}5 \text{ sec.}$$

Total time for going up and coming down = $2t$
$= 2 \times 1{\cdot}5 = 3$ s

**Q. 7. Two discs of moments of inertia $I_1$ and $I_2$ about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed $\omega_1$ and $\omega_2$ are brought into contact face to face with their axes of rotation coincident.**

**(a) Does the law of conservation of angular momentum apply to the situation? Why?**

**(b) Find the angular speed of the two-disc system.**

**(c) Calculate the loss in kinetic energy of the system in the process.**

**(d) Account for this loss.**

**[NCERT Ex. Q. 7.24, Page 55]**

**Ans. (a)** Yes, the law of conservation of angular momentum can be applied to the situation because there is no net external torque on the system. External forces, gravitation and normal reaction, act through the axis of rotation. Hence, produce no torque.

(b) By angular momentum conservation law,

$$L_f = L_i$$

$$I\omega = I_1\omega_1 + I_2\omega_2 \text{ or } \omega = \frac{I_1\omega_1 + I_2\omega_2}{I}$$

$$\therefore\ \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} \quad [\because\ I = I_1 + I_2]$$

(c) Final kinetic energy = Rotational K.E + Translational K.E.

$$(K.E.)_f = (K.E.)_R + (K.E.)_T$$

As, there is no translational energy, $\Sigma\,(K.E)_T=0$

$$(K.E.)_f = (K.E.)_R$$

$$K_f = \frac{1}{2}(I_1 + I_2)\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)^2} = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)}$$

$$K_i = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2)$$

$$\Delta K = K_f - K_i = -\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2 < 0$$

(d) Negative sign indicates that there is loss in kinetic energy of the system due to work against the friction between moving surface of discs.

**Q. 8. Prove the theorem of perpendicular axes.**

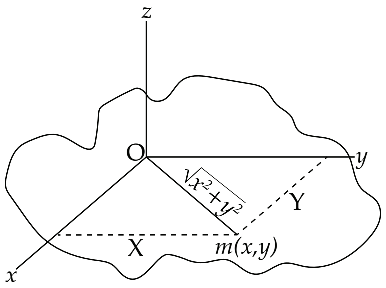
**(Hint: Square of the distance of a point (x, *y*) in the *x-y* plane from an axis through the origin perpendicular to the plane is $x^2 + y^2$).**

**Prove the theorem of parallel axes.**

**(Hint: If the center of mass is chosen to be the origin $\sum m_i r_i = 0$).[NCERT Ad. Ex. Q. 7.26, Page 180]**

**Ans. (a)** The theorem of perpendicular axes states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

A physical body with center O and a point mass *m*, in the *x-y* plane at *(x, y)* is shown in the following figure.




Moment of inertia about *x*-axis, $I_x = mx^2$
Moment of inertia about *y*-axis, $I_y = my^2$

Moment of inertia about *z*-axis, $I_z = m\left(\sqrt{x^2+y^2}\right)^2$

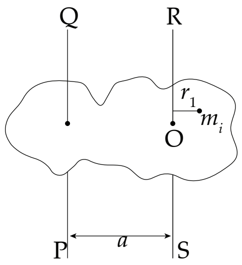
$$I_x + I_y = mx^2 + my^2 = m(x^2 + y^2)$$

$$= m\left(\sqrt{x^2+y^2}\right)^2$$

$$I_x + I_y = I_z$$

Hence, the theorem is proved.

**(b)** The theorem of parallel axes states that the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its center of mass and the product of its mass and the square of the distance between the two parallel axes.




Suppose a rigid body is made up of *n* particles, having masses $m_1, m_2, m_3, \ldots, m_n$, at perpendicular distances $r_1, r_2, r_3, \ldots, r_n$ respectively from the center of mass O of the rigid body.

The moment of inertia about axis RS passing through the point O:

$$I_{RS} = \sum_{i=1}^{n} m_i r_i^{\ 2}$$

The perpendicular distance of mass $m_i$, from the axis $= a + r_i$

Hence, the moment of inertia about axis QP:

$$I_{QP} = \sum_{i=1}^{n} m_i \left(a + r_i\right)^2$$

$$= \sum_{i=1}^{n} m_i \left(a^2 + r_i^{\ 2} + 2ar_i\right)^2$$

$$= \sum_{i=1}^{n} m_i a^2 + \sum_{i=1}^{n} m_i 2ar_i^{\ 2} + 2\sum_{i=1}^{n} m_i 2ar_i$$

$$= I_{RS} + \sum_{i=1}^{n} m_i a^2 + 2\sum_{i=1}^{n} m_i ar_i^2$$

Now, at the centre of mass, the moment of inertia of all the particles about the axis passing through the centre of mass is zero, that is,

$$2\sum_{i-1}^{n} m_i ar_i^{\ 2} = 0$$

$$\because a \neq 0$$

$$\therefore \sum m_i r_i^{\ 2} = 0$$

Also,

$$\sum_{n=1}^{n} m_i = \text{M};$$

M = Total mass of the rigid body

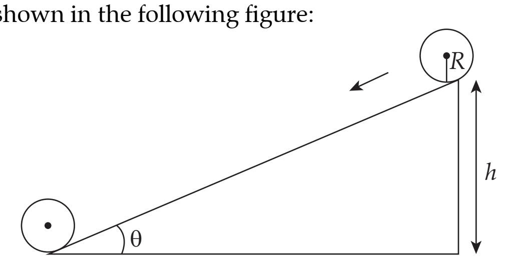
$\therefore I_{QR} = I_{RS} + Ma^2$

Hence, the theorem is proved.

**Q. 9. Prove the result that the velocity $v$ of translation of a rolling body (like a ring, disc, cylinder or cylinder or sphere) at the bottom of an inclined plane of a height $h$ is given by $v^2 = \dfrac{2gh}{\left(1 + K^2/R^2\right)}$. using dynamical consideration (i.e. by consideration of forces and torques).**

**Note : $K$ is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane. [NCERT Ad. Ex. Q. 7.27, Page 180]**

**Ans.** A body rolling on an inclined plane of height $h$, is shown in the following figure:




$m$ = Mass of the body

$R$ = Radius of the body

$K$ = Radius of gyration of the body

$v$ = Translational velocity of the body

$h$ = Height of the inclined plane

$g$ = Acceleration due to gravity

Total energy at the top of the plane, $E_l = mgh$

Total energy at the bottom of the plane,

$$E_b = KE_{rot} + KE_{trans}$$

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

But $I = mK^2$ and $\omega = \dfrac{v}{R}$

$$\therefore E_b = \frac{1}{2}\left(mK^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2$$

$$= \frac{1}{2}mv\frac{K^2}{R^2} + \frac{1}{2}mv^2$$

$$= \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

From the law of conservation of energy, we have:

$$E_T = \text{E}_b$$

$$mgh = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

$$\therefore v^2 = \frac{2gh}{\left(1 + K^2/R^2\right)}$$

Hence, the given result is proved.

**Q. 10. A solid disc and ring, both of radius 10 cm are placed on a horizontal table simultaneously with initial angular speed equal to 10π rad s$^{-1}$. Which of the two will start to roll earlier ? The coefficient of kinetic friction is $\mu_k = 0{\cdot}2$.**

**[NCERT Ad. Ex. Q. 7.30, Page 181]**

**Ans.** The initial angular speed

$$\omega_0 = 10\pi \text{ rads}^{-1}$$

The force due to friction

$$= \mu_K \times \text{N}$$

$$= \mu_K mg$$

where N is the normal reaction of the table. The centre of mass of body will move with an acceleration '$a$', according to Newton's second law of motion,

$$\mu_K mg = m_a$$

$$a = \mu_K g$$

The torque due to friction ($\mu_R mg$R) will decrease the initial angular speed $\omega_0$ and hence will produce angular retardation,

$$\therefore \quad \mu_K mg\text{R} = \text{I}\alpha$$

$$\alpha = \frac{\mu_K mg\text{R}}{\text{I}} \quad ...(1)$$

From equation (1) the centre of mass will have a speed $v$ (its initial speed $u = 0$ given by $v = u + at$)

$$v = 0 + \mu_K gt$$

$$= \mu_K gt \quad ...(2)$$

Also the angular speed after a time $t$ is given by [using eq. (2)]

$$\omega = \omega_0 + \alpha t$$

$$= \omega_0 - \frac{\mu_K mg\text{R}t}{\text{I}} \quad ...(3)$$

For a ring $I = MR^2$ and rolling begins when $v = R\omega$.

Now equation (3) gives

$$v = R\omega = \mu_K g t \quad ..(4)$$

and equation (3) gives $\omega = \omega_0 - \dfrac{\mu_K mgRt}{mR^2}$

$$\omega = \omega_0 - \frac{\mu_K g t}{R} \quad ...(5)$$

Dividing equation (4) by equation (5),

$$\frac{v}{\omega} = \frac{\mu_K g t}{\omega_0 - \frac{\mu_K g t}{R}}$$

$$\omega_0 R - \mu_K g t = \mu_K g t$$

$$2\mu_K g t = \omega_0 R$$

or $$t = \frac{\omega_0 R}{2\mu_K g}$$

For disc $$I = \frac{1}{2} mR^2$$

and rolling starts at a time

$$t_2 = \frac{\omega_0 R}{3\mu_K g}$$

∴ The disc begins to roll earlier because $\omega_0 R$ and $\mu_K$ are the same for both the ring and the disc.

The ring will start rolling after the time.

$$t_1 = \frac{\omega_0 R}{2\mu_K g}$$

$$= \frac{10\,\pi \times 0\cdot 1}{2 \times 0\cdot 2 \times 9\cdot 8} = \frac{\pi}{9\cdot 8 \times 0\cdot 4}$$

$$= 0\cdot 80 \text{ s.}$$

The disc will starts rolling after a time

$$t_2 = \frac{\omega_0 R}{3\mu_K g}$$

$$= \frac{10\,\pi \times 0\cdot 1}{3 \times 0\cdot 2 \times 9\cdot 8}$$

$$= \frac{\pi}{9\cdot 8 \times 0\cdot 6}$$

$$= 0\cdot 53 \text{ s.}$$

Since $t_2 < t_1$, the disc will starts rolling before the ring.

**Q. 11. A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The coefficient of static friction $\mu_s = 0.25$.**

**(a) How much is the force of friction acting on the cylinder?**

**(b) What is the work done against friction during rolling?**

**(c) If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid, and not roll perfectly?**

**[NCERT Ad. Ex. Q. 7.31, Page 181]**

**Ans. (a)** Mass of the cylinder, $m = 10$ kg

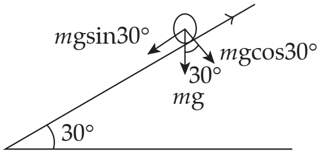
Radius of the cylinder, $r = 1$ cm $= 0.15$ m

Co-efficient of kinetic friction, $\mu_k = 0.25$

Angle of inclination, $\theta = 30°$

Moment of inertia of a solid cylinder about its geometric axis, $I = \frac{1}{2} mr^2$

The various forces acting on the cylinder are shown in the following figure:




The acceleration of the cylinder is given as:

$$a = \frac{mg\sin\theta}{m + \frac{I}{r^2}}$$

$$= \frac{mg\sin\theta}{m + \frac{1.mr^2}{2r^2}}$$

$$= \frac{2}{3} \times 9.8 \times 0.5 = 3.27 \text{m/s}^2$$

Using Newton's second law of motion, we can write net force as:

$f_{net} = ma$

$mg \sin 30° - f = ma$

$f = mg \sin 30° - ma$

$= 10 \times 9.8 \times 0.5 - 10 \times 3.27$

$= 49 - 32.7 = 16.3N$

**(b)** During rolling, the instantaneous point of contact with the plane comes to rest. Hence, the work done against frictional force is zero,

**(c)** For rolling without skid, we have the relation:

$$\mu = \frac{1}{3} \tan\theta$$

$\tan \theta = 3\mu = 3 \times 0.25 = 0.75$

$\theta = \tan^{-1} 0.75 = 36.87°$

**Q. 12. A disc of radius R is rotating with an angular speed $\omega_0$ about a horizontal axis. It is placed on a horizontal table. The coefficient of kinetic friction is $\omega_k$.**

**(a) What was the velocity of its centre of mass before being brought in contact with the table?**

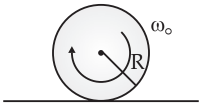
**(b) What happens to the linear velocity of a point on its rim when placed in contact with the table?**

**(c) What happens to the linear speed of the centre of mass when disc is placed in contact with the table?**

**(d) Which force is responsible for the effects in (b) and (c) ?**

**(e) What condition should be satisfied for rolling to begin?**

**(f) Calculate the time taken for the rolling to begin.**




**[NCERT Exemp. Q. 7.25, Page 55]**

**Ans. (a)** Before being brought in contact with the table the disc was in pure rotational motion (only) about its axis passing through centre-

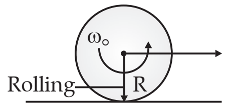
$\therefore$ $v_{cm}=0$ as the point on axis are considered at rest.

**(b)** When the rotating disc is placed in contact with table's surface due to friction, linear velocity of a point on the rim decreases.

**(c)** When disc is placed in contact with the surface of the table, due to frictional force, centre of mass acquires some linear velocity. The linear velocity increases.

**(d)** Frictional force

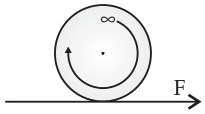
**(e)** When rolling of disc starts on table, then $v_{cm}= \omega R$ due to reaction force.




**(f)** Acceleration produced in centre of mass due to reaction force F because of frictional force in disc

$$F = mA \text{ or } a_{cm} = \frac{F}{m} = \frac{\mu_k mg}{m} = \mu_k g.$$

Angular retardation produced by the torque due to friction.




$$\tau = I\alpha$$

$$\text{or} \quad \alpha = \frac{\tau}{I} = \frac{\mu_k mgR}{I} \qquad [\because \tau = (\mu_k N)R = \mu_k mgR]$$

$$\therefore \quad v_{cm} = u_{cm} + a_{cm}t \text{ or } v_{cm} = \mu_k gt$$

$$\text{and} \quad \omega = \omega_0 + \alpha t \text{ or } \omega = \omega_0 - \frac{\mu_k mgR}{I}t$$

For rolling without slipping, $\frac{v_{cm}}{R} = \omega$

$$\frac{v_{cm}}{R} = \omega_0 - \frac{\mu_k mgR}{I}t$$

$$\frac{\mu_k gt}{R} = \omega_0 - \frac{\mu_k mgR}{I}t$$

$$t = \frac{R\omega_0}{\mu_k g\left(1 + \frac{mR^2}{I}\right)}$$

Here, frictional force helps in rolling motion.

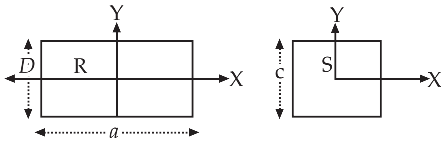
**Q. 13. A uniform square plate S(slide c) and a uniform rectangular plate R (sides b, a) have identical areas and masses (as shown in figure)**

**Show that**

**(i)** $I_{xR} / I_{xS} < 1;$

**(ii)** $I_{yR} / I_{yS} > 1;$

**(iii)** $I_{zR} / I_{zS} > 1$




**[NCERT Exemp. Q. 7.27, Page 56]**

**Ans.** Given: $m_R = m_S = m$

Area of surface= Area of rectangular plate

or $c^2 = a \times b = ab.$

By definition,

$$I = mr^2$$

**(i)**

$$\frac{I_{xR}}{I_{xS}} = \frac{m\left(\frac{b}{2}\right)^2}{m\left(\frac{c}{2}\right)^2} = \frac{b^2}{4} \times \frac{4}{c^2} = \frac{b^2}{c^2}$$

From diagram, $c > b, c^2 > b^2$

$$\therefore \frac{b^2}{c^2} < 1 \text{ or } \left(\frac{b}{c}\right)^2 < 1 \text{ or } I_{xR} < I_{xS}$$

**(ii)**

$$\frac{I_{yR}}{I_{yS}} = \frac{m\left(\frac{a}{2}\right)^2}{m\left(\frac{c}{2}\right)^2} = \frac{a^2}{c^2}$$

as $a > c, \therefore a^2 > c^2 \text{ or } \left(\frac{a}{c}\right)^2 > 1$

Hence, $\frac{I_{yR}}{I_{yS}} > 1$

**(iii)**

$$I_{zR} - I_{zS} = m\left(\frac{d_R}{2}\right)^2 - m\left(\frac{d_S}{2}\right)^2$$

$$= \frac{m}{4}\left[d_R^2 - d_S^2\right]$$

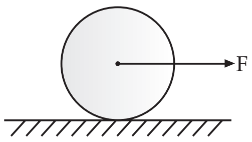
$$= \frac{m}{4}\left(a^2 + b^2 - 2c^2\right)$$

$$= \frac{m}{4}\left(a^2 + b^2 - 2ab\right) \qquad [from\, c^2 = ab]$$

$$= \frac{(a-b)^2 m}{4} > 0$$

$$or\ (I_{zR} - I_{zS}) > 0\ or\ \frac{I_{zR}}{I_{zS}} > 1$$

**Q. 14. A uniform disc of radius R, is resting on a table on its rim. The coefficient of friction between disc and table is $\mu$ (Fig 7.12). Now the disc is pulled with a force F as shown in the figure. What is the maximum value of F for which the disc rolls without slipping?**




**[NCERT Exemp. Q. 7.28, Page 56]**

**Ans.** Let us consider the given diagram, Frictional force (f) is

acting in the opposite direction of F.

Now, let 'a' and $\alpha$ be the linear and angular acceleration respectively.

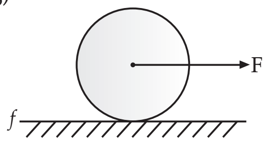
Let the acceleration of the center of mass of disc be 'a', then

$F - f = Ma$ ...(1)

where m is mass of disc.

Force of friction applies torque about centre.

The angular acceleration of the disc is $\alpha = a/R$. (pure rolling)




Then $\tau = I\alpha$ or $f \times R = \left(\frac{1}{2}MR^2\right) \times \alpha$

$$f \times R = \left(\frac{1}{2}MR^2\right)\frac{a}{R}$$

or $Ma = 2f$

$F - f = 2f$ or $F = 3f$ ...(2)

from (1) and (2), we get

$$f \leq \mu N$$

Thus, $f = F/3$.

Since there is no sliding.

or $f \leq \mu mg$

or $F \leq 3\mu Mg$.

**TIPS... & TRICKS...**

- *Understand about center of mass of various regular shapes.*
- *Study about moment of force and its direction.*
- *Drive formula for rotational motion.*
- *Study law of conservation of Angular momentum.*
- *Understand about moment of Inertia.*
- *Learn formula of Momental Inertia to Regular shapes about their axis.*
- *Study and understand the theorems of moment at Inertia.*

## Some Commonly Made Errors

- The concept of center of mass is generally not clear to the students.
- Generally students got confused between rolling motion with and without slipping.

### EXPERT ADVICE

- *Learn the moment of inertia for different objects and shapes by preparing chart or table.*
- *Revise and practice the relations between the physical quantities in the chapter.*

## OSWAAL LEARNING TOOLS

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**Or Scan the Code**

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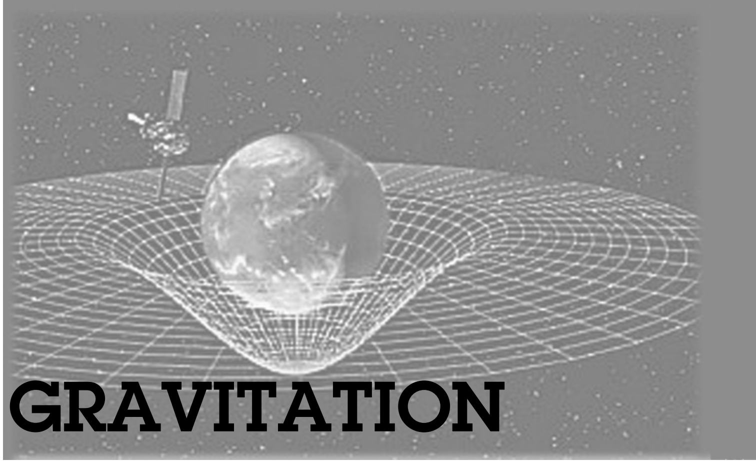


CHAPTER

# 8 GRAVITATION

## Chapter Objective

**This chapter will help you understand :**

- *Kepler's law of Planetary Motion, universal law of Gravitation, acceleration due to gravity and its variation with altitude and depth.*
- *Gravitational Potential energy and Gravitational Potential. Escape velocity, orbital velocity of a satellite, Geo-stationary satellites.*

# TOPIC-1
## Kepler's Laws, Universal law of Gravitation, Acceleration Due to Gravity



### Quick Review

- **Kepler's Laws of Planetary Motion :**
  - **(a) Kepler's I Law (Law of Orbits) :** Each planet revolves around the Sun in an elliptical orbit. The Sun is situated at one foci of the ellipse.
  - **(b) Kepler's II Law (Law of Areas) :** The position vector of the planet from the Sun sweeps out equal area in equal interval of time. That is the areal velocity of the planet around the Sun is constant.
  - **(c) Kepler's III Law (Law of Periods) :** The square of the time period of any planet about the Sun is proportional to the cube of the semi-major axis of the elliptical orbit.

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

**Universal Law of Gravitation :** It states that every body in universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

$$F \propto \frac{m_1 m_2}{r^2} \text{ or } F = \frac{G m_1 m_2}{r^2}$$

- **Gravitational constant (G) :** It is defined as the force of attraction acting between two bodies each of unit mass, whose centres are placed at unit distance apart. The value of G is constant throughout the universe. It is a scalar quantity. The dimensional formula of G = $[M^{-1}L^3T^{-2}]$. The value of G = $6.67 \times 10^{-11}$ N$m^2$kg$^{-2}$.

  The value of G being too small, we do not experience it in our daily life.
- **Gravity :** It is the force of attraction exerted by Earth towards its centre on a body lying on or near the surface of the earth.

  Gravity is the measure of weight of the body.

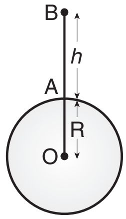
  The weight of the body = mass ($m$) × acceleration due to gravity ($g$) = $mg$.

  The unit of weight of the body will be the same as those of force.

  Gravity is a vector quantity. It is always directed towards the centre of earth. Gravity holds the atmosphere around the earth.

- **Acceleration due to gravity ($g$) :** It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone.
  Acceleration due to gravity is a vector quantity. It is directed towards the centre of Earth.
  The unit of $g$ is $ms^{-2}$ and its dimensional formula is $[M^0LT^{-2}]$.
  **(a)** The acceleration due to gravity does not depend upon (a) the mass of body, (b) shape or size of the body.
- **Variation of acceleration due to gravity.**

**(a) Effect of altitude :** $$g' = \frac{gR^2}{(R+h)^2}$$




when $h$ is comparable with R and $h << R$.

$$g' = g\left(1 - \frac{2h}{R}\right)$$

From these relations, we conclude that acceleration due to gravity decreases with increase in height from the surface of earth.

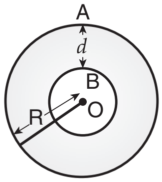
**(i)** Fractional decrease in the value of $g$ with height $= \frac{g - g'}{g} = \frac{2h}{R}$

**(ii)** % decrease in the value of $g$ $= \left(\frac{g - g'}{g}\right) \times 100 = \frac{2h}{R} \times 100\%$

**(b) Effect of depth :** $g' = g\left(1 - \frac{d}{R}\right)$

**(i)** The acceleration due to gravity decreases with increase in depth $d$ and becomes zero at the centre of the earth.

**(ii)** Decrease in the value of $g$ with depth, $\Delta g = g - g' = \frac{gd}{R}$




$\therefore$ Fractional decrease in the value of $g$ with depth $= \frac{g - g'}{g} = \frac{d}{R}$

**(iii)** % decrease in the value of $g$ with depth,

$$\frac{g - g'}{g} \times 100 = \frac{d}{R} \times 100\%$$

**(c) Rotation of earth :** $g = g' + R\omega^2 \cos^2 \lambda$

or $g' = g - R\omega^2 \cos^2 \lambda$

where, $\omega$ is the angular velocity of rotation of earth about its polar axis and $\lambda$ is the latitude of a place.

**(i)** At the equator, $\lambda = 0°$,

**so** $g' = g - R\omega^2 \cos^2 0° = g - R\omega^2$

**(ii)** At the poles, $\lambda = 90°$,

so $g' = g - R\omega^2 \cos^2 90° = g$

Hence, the value of acceleration due to gravity increases from equator to pole due to rotation of earth. It means the value of $g$ increases with latitude.

**(iii)** If a body of mass $m$ is moved from equator to pole its weight increases by an amount

$$= m(g_p - g_e) = mR\omega^2.$$

**(iv)** Decrease in $g$ in going from pole to equator is about 0·35%.

**(v)** When the earth stops rotating about its own axis, there will be no change in the value of $g$ on the poles, but there will be increase in the value of $g$ by about 0·35% at the equator.

**(vi)** When the earth starts rotating 17 times faster than its present rate, the value of $g$ on the equator will become zero, but it will remain unchanged at the poles. In this situation, the duration of day will be 1 hour 24 minutes.

**(d)** Shape of the earth. Earth is not a perfect sphere. It is flattened at the poles and bulges out at the equator. The polar radius of Earth is smaller than its equatorial radius by 21 km. As $g = GM/R^2$, so $g \propto 1/R^2$.

It means the value of acceleration due to gravity increases as we go from equator to pole. In fact, due to shape of the earth, at sea level, the value of acceleration due to gravity at pole is greater than at equator by 1·80 $cms^{-2}$.

## Know the Terms

- **Areal velocity** may be defined as the area swept by the radius vector in unit time.
- **Cavendish method** determines the value of G.
- **Geodesic** is the shortest distance between two points on Earth.
- **Aphelion** is the nearest point from Sun.
- **Perihelion** is the farthest point from the Sun.
- **Central forces** is a force that points from the particle directly towards a fixed point in space, the centre, and whose magnitude only depends on the distance of the object to the centre.
- **Riemann Geometry** is geometry of curve space.

## Know the Formulae

- **Kepler's Law :**

$$T^2 = kr^3$$
$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

Here, T = Time period and r = Orbital radius.

- **Newton's Gravitational Law :**

**(a)** $$F = \frac{Gm_1m_2}{r^2}$$

**Units :** F = Newton, m = kg, $r$ = metre

**(b)** **For system of mass bodies** $$\vec{F} = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + ..... + \vec{F}_{0n}$$

- **Relation between g & G** $$g = \frac{GM}{R^2}$$

- **Variation of $g$ with height :**

**(a)** $$g' = \frac{gR^2}{(R+h)^2}$$

**(b)** $$g' = g\left(1-\frac{2h}{R}\right) \qquad \textit{if } h << R.$$

**(c)** % Change $= \frac{2h}{R} \times 100\%$

- **Variation of $g$ with depth :**

(a) $$g' = g\left(1-\frac{d}{R}\right)$$

**(b)** % Change in $g$ with depth $d$, $= \frac{dg}{R} \times 100 = \frac{d}{R} \times 100\%$

## Know the Links

- *www.learncbse.in*
- *www.vedantu.com*
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# MCQ/Fillups/True or False

**(1 mark each)**

## (A) Multiple Choice Questions

**Q. 1. The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity**

**(a) will be directed towards the centre but not the same everywhere.**

**(b) will have the same value everywhere but not directed towards the centre.**

**(c) will be same everywhere in magnitude directed towards the centre.**

**(d) cannot be zero at any point.**

**[NCERT Exemp. Q. 8.1, Page 57]**

**Ans. Correct option:** (d)

*Explanation:* The acceleration due to gravity cannot be zero on the surface of earth because the centre of gravity is inside the earth.

**Q. 2. As observed from earth, the sun appears to move in an approximate circular orbit. For the motion of another planet like mercury as observed from earth. this would**

**(a) be similarly true.**

**(b) not be true because the force between earth and mercury is not inverse square law.**

**(c) not be true because the major gravitational force on mercury is due to sun.**

**(d) not to be true because mercury is influenced by forces other than gravitational forces.**

**[NCERT Exemp. Q. 8.2, Page 57]**

**Ans. Correct option:** (c)

*Explanation:* Because the gravitational force on mercury is due to sun. So not similar condition for sun and Mercury.

**Q. 3. Different points in earth are at slightly different distances from the sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the c.m. (centre of mass) causing translation and a net torque at the c.m. causing rotation around an axis through the c.m. For the earth-sun system (approximating the earth as a uniform density sphere)**

**(a) the torque is zero.**

**(b) the torque causes the earth to spin.**

**(c) the rigid body result is not applicable since the earth is not even approximately a rigid body.**

**(d) the torque causes the earth to move around the sun. [NCERT Exemp. Q. 8.3, Page 57]**

**Ans. Correct option:** (a)

*Explanation:* Around centre of mass total torque is zero for earth-sun system.

**Q. 4. Both earth and moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon**

**(a) will be elliptical**

**(b) will not be strictly elliptical because the total gravitational force on it is not central.**

**(c) is not elliptical but will necessarily be a closed curve.**

**(d) deviates considerably from being elliptical due to influence of planets other than earth.**

**[NCERT Exemp. Q. 8.5, Page 58]**

**Ans. Correct option:** (b)

*Explanation:* If force is central then orbit is elliptical.

**Q. 5. In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They**

**(a) will not move around the sun since they have very small masses compared to the sun.**

**(b) will move in an irregular way because of their small masses and will drift away into outer space.**

**(c) will move around the sun in closed orbits but not obey Kepler's laws.**

**(d) will move in orbits like planets and obey Kepler's laws. [NCERT Exemp. Q. 8.6, Page 58]**

**Ans. Correct option:** (d)

*Explanation:* Chunks of matter also like a planet and thing follows Kepler's law.

**Q. 6. Choose the wrong option.**

**(a) Inertial mass is a measure of difficulty of accelerating a body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass.**

**(b) That the gravitational mass and inertial mass are equal is an experimental result.**

**(c) That the acceleration due to gravity on earth is the same for all bodies is due to the equality of gravitational mass and inertial mass.**

**(d) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot.**

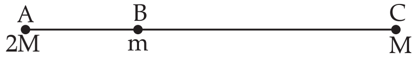
**[NCERT Exemp. Q. 8.7, Page 59]**

**Ans. Correct option:** (d)

*Explanation:* Inertial mass can not depend on gravitational force.

**Q. 7. Particles of masses 2M, *m* and M are respectively at points A, B and C with AB=1/2 (BC). *m* is much-much smaller than M and at time t = 0. They are all at rest (in figure).**

**At subsequent times before any collision takes place:**




**(a) *m* will remain at rest.**

**(b) *m* will move towards M.**

**(c) *m* will moves towards 2 M.**

**(d) *m* will have oscillatory motion.**

**[NCERT Exemp. Q. 8.8, Page 59]**

**Ans. Correct option:** (c)

*Explanation:*


Gravitational force, $F = \frac{GMm}{r^2}$

Let AB=r

∴ Force on B due to A,

$$\therefore F_{BA} = \frac{G(2Mm)}{r^2} = 2F$$

Now, Force on B due to C, -

$$\therefore F_{BC} = \frac{GMm}{(BC)^2} = \frac{GMm}{4r^2} \quad [\because BC = 2AB]$$

$$F_{BC} = \frac{F}{4} \text{ or } F_{BA} > F_{BC}$$

Hence, $m$ will move towards BA *i.e.* 2M.

**Q. 8. Which of the following options are correct?**

**(a) Acceleration due to gravity decreases with increasing altitude.**

**(b) Acceleration due to gravity increases with increasing depth (assume the earth to be a sphere of uniform density)**

**(c) Acceleration due to gravity increases with increasing latitude.**

**(d) Acceleration due to gravity is independent of the mass of the earth. [NCERT Exemp. Q. 8.9, Page 59]**

**Ans. Correct option:** (a) and (c)

*Explanation:*

**(a)** $g' = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$ here $g' < g$

$h \Rightarrow$ altitude (Increasing)

**(c)** If $h \rightarrow$ latitude (decreasing)

then $g' > g$.

**Q. 9. If the law of gravitation, instead of being inverse-square law, becomes an inverse-cube law**

**(a) planets will not have elliptic orbits.**

**(b) circular orbits of planets is not possible.**

**(c) projectile motion of a stone thrown by hand on the surface of the earth will be approximately parabolic.**

**(d) there will be no gravitational force inside a spherical shell of uniform density.**

**[NCERT Exemp. Q. 8.10, Page 59]**

**Ans. Correct option:** (a) and (c)

*Explanation:*

From question –

**(a)** $F = \dfrac{GMm}{a^3} = \dfrac{mv^2}{a}$

or $v \propto \dfrac{1}{a} \left[v = \sqrt{\dfrac{GM}{a}}\right]$

Time period, $T = \dfrac{2\pi a}{v}$

$T = \dfrac{2\pi a^2}{\sqrt{GM}}$

or $T^2 \propto a^4$

Hence, orbit will not be elliptical.

**(c)** Force, $F = \left(\dfrac{GM}{a^3}\right)m = g'm$

$g' = \dfrac{GM}{a^3}$ $[g' = \text{acceleration due to gravity}]$

As g' is constant,

∴ path of projectile will be approximately parabolic

**Q. 10. If the mass of the sun were ten times smaller and gravitational constant G were ten times larger in magnitudes, then**

**(a) walking on the ground would become more difficult.**

**(b) the acceleration due to gravity on earth will not change.**

**(c) raindrops will fall much faster.**

**(d) airplanes will have to travel much faster.**

**[NCERT Exemp. Q. 8.11, Page 60]**

**Ans. Correct option:** (a), (c) and (d)

*Explanation:* According to the problem G′ = 10 G, then gravitational acceleration due to gravity

$$g' = \frac{G'M}{R^2} = \frac{10\,GM}{R^2} = 10g$$

Now, Weight of person = $mg' = m \times 10g = 10\,mg$

Force on the man due to the sun

$$F = \frac{G'M_s m}{r^2} = \frac{G'M_s m}{10r^2} \qquad \left(\because 1M_s = \frac{m_s}{10}\right)$$

as $r >> R$ so F will be very small so, the effect at the sun will be neglected. Due to this region gravity pull on the person will increase. Due to it, walking on ground would become more difficult.

Critical velocity, $v_c$ is proportional to $g_c$ *i.e.*, $v_c g$

as $g' > g \geq v_c' > v_c$

Hence, rain drops will fall much faster.

To overcome the increased gravitational force of the earth, the aeroplanes will have to travel much faster.

**Q. 11. If the sun and the planets carried huge amounts of opposite charges.**

**(a) all three of Kepler's laws would still be valid.**

**(b) only the third law will be valid.**

**(c) the second law will not change**

**(d) the first law will still be valid**

**[NCERT Exemp. Q. 8.12, Page 60]**

**Ans. Correct option:** (a), (c) and (d)

*Explanation:* From electrostatic force of attraction will produce due to opposite charges. If the sun and the planets carries huge amount of opposite charges. Then electrostatic force of attraction will be large, gravitational force is also attraction in nature haves both forces will added and both are radial in nature.

Both the forces obey inverse source law and an central forces. As both the forces are at same nature, hence all the their kepler's laws will be valid.

**Q. 12. There have been suggestions that the value of the**

**gravitational constant G becomes smaller when considered over very large time period (in billions of years) in the future. If that happens, for our earth,**

**(a) nothing will change**

**(b) we will become hotter after billions of years.**

**(c) we will be going around but not strictly in closed orbits.**

**(d) after sufficiently long time we will leave the solar system [NCERT Exemp. Q. 8.13, Page 60]**

**Ans. Correct option:** (c) and (d)

*Explanation:* Gravitational force between earth and sun.

$$Fg = G\left(\frac{m_s \times m_e}{r^2}\right)$$

This force provides the necessary centripetal force for the circular orbit of the earth around the sun. As G decreases with time. The gravitational force $F_g$ will become weaken with time. As $F_g$ is changing with time due to it, the earth will be going around the sun not strictly in closed orbit and radius also increases, since the attraction force is getting weaker Hence, after long time the earth will leave the solar-system.

**Q. 13. Supposing Newton's law of gravitation for gravitation forces $F_1$ and $F_2$ between two masses $m_1$ and $m_2$ at positions $r_1$ and $r_2$ read**

$$F_1 = -F_2 = -\frac{r_{12}}{r^3_{12}} G{M_0}^2\left(\frac{m_1 m_2}{{M_0}^2}\right)^n$$ **where $M_0$ is a constant of dimension of mass, $r_{12}=r_1-r_2$ and n is a number. In such a case,**

**(a) the acceleration due to gravity on earth will be different for different objects.**

**(b) none of the three laws of Kepler will be valid.**

**(c) only the third law will become invalid.**

**(d) for n negative, an object lighter than water will sink in water. [NCERT Exemp. Q. 8.14, Page 60]**

**Ans. Correct option:** (a), (c) and (d)

*Explanation:* Given:

$$F_1 = -F_2$$

$$= -\frac{r_{12}}{r^3_{12}} GM_0^2\left(\frac{m_1 m_2}{M_0^2}\right)^n$$

$$r_{12} = r_1 - r_2$$

$$g = \frac{F}{mass}$$

$$= \frac{GM_0^2(m_1 m_2)^n}{r_{12}^2(M_0)^{2n}} \times \frac{1}{mass}$$

As g depends on position vector, it is different for different objects.

As g is variable, so proportionality constant will not be constant in Kepler's III law.

Now, the nature of force is central force Hence, Kepler's I and II law will be valid

Now, $n = (-)ve$

$$g = \frac{GM_0^2(m_1 m_2)^n}{r_{12}^2(M_0)^{-2n}} \times \frac{1}{mass}$$

$$g = \frac{GM_0^2}{r_{12}^2}\left(\frac{M_0^2}{m_1 m_2}\right)^n \times \frac{1}{mass}$$

$$M_0 > m_1 \text{ or } m_2$$

$\therefore g > 0$, hence, object lighter than water will sink in water.

**Q. 14. The centre of mass of an extended body on the surface of the earth and its centre of gravity**

**(a) are always at the same point for any size of the body.**

**(b) are always at the same point only for spherical bodies.**

**(c) can never be at the same point.**

**(d) is close to each other for objects, say of sizes less than 100 m.**

**(e) both can change if the object is taken deep inside the earth. [NCERT Exemp. Q. 8.16, Page 61]**

**Ans. Correct option:** (d)

*Explanation:* For small objects, say of sizes less than 100 m placed in uniform gravitational field then centre of mass is very close with the centre of gravity at the body. But when the size of object increases, its weight changes and its C.M. an C.G. become far from each other. Like in the case of spherical ball, the CM and the CG are the same, but in case of mount Everest, its CM less a bit above its CG.

## Very Short Answer Type Questions (1 mark each)

**Q. 1. Molecules in air in the atmosphere are attracted by gravitational force of the earth. Explain why all of them do not fall into the earth just like an apple falling from a tree.**

**[NCERT Exemp. Q. 8.17, Page 61]**

**Ans.** Air molecules in the atmosphere experience the vertically downward force due to gravity just like an apple falling from a tree. Due to thermal motion, air molecules move randomly, their velocity is not in the vertical direction. The downward force gravity causes the density of air in the atmosphere close to earth higher than the density as we go up. But in apple's case, only vertical motion dominates because of heavier molecules than air molecules.

**Q. 2. Give one example each central force and non-central force. [NCERT Exemp. Q. 8.18, Page 61]**

**Ans.** Examples of

**Central force :** gravitational force of a point mass, electrostatic force due to a point charge. **(Any one)**

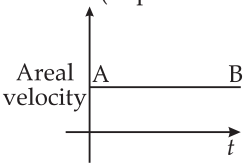
Examples of

**Non-central force:** spin-dependent nuclear forces, magnitude force between two current carrying loops. **(Any one)**

**Q. 3. Draw areal velocity versus time graph for mars.**

**[NCERT Exemp. Q. 8.19, Page 61]**

**Ans.** The graph between areal velocity and time is a straight line parallel to time axis because areal velocity of a planet revolving around the sun is constant with time. (Kepler's second law).




**Q. 4. What is the direction of areal velocity of the earth around the sun?**

**[NCERT Exemp. Q. 8.20, Page 61]**

**Ans.** Areal velocity of earth around the sun is –

$$\frac{dA}{dt} = \frac{L}{2m}$$

[L=angular momentum, $m$=mass of earth]

But angular momentum, $L = \vec{r} \times \vec{p}$

$$= \vec{r} \times m\vec{v}$$

$\therefore$ Areal velocity, $\left(\frac{dA}{dt}\right) = \frac{1}{2m}\left(\vec{r} \times m\vec{v}\right)$

$$= \frac{1}{2}\left(\vec{r} \times \vec{v}\right)$$

Therefore, the direction of $\left(\vec{r} \times \vec{v}\right)$ areal velocity is in direction of *i.e.* perpendicular to the plane of $\vec{r}$ and $\vec{v}$ (as by Maxwell's right hand grip rule).

**Q. 5. How is the gravitational force between two point masses affected when they are dipped in water keeping the separation between them the same?**

**[NCERT Exemp. Q. 8.21, Page 61]**

**Ans.** By Universal law of gravitation,

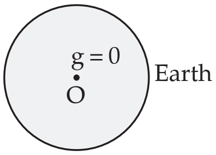
$$F = \frac{Gm_1m_2}{r^2}$$

The force acting between two point masses $m_1$ and $m_2$, is independent of the nature of medium between them. Therefore, Gravitational force acting between two masses will remain unaffected when they are dipped in water.

**Q. 6. Is it possible for a body to have inertia but no weight?**

**[NCERT Exemp. Q. 8.22, Page 61]**

**Ans.** Yes, because weight (mg) of a body can be zero as it depend upon acceleration due to gravity but every body will always have inertial (*i.e.* mass)




For example: At O, $g = 0$, at the centre of the earth.

**Q. 7. We can shield a charge from electric fields by putting it inside a hollow conductor. Can we shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?**

**[NCERT Exemp. Q. 8.23, Page 61]**

**Ans.** No, because gravitational force between two point mass bodies is independent of the intervening medium between them.

**Q. 8. An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?**

**[NCERT Exemp. Q. 8.24, Page 61]**

**Ans.** The value of acceleration due to gravity, g can be considered as constant inside small spaceship orbiting around the earth, so astronaut feels weightlessness.

If space station orbiting around has a large size, the variation in $g$ matters and astronaut will experience gravitational force, hence he can detect gravity. On Moon, due larger size, gravity can be detected.

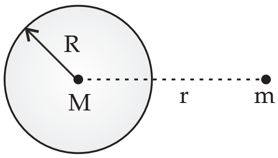
**Q. 9. The gravitational force between a hollow spherical shell (of radius R and uniform density) and a point mass is *F*. Show the nature of F *vs r* graph where *r* is the distance of the point from the centre of the hollow spherical shell of uniform density.**

**[NCERT Exemp. Q. 8.25, Page 61]**

**Ans.** Gravitational force is *F*.

Let us consider the diagram, shell's density is constant and it is $\rho$

As density of shell is uniform and it can be treated as point mass.




Mass, M (Shell)= $\rho \times \frac{4}{3}\pi R^3$

Gravitational force between M & m,

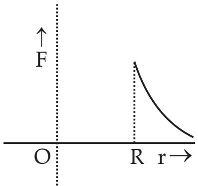
$$F = \frac{GMm}{r^2}$$

But force inside the shell is zero.

$F = 0$ for $r < R$

$$= \frac{GM}{r^2} \text{ for } r > R$$

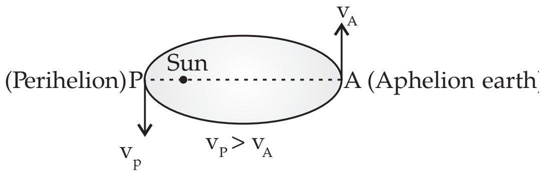
Variation of F versus r.



**Q. 10. Out of aphelion and perihelion, where is the speed of the earth more and why?**

**[NCERT Exemp. Q. 8.26, Page 61]**

**Ans.** Areal velocity of earth around the sun is constant from Kepler's II law, so the speed of the earth is more at the perihelion than at the aphelion. The earth has to cover greater linear distance to keep the areal velocity constant, at aphelion.




**Q. 11. What is the angle between the equatorial plane and the orbital plane of**

**(a) Polar satellite?**

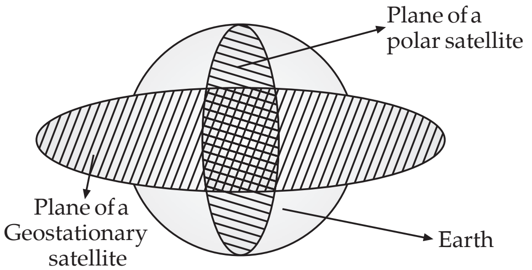
**(b) Geostationary satellite?**

**[NCERT Exemp. Q. 8.27, Page 61]**

**Ans.** According to the diagram where plane of geostationary and polar satellite are shown :

**(a)** Angle between equatorial plane and orbital plane is 90° .

**(b)** Angle between equatorial and orbital plane is 0°.




**Q. 12. Answer the following :**

**(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?**

**(b) An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size can he hope to detect gravity ?**

**(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the sun's pull is greater than moon's pull. However, the tidal effect of the moon's pull is greater than the tidal effect of the sun. Why ?**

**[NCERT Ex. Q. 8.1, Page 200]**

**Ans. (a)** No. Electrical forces depend upon the nature of the intervening medium while the gravitational forces don't depend upon the nature of the intervening medium. So, such shielding acts are not possible in case of gravitation, i.e., gravity screens are not possible.

**(b)** Yes, Astronaut can hope to detect gravity if the size of the spaceship is extremely large, then the magnitude of the gravity will become appreciable and hence, the gravitational effect of the spaceship may become measurable.

**(c)** Earth-moon distance is very small as compared to earth-sun distance. Tidal effect is inversely proportional to the cube of the distance it means it is not governed by inverse square law like the gravitational force (which obeys inverse square law). Hence, tidal effect of moon is larger than that due to the sun.

**Q. 13. Choose the correct alternative**

**(a) Acceleration of gravity increases/decreases with increasing altitude.**

**(b) Acceleration due to gravity increases/decreases with increasing depth.**
**(assume the Earth to be a sphere of uniform density).**

**(c) Acceleration due to gravity is independent of the mass of the Earth/mass of the body.**

**(d) The formula $-GMm\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$ is more/less accurate than the formula $mg(r_2-r_1)$ for the difference of potential energy between two points $r_2$ and $r_1$ distance away from the centre of the Earth.** **[NCERT Ex. Q. 8.2, Page 201]**

**Ans. (a)** decreases

(b) decreases

(c) independent of mass of the body

(d) more

## Short Answer Type Questions

**(2 or 3 marks each)**

**Q. 1. Mean solar day is the time interval between two successive noon when sun passes through zenith point (meridian).**

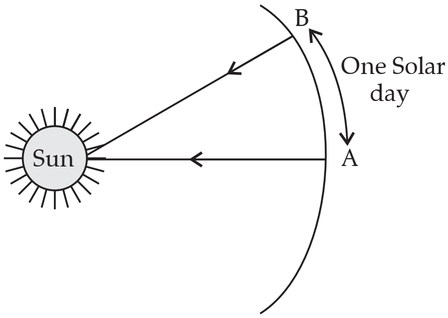
**Sidereal day is the time interval between two successive transit of a distant star through the zenith point (meridian).**

**By drawing appropriate diagram showing earth's spin and orbital motion, show that mean solar day is 4 minutes longer than the sidereal day. In other words, distant stars would rise 4 minutes early every successive day.**

**(Hint: you may assume circular orbit for the earth).**

**[NCERT Exemp. Q. 8.28, Page 62]**

**Ans.** Let us consider the following diagram, the earth moves from the point A to B in one solar day.




Every day the earth advances in the orbit by approximately 1°. Then, it will have to rotate by 361° (which we define as 1 day) to have sun at zenith point again. Since 361° corresponds to 24 hours:

$\therefore$ extra 1° corresponds to approximately 4 minutes [3 min 59 seconds].

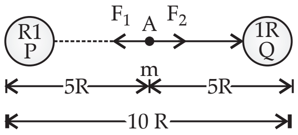
*i.e.* $\frac{24}{361} \times 1 = 0.066 \text{ h} = 3.99 \text{ min} = 4 \text{ min}$

Thus, distant stars would rise 4 minutes early every successive day.

**Q. 2. Two identical heavy spheres are separated by a distance 10 times their radius. Will an object placed at the mid point of the line joining their centres be in stable equilibrium or unstable equilibrium? Give reason for your answer.**

**[NCERT Exemp. Q. 8.29, Page 62]**

**Ans.** Given: $m_1 = m_2 = M$, $r = 10R$




Let mass m is placed at mid-point A (line joining the centres of P & Q sphere)

Now, $|F_2| = |F_1| = \frac{GMm}{(5R)^2}$

$|F_1| = |F_2| = \frac{GMm}{25R^2}$

$F_1$ & $F_2$ are equal and opposite forces are acting on m at A.

Net force $F_1 = -F_2$ or $F_1 + F_2 = 0$

So, mass is in equilibrium.

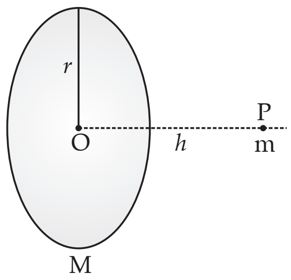
If $m$ is slightly displaced from A to P then AP $= (5R - x)$

AQ $= (5R + x)$

$\therefore F_1 = \frac{GMm}{(5R-x)^2}$ & $F_2 = \frac{GMm}{(5R+x)^2}$ or $F_1 > F_2$

That means resultant force acting on A is towards P. Hence, equilibrium is unstable equilibrium.

**Q. 3. A mass $m$ is placed at P a distance $h$ along the normal through the centre O of a thin circular ring of mass M and radius r in figure.**



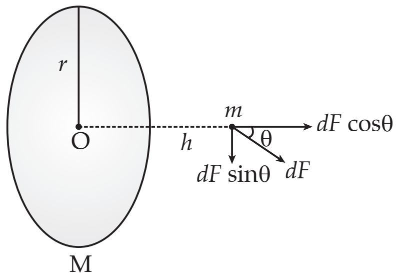

**If the mass is removed further away such that OP becomes $2h$, by what factor the force of gravitation will decrease, if $h = r$ ?**

**[NCERT Exemp. Q. 8.33, Page 63]**

**Ans.** Gravitational force F at P

$$F_h = \frac{GMm\cos\theta}{AP^2} = \frac{GMmh}{(r^2+h^2)^{3/2}}$$

$$\because \cos\theta = \frac{h}{(r^2+h^2)^{1/2}}$$




$$\frac{F_r}{F_{2r}} = \frac{\frac{GMm.r}{(r^2+r^2)^{3/2}}}{\frac{GMm2r}{\left[r^2+(2r)^2\right]^{3/2}}}$$

or $$= \frac{(r^2+4r^2)^{3/2}}{2(2r^2)^{3/2}}$$

$$\frac{F_r}{F_{2r}} = \frac{(5r^2)^{3/2}}{2\sqrt{2}\,r^3}$$

$$= \frac{5\sqrt{5}\,r^3}{4\sqrt{2}\,r^3}$$

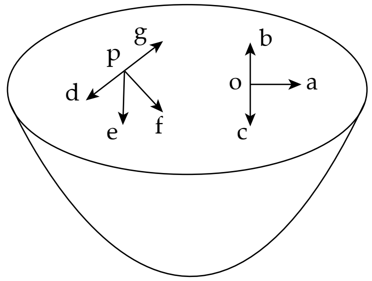
$$\frac{F_r}{F_{2r}} = \frac{5}{4}\sqrt{\frac{5}{2}}$$

or $$\frac{F_{2r}}{F_r} = \frac{4}{5}\sqrt{\frac{2}{5}}$$

or $$F_{2r} = \frac{4}{5}\sqrt{\frac{2}{5}}\,F_r$$

**Q. 4. In the following two exercises, choose the correct answer from the given ones. The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig).**

**(i) a, (ii) b, (iii) c, (iv) o**




**[NCERT Ex. Q. 8.10, Page 201]**

**Ans.** Inside a hollow spherical shell, potential will be same at all points. So, gravitational intensity, being negative of potential gradient will be zero. Due to zero gravitational intensity, the gravitational forces acting on any particle at any point inside a spherical shell will be symmetrically placed. So, if we remove the upper hemispherical shell, the net gravitational force acting on a particle at P will be downwards. Since gravitational intensity is gravitational force per unit mass, so the direction of gravitational intensity will be along e. So, option (iii) is correct.

**Q. 5. For the above problem, the direction of the gravitational intensity at any arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.**

**[NCERT Ex. Q. 8.11, Page 201]**

**Ans.** Using the above explanation, the direction of gravitational field intensity at P will be along e. So option (ii) is correct.

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. Six point masses of mass *m* each are at the vertices of a regular hexagon of side L. Calculate the force on any of the masses.**

**[NCERT Exemp. Q. 8.35, Page 63]**

**Ans.** Let us consider the following diagram in which size point masses are placed at six sides,

A, B, C, D, E, F.

AC = AG + GC = 2AG

$= 2l\cos 30°$

$=2l\sqrt{3/2}=\sqrt{3}l=AE$

AD = AH + HJ + JD

$= l\sin 30° + l + l\sin 30°$

$= 2l$

Force on A due to B, $f_1=\dfrac{GMm}{l^2}$ along A to B

$=\dfrac{Gm^2}{l^2}$

Force on A due to C, $F_2=\dfrac{Gm.m}{(\sqrt{3}l)^2}$

$=\dfrac{Gm^2}{3l^2}$ along A to C

$[\because AC=\sqrt{3}1]$

Force on A due to D, $F_3=\dfrac{Gm.m}{(2l)^2}$

$=\dfrac{Gm^2}{4l^2}$ along A to D

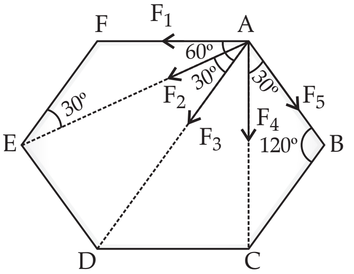
$[\because AD=2l]$

Force on A due to $E$, $F_4=\dfrac{Gm.m}{(\sqrt{3}l)^2}$

$=\dfrac{Gm^2}{3l^2}$ along A to E

Force on A due to F, $F_5=\dfrac{Gm.m}{(l)^2}$

$=\dfrac{Gm^2}{l^2}$ along A to F




Resultant force due to $F_1$ and $F_5$,

$F_1=\sqrt{F_1^2+F_5^2+2F_1F_5\cos 120°}$

$=\dfrac{Gm^2}{l^2}$ along A to D [Angle between $F_1$ and $F_5=120°$]

Resultant force due to $F_2$ and $F_4$,

$F_1=\sqrt{F_2^2+F_4^2+2F_2F_4\cos 60°}$

$=\dfrac{\sqrt{3}Gm^2}{3l^2}=\dfrac{Gm^2}{\sqrt{3}l^2}$ along A to D

$\therefore$ Net force along A to D = $F_1 + F_2 + F_3$

$=\dfrac{Gm^2}{l^2}+\dfrac{Gm^2}{\sqrt{3}l^2}+\dfrac{Gm^2}{4l^2}$

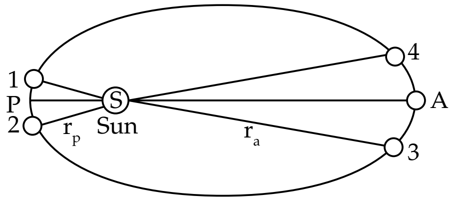
$=\dfrac{Gm^2}{l^2}\left(l+\dfrac{1}{\sqrt{3}}+\dfrac{1}{4}\right)$

**Q. 2. Earth's orbit is an ellipse with eccentricity 0.0167. Thus, earth's distance from the sun and speed as it moves around the sun varies from day to day. This means that the length of the solar day is not constant through the year. Assume that earth's spin axis is normal to its orbital plane and find out the length of the shortest and the longest day. A day should be taken from noon to noon. Does this explain variation of length of the day during the year?** **[NCERT Exemp. Q. 8.37, Page 64]**

**Ans.** Let us consider the following diagram.

Let the mass of earth be $m$, $v_p$, $v_a$ be velocity of earth at perigee and apogee respectively.

$\omega_p$ and $\omega_a$ are angular velocities.




Angular momentum and areal velocity are constant as the earth orbits the sun.

At perigee $r_p^2\omega_p = r_p^2\omega_a$ at apogee.

If 'a' is the semi-major axis of earth's orbit, then $r_p = a(1 - e)$ and $r_a = a(1 + e)$.

$$\therefore \frac{\omega_p}{\omega_a} = \left(\frac{1+e}{1-e}\right)^2, \qquad e = 0.0167$$

$$\therefore \frac{\omega_p}{\omega_a} = 1.0691$$

Let ω be angular speed which is geometric mean of $\omega_p$ and $\omega_a$ and corresponds to mean solar day.

$$\therefore \left(\frac{\omega_p}{\omega}\right) = \left(\frac{\omega}{w_a}\right) = 1.0691$$

$$\therefore \quad \frac{\omega_p}{\omega} = \frac{\omega}{\omega_a} = 1.034.$$

If ω corresponds to 1° per day (mean angular speed), then $\omega_p = 1.034°$ per day and $\omega_a = 0.967°$ per day. Since 361° = 24 hrs: mean solar day, we get 361.034° which corresponds to 24 hrs 8.14″ (8.1″ longer) and 360.967 corresponds to 23 hrs 59 min 52″ (7.9″ smaller).

This does not explain the actual variation of the length of the day during the year.

**Q. 3. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of earth ?**

**[NCERT Ex. Q. 8.3, Page 201]**

**Ans.** From Kepler's third law,

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Let 1 denotes planet and 2 denotes earth

$$R_1^3 = \frac{T_1^2}{T_2^2} \times R_2^3 \qquad \text{(i)}$$

As the planet is revolving twice as fast as earth.

$$T_1 = \frac{T_2}{2}$$

Using equation (i),

$$\text{or} \qquad R_1^3 = \frac{\left(\frac{T_2}{2}\right)^2}{T_2^2} \times (1\text{ A.U.})^3$$

$$\text{or} \qquad R_1 = \left(\frac{1}{4}\right)^{1/3}\text{ A.U.}$$

$$R_1 = \frac{1}{\sqrt[3]{4}}\text{ A.U.}$$

$$R_1 = 0.62996\text{ A.U.} = 0.63\text{ A.U.}$$

This result can be stated as the orbital radius of planet is 0.63 times the orbital radius of earth.

**Q. 4. $I_0$, one of the satellites of Jupiter has an orbital period of 1.769 days and the radius of the orbit is $4.22 \times 10^8$ m. Show the mass of Jupiter is about the thousandth that of sun.**

**[NCERT Ex. Q. 8.4, Page 201]**

**Ans.** Suppose,

$M_J$ = Mass of Jupiter = ?

$M_S$ = Mass of Sun $= 2 \times 10^{30}$ kg

T = Time period of $I_0$ satellite of Jupiter
= 1.769 days
= 1.769 × 24 × 3600 s
$= 15.2841 \times 10^4$ s

$r$ = Radius of its orbit around Jupiter $= 4.22 \times 10^8$ m.

$G = 6.67 \times 10^{-11}\text{ Nm}^2\text{ kg}^{-2}$

We have to prove

$$M_J \approx \frac{1}{1000} M_S$$

Using the formula,

$$\frac{GM}{r^3} = \omega^2$$

we get

$$\frac{GM_J}{r^3} = \left(\frac{2\pi}{T}\right)^2$$

$$M_J = \frac{4\pi^2 r^3}{T^2 G}$$

$$= \frac{4 \times 9.87 \times (4.22 \times 10^8)^3}{(1.52841 \times 10^4)^2 \times 6.67 \times 10^{-11}}$$

$$M_J = 1.9 \times 10^{27}\text{ kg} = 2 \times 10^{27}\text{ Kg}$$

$$\frac{M_J}{M_S} = \frac{2 \times 10^{27}}{2 \times 10^{30}} = \frac{1}{1000}$$

$$M_J = \frac{1}{1000} M_S$$

It means mass of Jupiter is about one thousandth of the mass of sun.

**Q. 5. Let us assume that our galaxy consists of $2.5 \times 10^{11}$ stars each of one solar mass. How long will a star at a distance of 50,000 light years from the galactic centre take to complete one revolution ? Take the diameter of milky way to be $10^5$ light years.**

**[NCERT Ex. Q. 8.5, Page 201]**

**Ans.** Let us assume our galaxy to be spherical and the star in question is revolving around the centre of mass of our galaxy.

Now radius of orbit of star = $r$

$$= 50{,}000 \text{ light years.}$$

$$r = 5 \times 10^4 \times 9.46 \times 10^{15} \text{ m}$$

$$= 4.73 \times 10^{20} \text{ m}$$

Mass of stars present in the galaxy

$$(\text{M}) = n.m$$

$$= 2.5 \times 10^{11} \times 2 \times 10^{30} \text{ kg}$$

where, $n$ = total number of stars present in the galaxy

$$\text{M} = 5 \times 10^{41} \text{ kg}$$

Now the gravitational force of attraction will provide the necessary central force to the star,

$$\frac{\text{GM}m}{r^2} = m\omega^2 r$$

$$\frac{\text{GM}}{r^3} = \omega^2$$

using, $\omega = \frac{2\pi}{\text{T}}$

$$\text{T} = \sqrt{\frac{4\pi^2 \text{R}^3}{\text{GM}}}$$

$$= \sqrt{\frac{4\pi^2 \times (4.73 \times 10^{20})^3}{6.67 \times 10^{11} \times 5 \times 10^{41}}} \text{ s}$$

$$= 11.19 \times 10^{15} \text{ s}$$

$$= 111.9 \times 10^{14} \text{ s}$$

$$= \frac{111.9 \times 10^{14}}{365 \times 24 \times 3600} \text{ years}$$

$$= 3.55 \times 10^8 \text{ years.}$$

**Q. 6. A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2 × $10^{30}$ kg, mass of the earth = 6 × $10^{24}$ kg. Neglect the effect of other planets etc. (orbital radius = 1.5 × $10^{11}$ m.)**

**[NCERT Ex. Q. 8.12, Page 201]**

**Ans.** Here,

$$\text{M}_s = 2 \times 10^{30} \text{ kg};$$

$$\text{M}_e = 6 \times 10^{24} \text{ kg};$$

$$r = 1.5 \times 10^{11} \text{ m}$$

Let $x$ be the distance of a point from the earth where gravitational forces on the rocket due to sun and earth become equal and opposite. Then distance of rocket from the sun = $(r - x)$

If $m$ is the mass of rocket then

$$\frac{\text{GM}_s m}{(r-x)^2} = \frac{\text{GM}_e m}{x^2}$$

or

$$\frac{(r-x)^2}{x^2} = \frac{M_s}{M_e}$$

$$\frac{r-x}{x} = \sqrt{\frac{M_s}{M_e}}$$

$$= \sqrt{\frac{2 \times 10^{30}}{6 \times 10^{34}}}$$

$$= \frac{10^2}{\sqrt{3}}$$

$$\frac{r}{x} - 1 = \frac{10^3}{\sqrt{3}}$$

$$\frac{r}{x} = 1 + \frac{10^3}{\sqrt{3}}$$

$$= \frac{\sqrt{3} + 10^3}{\sqrt{3}}$$

so,

$$x = \frac{r\sqrt{3}}{\sqrt{3} + 10^3}$$

$$= \frac{1.5 \times 10^{11} \times \sqrt{3}}{\sqrt{3} + 10^3} \text{ m}$$

$$= \frac{1.5 \times 1.732 \times 10^{11}}{(1.732 + 100)} \text{ m}$$

$$= 2.59 \times 10^8 \text{ m}$$

**Q. 7. A body weight 63 N on the surface of earth. What is the gravitational force on it due to the earth at a height equal to half the radius of earth ?**

**[NCERT Ex. Q. 8.15, Page 201]**

**Ans.** Let $h$ = height above earth's surface = $\frac{\text{R}}{2}$

where, R = radius of earth

Now, the acceleration due to gravity at a height '$h$' above earth's surface ($g$) is given by

$$g_h = g\left(1 + \frac{h}{\text{R}}\right)^{-2}$$

Given,

$$h = \frac{\text{R}}{2}$$

$\therefore$

$$g_h = g\left(1 + \frac{\text{R}/2}{\text{R}}\right)^{-2}$$

or

$$\frac{g_h}{g} = \frac{1}{\left(1 + \frac{1}{2}\right)^2} = \frac{1}{\left(\frac{3}{2}\right)^2}$$

$$= \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

or

$$g_h = \frac{4}{9} \times g = \frac{4}{9} g \quad \text{...(i)}$$

Let $m$ = mass of the body

If W and $\text{W}_h$ be its weight at earth's surface and at a height $h$ above earth's surface respectively, then

$$\text{W} = mg = 63 \text{ N} \quad \text{(given)}$$

and

$$\text{W}_h = mg_h = m \times \frac{4}{9} g = \frac{4}{9} mg$$

or

$$\text{W}_s = \frac{4}{9} \times 63 = 28 \text{ N}$$

So,

$$\text{W}_h = 28 \text{ N}$$

**Q. 8. Assuming the earth to be a sphere of uniform mass density, how much would a body weight half way down to the centre of earth, if it weighed 250 N on the surface ?**

**[NCERT Ex. Q. 8.16, Page 201]**

**Ans.** Suppose $g$, $g_d$ be the acceleration due to gravity on earth's surface and at a depth '$d$', from surface respectively.

Also, suppose W and $W_d$ be the weight of a body on earth's surface and at depth '$d$', respectively,

$\therefore$ $W = mg = 250$ N ...(i)

and $W_d = mg_d$ ...(ii)

Now, we know that

$$g_d = g\left(1 - \frac{d}{R}\right) \quad ...(iii)$$

Here, $d = \frac{R}{2}$, R = radius of earth...(iv)

$\therefore$ From equations (iii) and (iv), we get

$$g_d = g\left(1 - \frac{R/2}{R}\right)$$

$$= g\left(1 - \frac{1}{2}\right) = g \times \left(\frac{1}{2}\right)$$

$$= \frac{g}{2} \quad ...(v)$$

$$\therefore \quad W_d = mg_d$$

$$= m\frac{g}{2} \quad \text{[by using equation (v)]}$$

$$= \frac{1}{2}mg = \frac{1}{2}W$$

$$= \frac{1}{2} \times 250 = 125 \text{ N.}$$

$\therefore$ Weight of the body half way down to the centre of earth = 125 N.

# TOPIC-2
## Gravitational Potential Energy and Satellites

## Extra Info

- **Gravitational potential energy (U) :** The amount of work done in bringing a body from infinity to that point.

$$U = -\frac{GMm}{r}$$

- **Escape Velocity :** The minimum velocity with which a body must be projected up in the space, so as to enable it to just overcome the gravitational pull.

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}. \qquad \text{as } \therefore \left[g = \frac{GM}{R^2}\right]$$

- **Satellite :** A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.
  - **(i) Natural satellites :** All those satellites were made by nature. *e.g.*, Jupiter–16 moons. Saturn–18 moons.
  - **(ii) Artificial satellites :** All man-made satellites *e.g.*, Aryabhatta, etc.
  - **(iii) Geostationary satellites :** A satellite which appears to be at a fixed position at a definite height to an observer on earth. It is also known as geosynchronous satellite.
- **Essential conditions for Geostationary satellites:**
  - **(a)** It should be at 36000 km above equator of earth.
  - **(b)** Its revolution period should be 24 hours about its axis.
  - **(c)** It should revolve in an orbit concentric and coplanar with equatorial plane.
  - **(d)** Its orbital speed is nearly 3.11 km/s and should be same sense of rotation as earth.
- **Polar satellite :** These satellites revolve in polar orbits around earth. A Polar orbit is that orbit whose angle of inclination with equatorial plane of earth is 90°.
- Gravitational forces are **central forces** as they act along the line joining the centers of the two bodies. The gravitational forces are **conservative forces.**

## Know the Terms

- **Gravitational field** is the space around a material body in which its gravitational pull can be experienced.
- **Gravitational field intensity** of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.

- **Gravitational potential** at a point in a gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.
- **Mass** of a body is the quantity of matter possessed by body.
- **Inertial mass of a body** is equal to the magnitude of external force required to produce unit acceleration in the body.
- **Gravitational mass** of a body is defined as the magnitude of gravitational pull experienced by the body in a gravitational field of unit intensity.
- **Centre of Gravity (C.G.)** of a body placed in the gravitational field is that point where the net gravitational force of the field acts.

## Know the Formulae

- **Gravitational Intensity :** $I = \frac{F}{m_0} = \frac{GMm}{r^2 m_0} = g$

  **Unit**

  N/kg in SI, dyne/g in CGS Dimensions $[M^0LT^{-2}]$
- **Gravitational Potential :** $V = \frac{W}{m_0}$

  $$V = -\frac{GM}{r}$$

  Unit — J/kg in S.I. erg/g in C.G.S. system where W = Work done, $m_0$ = mass of body

  **Dimensions** – $[M^0L^2T^{-2}]$.
- **Gravitational Potential Energy :** $U = \frac{-GMm}{r}$
- **Satellite :**

  **(a) Orbital speed :** $v = R\sqrt{\frac{g}{R+h}} = \sqrt{\frac{GM}{R+h}}$

  **(b) Time period of revolution :** $T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$

  **(c) Height of satellite :** $h = \left(\frac{T^2R^2g}{4\pi^2}\right) - R$
- **Escape speed**

  $$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \qquad \therefore \left[g = \frac{GM}{R^2}\right]$$

## Know the Links

- *www.vedantu.com*
- *www.studyrankers.com*
- *www.learncbse.in*

## MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. Satellites orbiting the earth have finite life and sometimes debris of satellites fall to the earth. This is because,**

**(a) the solar cells and batteries in satellites run out.**

**(b) the laws of gravitational predict a trajectory spiralling inwards.**

**(c) of viscous forces causing the speed of satellite and hence height to gradually decrease.**

**(d) of collisions with other satellites.**

**[NCERT Exemp. Q. 8.4, Page 58]**

**Ans. Correct option:** (c)

*Explanation:* Total potential energy $= -\frac{GMm}{2R}$

Due to the atmospheric friction (viscous force) acting on satellite, energy decreases continuously, radius of the orbit or hight decreases gradually and the satellite spirals down with decreasing speed fill it burns in the denser layers of the atmosphere.

**Q. 2. Which of the following are true?**

**(a) A polar satellite goes around the earth's pole in north-south direction.**

**(b) A geostationary satellite goes around the earth in east-west direction.**

**(c) A geostationary satellite goes around the earth in west-east direction.**

**(d) A polar satellite goes around the earth in east-west direction.**

**[NCERT Exemp. Q. 8.15, Page 60]**

**Ans. Correct option:** (a) and (c)

*Explanation:* A geostationary satellite revolves around the earth will the same angular velocity and in the same sense as done by the earth about its own axis. i.e., west-east direction.

A polar satellite revolves around the earth's pole in north-south direction. It is independent of earth's rotation.

## Very Short Answer Type Questions (1 mark each)

**Q. 1. Choose the correct alternative :**

**(a) If the zero of potential energy is at infinity the total energy on orbiting satellite is negative of its kinetic/potential energy.**

**(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more / less than the energy required to project a stationary object at the same height (as can the satellite) out of earth's influence.**

**[NCERT Ex. Q. 8.6, Page 201]**

**Ans. (a)** Kinetic energy,

**(b)** Less.

**Q. 2. Does the escape speed of body from the earth depend upon :**

**(a) the mass of the body ?**

**(b) the location from where it is projected ?**

**(c) the direction of projection ?**

**(d) the height of location from where the body is launched ?**

**[NCERT Ex. Q. 8.7, Page 201]**

**Ans. (a)** No, from the formula $v_e = \sqrt{\frac{2GM}{R}}$, it is clear that escape velocity does not depend on the mass of the body.

**(b)** The escape velocity depends upon the value of gravitational potential at the point from where the body is projected. The gravitational potential energy of body $E = -\frac{GMm}{R}$ is slightly different at different points (the earth is not a perfect sphere and hence R is different at different points). Because of this escape velocity depend slightly on the latitude of the place from where the body is projected.

**(c)** The escape velocity of a body does not depend upon its direction of projection.

**(d)** Since the gravitational potential energy at a point at the height $h$ from the earth surface is $\frac{GM_s}{(R+h)}$, the escape velocity will be different for different values of $h$.

**Q. 3. A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant :**

**(a) Linear speed,**

**(b) Angular speed,**

**(c) Angular momentum,**

**(d) Kinetic energy,**

**(e) Potential energy,**

**(f) Total energy throughout it orbit ?**

**Neglect any mass loss of the comet when it comes very close to the sun ?**

**[NCERT Ex. Q. 8.8, Page 201]**

**Ans. (a)** The linear speed ($v = \omega R$) changes because the distance, *i.e.*, (R) of the comet from the sun changes due to it elliptical orbit around the sun.

**(b)** The angular speed of the comet also change because it covers different angle in equal interval of time.

**(c)** The angular momentum of the comet is same throughout due to the conservation of angular momentum in the absence of any torque.

**(d)** Kinetic energy changes because linear speed is different at different points.

**(e)** The potential energy at different points is different because the comet is not at the same distance from the sun (the orbit is not circular).

**(f)** The total energy of comet remain the same throughout the motion.

**Q. 4. Which of the following symptoms likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientation problem.**

**[NCERT Ex. Q. 8.9, Page 201]**

**Ans.** The astronaut in space will suffer from (b) swollen face, (c) headache and (d) orientation problem.

**(b)** We know that legs carry the weight of the body in the normal position due to gravity pull. The astronaut in space is in weightlessness state. Hence, swollen feet may not effect his working.

Due to weightlessness the astronaut may develop swollen face. As eyes, ears, nose are all embedded in face.

**(c)** Headache is due to mental strain. It will persist whether a person is an astronaut in space or earth.

**(d)** Space also has orientation, we also have the frames of reference.

## Short Answer Type Questions **(2 or 3 marks each)**

**Q. 1. Show the nature of the following graph for a satellite orbiting the earth.**

**(a) KE *vs* orbital radius R**

**(b) PE *vs* orbital radius R**

**(c) TE *vs* orbital radius R.**

**[NCERT Exemp. Q. 8.30, Page 62]**

**Ans.** Mass of earth = $M_e$

Mass of satellite = $m$

Radius of orbit of satellite = R

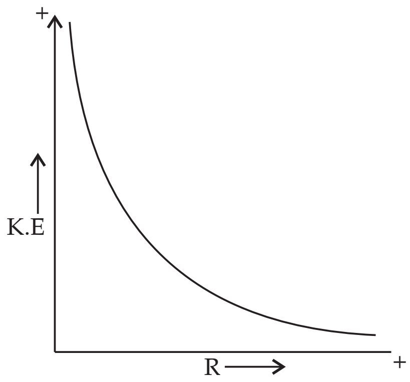
Orbital Velocity $v_o = \sqrt{\frac{GM}{R}}$

**(a)** K.E.versus orbital radius R

$$K.E = \frac{1}{2}mv_0^2 = \frac{1}{2}m.\frac{GM}{R}$$

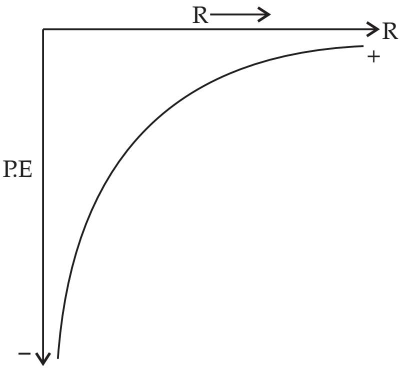
$$= \frac{GMm}{2R}$$

$$K.E. \propto \frac{1}{R} \text{ (It decreases with } R\text{)}$$




**(b)** P.E. versus orbital radius R.

$$P.E = -\frac{GMm}{R} \text{ or } P.E. \propto \frac{-1}{R}$$

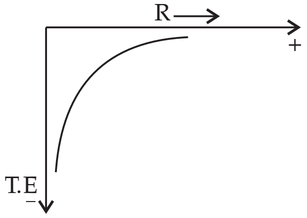



**(c)** Total Energy versus Orbital radius R

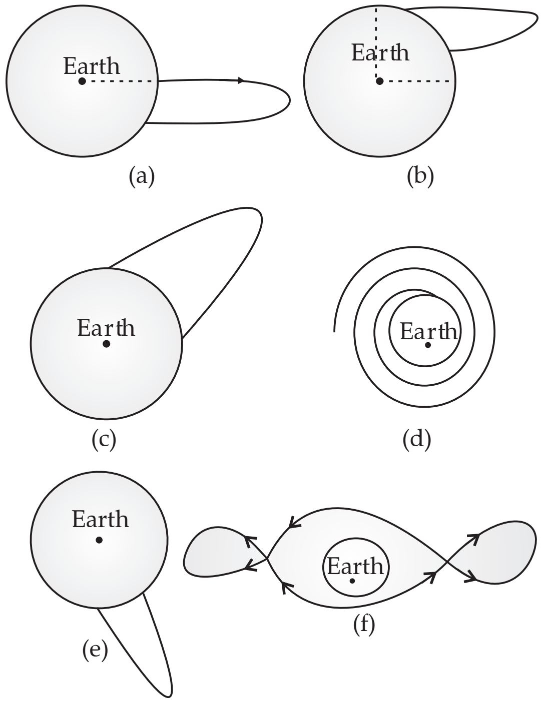
$$K.E = +\frac{1}{2}\frac{GMm}{R}, P.E = \frac{-GMm}{R}$$

$$T.E = \frac{1}{2}\frac{GMm}{R} - \frac{GMm}{R}$$

$$T.E = -\frac{1}{2}\frac{GMm}{R}$$




**Q. 2. Shown are several curves in below figures. Explain with reason which ones amongst them can be possible trajectories traced by a projectile (neglect air friction).**




**[NCERT Exemp. Q. 8.31, Page 62]**

**Ans.** The trajectory of a particle under gravitational force of the earth will be a conic section (for motion outside the earth) with the centre of the earth as a focus. Only (c) meets this requirement because in which centre of earth is the focus of trajectory.

**Q. 3. An object of mass *m* is raised from the surface of the earth to a height of equal to the radius of the earth, that is, taken from a distance R to 2R**

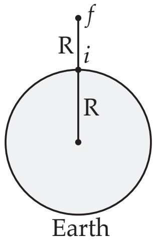
**from the centre of the earth. What is the gain in its potential energy?**

**[NCERT Exemp. Q. 8.32, Page 63]**

**Ans.** Given: An object is raised distance $R \rightarrow 2R$

Potential energy of body on the surface of earth $= \frac{-GMm}{R}$

P.E. of object at height equal to radius of earth $= \frac{-GMm}{2R}$




Gain in Potential Energy $= P.E_f - P.E_i$

$$= \frac{-GMm}{2R} - \left(\frac{-GMm}{R}\right)$$

$$= \frac{GMm}{R}\left[-\frac{1}{2}+1\right]$$

$$= \frac{GMm}{2R}$$

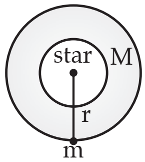
As, $GM = gR^2$

Gain in $P.E. = \frac{gR^2m}{2R} = \frac{1}{2}mgR.$

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. A star like the sun has several bodies moving around it at different distances. Consider that all of them are moving in circular orbits. Let r be the distance of the body from the centre of the star and let its linear velocity be v, angular velocity ω, kinetic energy K, gravitational potential energy U, total energy E and angular momentum *l*. As the radius r of the orbit increases, determine which of the above quantities increase and which ones decrease.**




**[NCERT Exemp. Q. 8.34, Page 63]**

**Ans.** Consider a body of mass m is rotating around the star S in circular path of radius r.

**(i)** Orbital velocity –

$$v_o = \sqrt{\frac{GM}{r}} \text{ or } v_0 \propto \frac{1}{\sqrt{r}}$$

Orbital Velocity decreases

**(ii)** Angular velocity $\frac{2\pi}{T}$

By Kepler's III law

$$T^2 \propto r^3 \text{ or } T^2 = Kr^3$$

$$\omega = \frac{2\pi}{Kr^{3/2}} \text{ or } \omega \propto \frac{1}{\sqrt{r^3}}$$

Hence, angular velocity decreases.

**(iii)** Kinetic Energy $K = \frac{1}{2}m\frac{GM}{r}$ or $K \propto \frac{1}{r}$

Hence K, decreases on increasing the radius.

**(iv)** Gravitational Potential Energy,

$$U = \frac{-GMm}{r}$$

or $$U \propto \frac{-1}{r}$$

So, on increasing radius of circular orbit the U increases.

**(v)** Total Energy,

$$E = K + U = \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right)$$

$$E = -\frac{GMm}{2r}$$

So, on increasing the radius, E will also be increased.

**(vi)** Angular momentum $L = mvr = mr\sqrt{\frac{GM}{r}}$

$L = m\sqrt{GMr}$ or $L \propto \sqrt{r}$ , increases

**Q. 2. A satellite is to be placed in equatorial geostationary orbit around earth for communication.**

**(a) Calculate height of such a satellite.**

**(b) Find out the minimum number of satellites that are needed to cover entire earth so that at least one satellite is visible from any point on the equator.**

**[$M = 6 \times 10^{24}$ kg, $R = 6400$ km. $T = 24$ h, $G = 6.67 \times 10^{-11}$ SI units]. [NCERT Exemp. Q. 8.36, Page 63]**

**Ans.** Mass of earth, $M = 6 \times 10^{24}$ kg

Radius of earth, $R = 6400$km

$$= 6.4 \times 10^6 \text{m}$$

Time, $T = 24 \times 3600s = 24 \times 36 \times 100s$

$$G = 6.67 \times 10^{-11} \text{N.m}^2\text{kg}^{-2}$$

Orbital Radius = R + $h$

**(a)** Orbital velocity, $v_0 = \sqrt{\dfrac{GM}{R+h}}$ or $v_0^2 = \dfrac{GM}{R+h}$

$$\therefore T = \frac{2\pi(R+h)}{v_0} \text{ or } T^2 = \frac{(2\pi)^2 (R+h)^2}{v_0^2}$$

$$T^2 = \frac{4\pi^2 (R+h)^2 (R+h)}{GM} = \frac{4\pi^2 (R+h)^3}{GM}$$

$$\text{or } (R+h)^3 = \frac{T^2 GM}{4\pi^2}$$

$$R+h = \left(\frac{T^2 GM}{4\pi^2}\right)^{1/3}$$

$$\text{or } h = \left(\frac{T^2 GM}{4\pi^2}\right)^{1/3} - R$$

$$h = \left(\frac{(24\times 3600)^2 \times (6.67\times 10^{-11}) \times (6\times 10^{24})}{4\times(3.14)^2}\right)^{1/3} - 6.4\times 10^6$$

$$= 4.23\times 10^7 - 6.4\times 10^6$$

$$= (42.3 - 6.4)\times 10^6$$

$$= 3.59\times 10^7 \text{ m}$$

**(b)** Let satellite S is at h metre above earth surface. Let angle subtended by satellite at centre of earth 2θ. Then,

In ΔCAO

$$\cos\theta = \frac{R}{R+h} = \frac{1}{\left(1+\dfrac{h}{R}\right)}$$

$$h = 3.59\times 10^7 \text{ m}$$

$$R = 6.4\times 10^6 \text{ m}$$

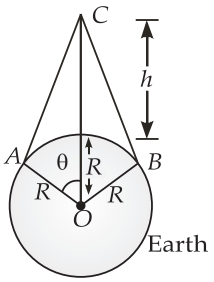
$$\cos\theta = \frac{1}{\left(1+\dfrac{h}{R}\right)}$$

$$\cos\theta = \frac{1}{\left(1+\dfrac{3.59\times 10^7}{6.40\times 10^6}\right)}$$

$$\cos\theta = \frac{1}{1+5.6} = \frac{1}{6.6}$$

$$\cos\theta = 0.1515 \text{ or } \theta = 81.28^\circ$$

$$2\theta = 81.28\times 2$$




∴ Number of satellite to cover $360^\circ = \dfrac{360^\circ}{81.28\times 2}$

$= 2.21$

∴ Minimum number of satellites = 3

**Q. 3. A satellite is in elliptic orbit around the earth with aphelion of 6R and perihelion of 2 R where R = 6400 km is the radius of the earth. Find eccentricity of the orbit. Find the velocity of the satellite at apogee and perigee. What should be done if this satellite has to be transferred to a circular orbit of radius 6R?**

**[G = 6.67 × 10⁻¹¹ SI units and M = 6 × 10²⁴ kg]**

**[NCERT Exemp. Q. 8.38, Page 64]**

**Ans.** Given

radius of perihelion, $r_p = 6R$

radius of aphelion, $r_a = 2R$

Hence,

$r_a = a(1+e) = 6R$ ...(i)

$r_p = a(1-e) = 2R$ ...(ii)

From (i) & (ii)

eccentricity, $e = \dfrac{1}{2}$

From law of conservation of angular momentum, angular momentum at perigee = angular momentum at apogee

$$\therefore m v_p r_p = m v_a r_a$$

$$\therefore \frac{v_a}{v_p} = \frac{1}{3}.$$

Applying low of Conservation of Energy :

Energy at perigee = Energy at apogee

(Where $M$ is mass of earth)

$$\frac{1}{2} m v_p^2 - \frac{GMm}{r_p} = \frac{1}{2} m v_a^2 - \frac{GMm}{r_a}$$

(where M is the mass of earth)

$$\frac{1}{2} m v_p^2 - \frac{GMm}{r_p} = \frac{1}{2} m v_a^2 - \frac{GMm}{r_a}$$

$$\therefore v_p^2\left(1-\frac{1}{9}\right) = -2GM\left[\frac{1}{r_a} - \frac{1}{r_p}\right] = 2GM\left[\frac{1}{r_p} - \frac{1}{r_a}\right]$$

Putting $v_a = \dfrac{v_p}{3}$

$$v_p = \frac{\left(2GM\left[\dfrac{1}{r_p} - \dfrac{1}{r_a}\right]\right)^{1/2}}{\left[1-\left(\dfrac{v_a}{v_p}\right)^2\right]^{1/2}} = \left[\frac{\dfrac{2GM}{R}\left[\dfrac{1}{2} - \dfrac{1}{6}\right]}{\left(1-\dfrac{1}{9}\right)}\right]^{1/2}$$

$$= \left(\frac{2/3}{8/9}\frac{GM}{R}\right)^{1/2}$$

$$= \sqrt{\frac{3}{4}\frac{GM}{R}} = 6.85 \text{ km/s}$$

$v_p = 6.85 \text{ km / s}$

$v_a = \frac{v_p}{3}$

$v_a = 2.28 \text{ km / s}$

For circular orbit of radius $r$,

$For\, r = 6R, v_c = \sqrt{\frac{GM}{6R}} = 3.23 \text{ km / s}.$

Hence to transfer to a circular orbit at apogee, we have to boost the velocity by $\Delta = (3.23 - 2.28) = 0.95$ km/s. This can be done by suitably firing rockets from the satellite.

**Q. 4. How will you weight the sun, i.e., estimate its mass ? The mean orbital radius of the earth around the sun is $1{\cdot}5 \times 10^8$ km. Estimate the mass of the sun. [NCERT Ex. Q. 8.13, Page 201]**

**Ans.** It is clear that earth revolves around the sun in an orbit of radius $1.5 \times 10^{11}$ m and completes on revolution around the sun in 365 days.

$\therefore$ R = radius of orbit of earth = $1.5 \times 10^{11}$ m

T = Time period of earth around the sun = 365 days = $365 \times 24 \times 60 \times 60$ s.

G = $6.67 \times 10^{11}$ Nm$^2$ kg$^{-2}$

As $M_s = \frac{4\pi^2 R^3}{T^2 G}$

or $M_s = \frac{4 \times 9.87 \times (1.5 \times 10^{11})^3}{(365 \times 24 \times 60 \times 60)^2 \times 6.67 \times 10^{-11}}$ kg

or $M_s = 2.0 \times 10^{30}$ kg.

$\therefore$ Mass of Sun $= 2.0 \times 10^{30}$ kg.

**Q. 5. A saturn year is 29·5 times the earth year. How far is the saturn from the sun if the earth is $1{\cdot}5 \times 10^8$ km away from the sun ? [NCERT Ex. Q. 8.14, Page 201]**

**Ans.** Here, $T_s = 29.5\, T_e$

$\mathbf{R_e = 1.5 \times 10^8}$ **km;**

$\mathbf{R_s = ?}$

Using the relation,

$$\frac{T_s^2}{R_s^3} = \frac{T_e^2}{R_e^3}$$

or $\mathbf{R_s} = R_e\left(\frac{T_s}{T_e}\right)^{2/3}$

$= 1.5 \times 10^8 \left[\frac{29.5\, T_e}{T_e}\right]^{2/3}$

$= 1.43 \times 10^9$ km

**Q. 6. A rocket is fired vertically with a speed of 5 kms$^{-1}$ from the earth's surface. How far from the earth does the rocket go before returning to the earth ?**
**Mass of the earth = $6{\cdot}0 \times 10^{24}$ kg, mean radius of earth = $6.4 \times 10^6$ m, G = $6.67 \times 10^{-11}$ Nm$^2$ kg$^{-2}$. [NCERT Ex. Q. 8.17, Page 202]**

**Ans.** Suppose $v$ be the initial speed of the rocket which reaches a height $h$ above the surface of earth where its velocity vanishes, *i.e.*, becomes zero.

When $m$ be the mass of the rocket, then its total energy at earth's surface is

$$\text{K.E.} + \text{P.E.} = \frac{1}{2}mv^2 - \frac{GMm}{R} \quad ...(i)$$

where, M = Mass of earth
R = Radius of earth
G = Universal gravitational constant

At highest point,

K.E. = 0 ($\because$ Velocity = 0)

and $\text{P.E.} = -\frac{GMm}{R+h}$ ...(ii)

$\therefore$ Total energy of the rocket at a height $h$ is given by

$= \text{K.E.} + \text{P.E.} = 0 + \text{P.E.}$

$= -\frac{GMm}{R+h}$ ...(iii)

From the law of conservation of energy.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\frac{1}{2}v^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$= \frac{GM}{R}\left(1 - \frac{R}{R+h}\right) \left(\because \frac{GM}{R^2} = g\right)$$

or $\frac{1}{2}v^2 = gR\left(\frac{R+h-R}{R+h}\right)$

$= \frac{gR}{R+h}h$

or $2gRh = v^2(R+h)$

or $Rv^2 = 2ghR - v^2h = h(2gR - v^2)$

$\therefore$ $h = \frac{Rv^2}{2gR - v^2}$ ...(iv)

Given, $v = 5$ kms$^{-1}$

$= 5000$ ms$^{-1}$ (given)

and R = $6{\cdot}4 \times 10^6$ m.

Putting these values in eqn. (iv), we have

$\therefore$ $h = \frac{6.4 \times 10^6 \times (5 \times 10^3)^2}{2 \times 9.8 \times 6.4 \times 10^6 - (5 \times 10^3)^2}$

$= 1.6 \times 10^6$ m

$\therefore$ Distance from centre of earth is given by

$= R + h = 6.4 \times 10^6 + 1.6 \times 10^6$

$= 8000$ km

**Q. 7. Escape speed of projectile on the earth's surface is 11.2 km s$^{-1}$. If a body is projected out with thrice its. What is the speed of the body far away from the earth ? Ignore the presence of the sun and other planets. [NCERT Ex. Q. 8.18, Page 202]**

**Ans.** Given $v_e$ = 11.2 from principle of conservation of energy

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_e^2$$

or $$v = \sqrt{v_i^2 - v_e^2}$$

$$= \sqrt{(2v_e)^2 - v_e^2}$$

$$= \sqrt{3}\, v_e$$

$$= 1.732 \times 11.2 \text{ km/s} \quad [\because \sqrt{3} = 1.732]$$

$$= 31.68 \text{ km s}^{-1}.$$

**Q. 8. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expanded to launch, the satellite out of the earth's gravitational influence ?**
**Mass of the satellite = 200 kg; mass of the Earth = 6.0 × $10^{24}$ kg, radius of earth = 6.4 × $10^6$ m; G = 6.67 × $10^{-11}$ $Nm^2$-$kg^{-2}$.**

**[NCERT Ex. Q. 8.19, Page 202]**

**Ans. Given :** $M = 6 \times 10^{24}$ kg

$m = 200$ kg

$r = 6.4 \times 10^6$ m

$x = 0.4 \times 10^6$ m

Using, total energy of satellite in orbit,

$$E = -\frac{GMm}{2(R + x)}$$

we get $$E = -\frac{(6.67 \times 10^{-11})(6 \times 10^{24})200}{2(6.4 \times 10^6 + 0.4 \times 10^6)} J$$

$$= -\frac{6.67 \times 6 \times 2 \times 10^{15}}{2 \times 6.8 \times 10^6} J$$

$$= -5.89 \times 10^9 \text{ J}$$

Energy required to send the satellite

$$= 5.9 \times 10^9 \text{ J}$$

**Q. 9. Two stars each of one solar mass ($= 2 \times 10^{30}$ kg) are approaching each other for a head on collision. When they are at distance $10^9$ km, their speeds are negligible. What is the speed with which they collide ? The radius of each star is $10^4$ km. Assume the stars to remain undistorted until they collide. (Use the known value of G).**

**[NCERT Ex. Q. 8.20, Page 202]**

**Ans.** Mass of each star, M = 2 × $10^{30}$ kg

Initial distance between two stars, r = $10^9$ km

$= 10^{12}$ m

Initial potential energy of the system = $\left(\frac{-GMm}{r}\right)$

Total kinetic energy of the system

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2$$

Where v is the speed of stars with which they collide. When the stars are about to collide the distance between their centres, r' = 2R, where R is their radius.

$\therefore$ Final potential energy of two stars = $-\frac{GMm}{2R}$

Since gain in K.E = loss in P.E

$$\therefore Mv^2 = -\frac{GMM}{r} - \left(\frac{-GMM}{2R}\right) = \frac{-GMM}{r} + \frac{GMM}{2R}$$

$$2 \times 10^{30} v^2 = \frac{-6.67 \times 10^{-17} \times (2 \times 10^{30})^2}{10^{12}} + \frac{6.67 \times 10^{-11} \times (2 \times 10^{30})^2}{2 \times 10^7}$$

$$= -2.668 \times 10^{38} + 1.334 \times 10^{43}$$

$$= 1.334 \times 10^{43} J$$

$$\therefore \quad v = \sqrt{\frac{1.334 \times 10^{43}}{2 \times 10^{30}}} = \sqrt{0.66 \times 10^{13}} \text{ m/s}$$

$$= \sqrt{6.67 \times 10^{12}} = 2.583 \times 10^6 \text{ m/s}$$

**Q. 10. Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres. Is an object placed at the point is in equilibrium ? If so, is the equilibrium stable or unstable ? [NCERT Ex. Q. 8.21, Page 202]**

**Ans.** Here $G = 6.67 \times 10^{-11}$ $Nm^2$ $kg^{-2}$

$M = 100$ kg

$R = 0.1$ m

Distance between the two spheres $d = 1.0$ m

Suppose that the distance of either sphere from the mid point of the line joining their centre is $r$. Then

$$r = \frac{d}{2} = 0.5 \text{ m}.$$

The gravitational field at the mid point due to two spheres will be equal and opposite.

Hence, the resultant gravitational field at the mid point = 0

The gravitational potential at the mid point.

$$= \left(-\frac{GM}{r}\right) \times 2$$

$$= \frac{6.67 \times 10^{-11} \times 100 \times 2}{0.5} \text{Jkg}^{-1}$$

$$= -2.668 \times 10^{-8} \text{ Jkg}^{-1}$$

Any object placed at point $x$ will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effecting force in that direction.

**Q. 11. As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite ? (Take the potential energy at infinity to be zero.) Mass of earth = 6.0 × $10^{24}$ kg, radius = 6400 km.**

**[NCERT Ad. Ex. Q. 8.22, Page 202]**

**Ans.** Mass of earth, $M = 6.0 \times 10^{24}$ kg

Radius of earth, $R = 6400$ km

Distance of the satellite from earth's surface,

$h = 36000$ km

$\therefore$ The gravitational potential due to earth's gravity at the site of the satellite

$$= -\frac{GM}{R+h}$$

$$= -\frac{6.67\times10^{-11}\times6\times10^{24}}{(6400+3600)\times10^{3}}$$

$$= -\frac{40.02\times10^{23}}{424\times10^{5}}\,\text{Jkg}^{-1}$$

Gravitational potential = $-9.43 \times 10^6$ Jkg$^{-1}$.

**Q. 12. A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.5 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain observed stellar objects called pulsars are believed to belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity ? (Mass of the sun = $2 \times 10^{30}$ kg)**

**[NCERT Ad. Ex. Q. 8.23, Page 202]**

**Ans.** For the object to remain stuck to the star, the gravitational force of the star must be equal to or greater than the centripetal force. Under this condition, the centrifugal force does not overcome the gravitational force and not fly off the object. It means,

$$mg \geq m\frac{v^2}{r}$$

or $$g \geq \frac{v^2}{r}$$

or $$g \geq a_c$$

where, $$a_c = \frac{v^2}{r}$$

= centripetal acceleration

Hence for the object to remain stuck, the acceleration due to gravity ($g$) on the star must be $\geq$ centripetal acceleration.

Now $r = 12$ km $= 12 \times 10^3$ m.

Frequency, $n = 1.5$ rps

$\therefore$ $\omega = 2\pi n = 2\pi \times 1.5$

$= 3\pi$ rads$^{-1}$.

$\therefore$ Centripetal acceleration is given by

$$a_c = \frac{v^2}{r} = r\omega^2$$

$$= 12 \times 10^3 \times (3\pi)^2 \quad \text{...(i)}$$

or $a_c = 12 \times 10^3 \times 9 \times 9.87$

or $a_c = 1065.97 \times 10^3$ ms$^{-2}$

$= 1.1 \times 10^6$ ms$^{-1}$.

Also, we know that acceleration due to gravity on the star ($g$) is given by the relation

$$g = \frac{GM}{r^2} \quad \text{..(ii)}$$

**Given :** $M$ = 2.5 times the mass of sun

$= 2.5 \times 2 \times 10^{30}$ kg

(Mass of sun = $2 \times 10^{30}$ kg)

$= 5 \times 10^{30}$ kg

$r = 12$ km ..(iii)

$G = 6.67 \times 10^{-11}$ Nm$^2$ kg$^{-2}$

$\therefore$ From equations (ii) and (iii), we get

$$g = \frac{6.67\times10^{-11}\times5\times10^{30}}{(12000)^2}$$

or $g = 0.2316 \times 10^{13}$ ms$^{-2}$

or $g = 23.16 \times 10^{11}$ ms$^{-2}$

or $g = 2.3 \times 10^{12}$ ms$^{-2}$ ...(iv)

Now form equations (i) and (iv), it is clear that $g >> a_c$. Hence, the object will remain stuck to the star.

**Q. 13. A spaceship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar ? Mass of the Mars = 1000 kg, Mass of the Sun = $2\times10^{30}\,kg$, Mass of the Mars = $6.4 \times 10^{23}$ kg, Radius of orbit of Mars $= 2.28\times10^{11}m, G = 6.67\times10^{-11}Nm^2kg^{-2}$.**

**[NCERT Ad. Ex. Q. 8.24, Page 202]**

**Ans.** Let R and R' be the radius of orbit of Mars and of Mass respectively. M be the mass of the sun and M' be that of Mars. If $m$ is the mass of the space-ship, then potential energy of space-ship due to gravitational attraction of the Sun = $-\frac{GM'm}{R'}$.

Since the kinetic energy of space ship is zero, therefore, total energy of spaceship

$$= \frac{-GMm}{R} - \frac{GM'm}{R'} = -Gm\left(\frac{M}{R}+\frac{M'}{R'}\right)$$

$\therefore$ Energy required to rocket out the spaceship from the

solar system = – (total energy of spaceship)

$$= -\left[\left(-Gm\frac{M}{R}+\frac{M'}{R'}\right)\right]$$

$$= Gm\left[\frac{M}{R}+\frac{M'}{R'}\right]$$

$$= 6.67 \times 10^{-11} \times 1000 \times \left[\frac{2 \times 10^{30}}{2.28 \times 10^{11}} + \frac{6.4 \times 10^{23}}{3395 \times 10^{3}}\right]$$

$$= 6.67 \times 10^{-8}\left[\frac{20}{2.28} + \frac{6.4}{33.95}\right] \times 10^{18} J = 5.98 \times 10^{11} J$$

**Q. 14. A rocket is fired ' vertically ' from the surface of mars with a speed of 2km/s. If 20% of its initial energy is lost due to motion atmospheric resistance, how far will the rocket go from the surface of mass before returning to it ? Mass of mars = 6.4 × $10^{23}$ kg, Radius of Mars = 3395 km, G = $6.67 \times 10^{-11}$ Nm$^2$kg$^{-2}$**

**[NCERT Ad. Ex. Q. 8.25, Page 202]**

**Ans.** Initial K.E = $\frac{1}{2}mv^2$, Initial P.E = $\frac{-GMm}{R}$

where $m$ = mass of the rocket, M = mass of the Mars,

R = radius of Mars

$\therefore$ Total initial energy = $\frac{1}{2}mv^2 - \frac{GMm}{R}$

Since 20% of K.E is lost, only 80% remains to reach the height.

$\therefore$ Total initial energy available

$$= \frac{4}{5} \times \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$= 0.4mv^2 - \frac{GMm}{R}$$

when the rocket reaches the height point, at a height $h$ above the surface, its K.E = 0 and

$$\Rightarrow \quad \frac{GMm}{R+h} = \frac{GMm}{R} - 0.4mv^2$$

$$\Rightarrow \quad \frac{GM}{R+h} = \frac{GM}{R} - 0.4v^2$$

$$\Rightarrow \quad \frac{GM}{R+h} = \frac{GM - 0.4Rv^2}{R}$$

$$\Rightarrow \quad \frac{R+h}{R} = \frac{GM}{GM - 0.4Rv^2}$$

or $$\frac{h}{R} = \frac{GM}{GM - 0.4Rv^2} - 1$$

$$= \frac{0.4Rv^2}{GM - 0.4Rv^2}$$

$$\Rightarrow \quad h = \frac{0.4R^2v^2}{GM - 0.4Rv^2}$$

or

$$h = \frac{0.4 \times (2 \times 10^3)^2 \times (3.395 \times 10^6)^2 \ m}{6.67 \times 10^{-11} \times 6.4 \times 10^{23} - 0.4(2 \times 10^3)^2 \times (3.395 \times 10^6)}$$

= 495 km.

**TIPS... & TRICKS...**

- *Study and understands about Kepler's law of Planetary Motion.*
- *Understand variations of acceleration due to gravity.*
- *Clear differences between g and G and Gravitational force and Electrical force.*
- *Gravitational potential is a scalar equality and gravitational intensity is a vector quantity.*
- *Gravitational potential is zero at infinite.*
- *At earth surface the potential energy is negative*
- *The positive value of potential energy at earth surface is equal to escape energy of the any body.*

## Some Commonly Made Errors

- Students gives most of their time in learning about satellites, geostationary topics.
- Some times students treat gravitational potential energy as a vector quantity.

### EXPERT ADVICE

- *Memorize the relations on a tip.*
- *Angular momentum and total mechanical energy remains conserved during the motion of an object under the gravitational force.*

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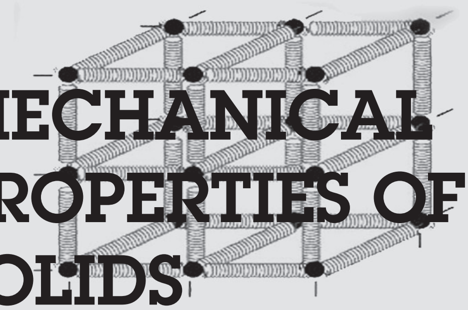


# CHAPTER 9 MECHANICAL PROPERTIES OF SOLIDS

## Chapter Objective

**This chapter will help you understand :**

- *Elastic behaviour, Stress-strain relationship, Hooke's law.*
- *Young's Modulus, Bulk modulus, Shear modulus, Modulus of rigidity. Poission's ratio, elastic energy.*

## TOPIC-1
## Elastic Behaviour of Solids



### Quick Review

- **Stress** is defined as the internal restoring force acting per unit area of a deformed body, *i.e.*,

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

The S.I. unit of stress is $N/m^2$ and its dimensional formula = $[ML^{-1}T^{-2}]$. Stress is a tensor quantity. Normal Stress have following three types :

**(a) Longitudinal stress :** If a body changes its length under a deforming force and the stress is normal to the surface of the body then the stress is called longitudinal stress. The longitudinal stress can be a tensile stress or compression stress. The longitudinal stress produced because of increase in length of body under deforming force is known as **tensile stress.** The longitudinal stress produced due to decrease in length of body under a deforming force is known as **compression stress.**

**(b) Volumetric or hydrostatic stress :** If a body changes its volume under a normal deforming force acting on every surface of the body, the stress set up in the body is volumetric stress.

**(c) Tangential stress :** It is also called shearing stress. When a deforming force applied tangentially to the surface of the body changes the shape of the body without changing its volume, the stress set up is known as tangential stress. The shape of the body changes or the body gets twisted due to tangential stress.

- **Strain** is defined as the ratio of change in configuration of the deformed body because of a deforming force on it, to the original configuration of the body it means

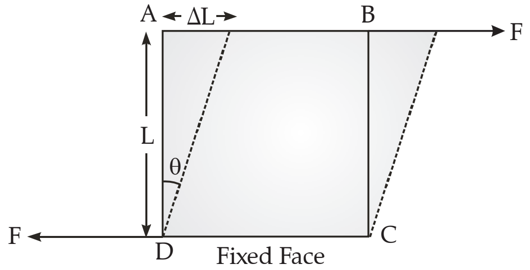
$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

Strain can be of following three types, depending upon the direction of force applied :

(a) $$\textbf{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l}$$

(b) $$\textbf{Volumetric strain} = \frac{\text{change of volume}}{\text{original volume}} \quad \frac{\Delta V}{V}$$

(c) **Shearing strain** is produced when the deforming force is applied parallel to the surface of a body and body changes its shape without changing its volume. Shearing strain is defined as the angle through which a vertical line perpendicular to the fixed surface gets rotated under the effect of a tangential deforming force.




Shearing strain is also defined as the ratio of displacement of a surface (ΔL) under the tangential deforming force to the perpendicular distance (L) of the displaced surface from the fixed surface, *i.e.*,

Shearing strain, $\theta = \frac{\Delta L}{L}$

Strain has no units and dimensions.

**(a)** If a beam is bent, both compression strain as well as extension strain are produced.

- Hooke's law states that within elastic limit, stress is directly proportional to strain, *i.e.*,

$$\text{Stress} \propto \text{Strain.}$$

## Know the Terms

- **Interatomic forces** are those forces which are acting between the atoms due to electrostatic interaction between the charges of the atoms.
- **Intermolecular forces** are the forces which are acting between the molecules due to electrostatic interaction between the charges of the molecules.
- **Plasma state :** It is a state of matter in which the medium is in the form of positive and negative ions.
- **Crystalline solids** are those solids which have a definite external geometrical form and whose constituent atoms/ions/molecules are arranged in a definite pattern in three dimensions within the solid.
- **Amorphous solids** are those solids which have no definite external geometrical form and whose constituent atoms/ions/molecules are not arranged in a definite pattern in three dimensions within the solid.
- **Deforming force** is that force which when applied changes the configuration of the body.
- **Elasticity** is the property of the body by virtue of which the body regains its original configuration (length, volume or shape) when the deforming forces are removed.
- **Perfectly elastic body** is that body which perfectly regains its original form on removing the external deforming force from it, *e.g.*, quartz.
- **Plastic body** is that body which does not regain its original form at all on the removal of deforming force, howsoever small the deforming force may be, *e.g.*, putty and paraffin wax.
- **Elastic limit** is the upper limit of deforming force up to which if deforming force is removed, the body regains its original form completely and beyond which if the force is increased, the body loses its property of elasticity and it gets permanently deformed. Elastic limit is the property of a body whereas elasticity is the property of material of a body.

## Know the Formulae

- Normal stress (S) = $F/A$, $A = \pi r^2$
- Breaking force = Breaking stress × area of cross-section
- Longitudinal strain = $\frac{\Delta l}{l}$
- Volumetric strain = $\frac{\Delta V}{V}$
- Shearing strain, $\theta = \frac{\Delta L}{L}$

## Know the Links

- *www.vedantu.com*
- *www.learncbse.com*
- *www.topper.com*

# MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will**

**(a) be double.** **(b) be half.**
**(c) be four times.** **(d) remain same.**

**[NCERT Exemp. Q. 9.2, Page 65]**

**Ans. Correct option:** (d)

*Explanation:* Breaking Force F = P × A ⇒ F ∝ A

It a wire of length L is cut into two and more parts, Then again its each put can hold the some weight as breaking force is independent of the length of wire.

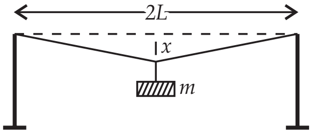
**Q. 2. A spring is stretched by applying a load to its free end. The strain produced in the spring is**

**(a) volumetric.**
**(b) shear.**
**(c) longitudinal and shear.**
**(d) longitudinal.** **[NCERT Exemp. Q. 9.4, Page 66]**

**Ans. Correct option:** (c)

*Explanation:* The change in length of spring corresponding to longitudinal strain and change in shape corresponds to shearing strain.

**Q. 3. A mild steel wire of length 2L and cross-sectional area A is stretched, well within elastic limit, horizontally between two pillars (as shown in fig.). A mass m is suspended from the mid point of the wire. Strain in the wire is**



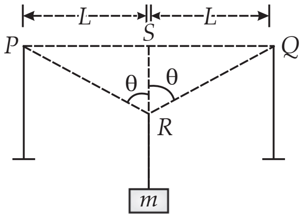

**(a)** $\frac{x^2}{2L^2}$ **(b)** $\frac{x}{L}$

**(c)** $\frac{x^2}{L}$ **(d)** $\frac{x^2}{2L}$

**[NCERT Exemp. Q. 9.6, Page 66]**

**Ans. Correct option:** (a)

*Explanation:*




Increase in length, ΔL= (PR + RQ) – PQ

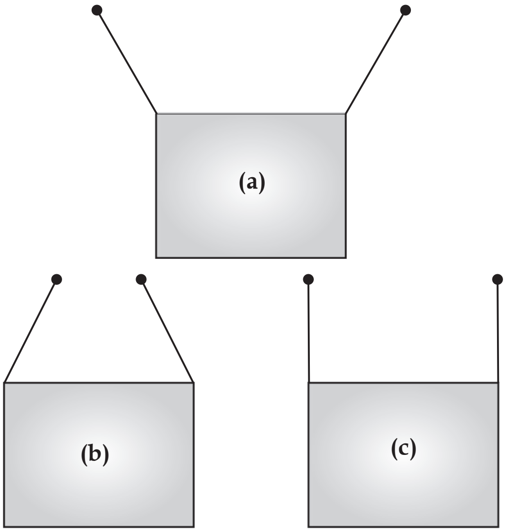
= 2 PR – PQ

$$\Delta L = 2(L^2 + x^2)^{1/2} - 2L = 2L\left(1 + \frac{x^2}{L^2}\right)^{1/2} - 2L$$

$$= 2L\left[1 + \frac{1}{2}\frac{x^2}{L^2}\right] - 2L \text{ (by binomial theorem)}$$

$$\text{strain} = \frac{\Delta L}{2L} = \frac{x^2}{2L^2}.$$

**Q. 4. A rectangular frame is to be suspended symmetrically by two strings of equal length on two supports (as shown in fig.). It can be done in one of the following three ways:**

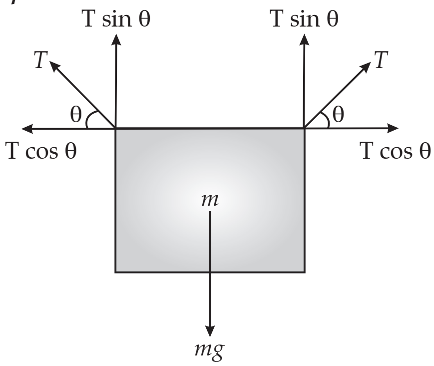



**The tension in the strings will be**

**(a) The same in all cases.**
**(b) least in (a).**
**(c) least in (b).**
**(d) least in (c).** **[NCERT Exemp. Q. 9.7, Page 67]**

**Ans. Correct option:** (c)

*Explanation:*



Let M be the mass and θ be the angle which the tension T in the string makes with the horizontal.

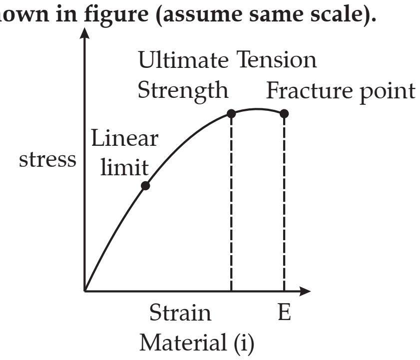
$2T \sin \theta = mg$

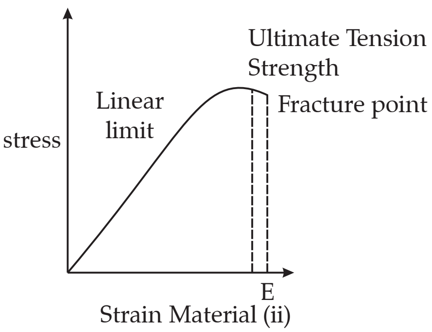
$$T = \frac{mg}{2\sin\theta} \text{ or } T \propto \frac{1}{\sin\theta}$$

T is least if sin θ has maximum value i.e.,

$\sin \theta = 1 = \sin 90°$

**Q. 5. The stress-strain graphs for two materials are shown in figure (assume same scale).**




**(a) Material (ii) is more elastic than material (i) and hence material (ii) is more brittle.**

**(b) Material (i) and (ii) have the same elasticity and the same brittleness.**

**(c) Material (ii) is elastic over a larger region of strain as compared to (i).**

**(d) Material (ii) is more brittle than material (i).**

**[NCERT Exemp. Q. 9.9, Page 68]**

**Ans. Correct option:** (c) and (d)

*Explanation:*

**(c)** from graph (i) linear limit vanishes soon and for small stress there is large strain as compared to graph (ii) hence, material (ii) is more elastic over a large region of strain as compared to (i).

**(d)** A material is said to be more brittle if its fracture paint is more closer to ultimate strength point, hence material (ii) is more brittle than material (i).

**Q. 6. A wire is suspended from the ceiling and stretched under the action of a weight F suspended from its other end. The force exerted by the ceiling on it is equal and opposite to the weight.**

**(a) Tensile stress at any cross section A of the wire is F/A.**

**(b) Tensile stress at any cross section is zero.**

**(c) Tensile stress at any cross section A of the wire is 2F/A.**

**(d) Tension at any cross section A of the wire is F.**

**[NCERT Exemp. Q. 9.10, Page 68]**

**Ans. Correct option:** (a), (d)

*Explanation:*

$$\text{Tension strength} = \frac{\text{Force Applied}}{\text{Area of cross section}} = \frac{F}{A}$$

Tension in wire = applied weight = F

## Very Short Answer Type Questions (1 mark each)

**Q. 1. Two identical solid balls, one of ivory and the other of wet-clay, are dropped from the same height on the floor. Which one will rise to a greater height after striking the floor and why?**

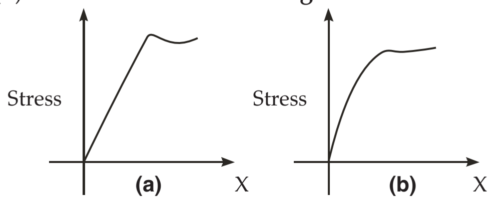
**[NCERT Exemp. Q. 9.23, Page 70]**

**Ans.** Since, the ivory ball is more elastic than the wet-clay ball, therefore, ivory ball will tend to retain its shape instantaneously after the collision. Hence, there will be a large energy and momentum transfer to the ivory ball as compared to the wet clay ball. Thus, the ivory ball will rise higher after striking the floor.

**Q. 2. The stress-strain graphs for materials A and B are shown in figure. The graphs are drawn on the same scale.**

**(a) Which of the materials has the greater Young's modulus ?**

**(b) Which of the two is stronger material ?**




**[NCERT Ex. Q. 9.3, Page 247]**

**Ans. (a)** From graph it is clear that for a given strain, stress for A is more than that of B. Hence Young's modulus (= stress/strain) is greater for A than that of B.

**(b)** A is stronger than B. Strength of a material is measured by the amount of stress required to cause fracture, corresponding to the point of fracture.

## Short Answer Type Questions

**(2 or 3 marks each)**

**Q. 1. A steel cable with a radius of 1.5 cm supports a chair lift at a ski area. If the maximum stress is not to exceed $10^8$ Nm$^{-2}$, what is the maximum load the cable can support ?**

**[NCERT Ex. Q. 9.9, Page 248]**

**Ans. Given :** Radius of steel cable,

$r = 1.5$ cm $= 1.5 \times 10^{-2}$ m.

Maximum Stress $= 10^8$ Nm$^{-2}$.

$\therefore$ Area of cross-section of cable

$A = \pi r^2 = \pi (1.5 \times 10^{-2})^2$

$$\text{Maximum stress} = \frac{\text{Maximum force}}{\text{Area of cross-section}}$$

or Maximum force = Maximum stress × Area of cross-section

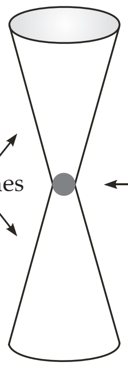
or $F_{max} = 10^8 \times \pi \times (1.5 \times 10^{-2})^2$

or $F_{max} = 3.142 \times 2.25 \times 10^8 \times 10^{-4}$ N.

or Maximum load the cable can withstand

$= 7.07 \times 10^4$ N.

**Q. 2. Anvils made of single crystals of diamond, with the shape as shown in figure, are used to investigate the behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil ?**




**[NCERT Ad. Ex. Q. 9.17, Page 249]**

**Ans. Given :** Compressional force, $F = 5 \times 10^4$ N

Diameter, $D = 5.0 \times 10^{-4}$ m

$\therefore$ radius, $r = \frac{D}{2} = 2.5 \times 10^{-4}$ m

Area, $A = \pi r^2$

$= \frac{22}{7} \times (2.5 \times 10^{-4})^2$ m$^2$

Pressure at the tip, P = ?

From the relation

$$P = \frac{F}{A},$$

we get, $P = \frac{5 \times 10^4}{(2.5 \times 10^{-4})^2}$ Pa

or $P = 0.255 \times 10^{12}$ Pa

or $P = 2.55 \times 10^{11}$ Pa.

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. Consider a long steel bar under a tensile stress due to forces F acting at the edges along the length of the bar (as shown in figure). Consider a plane making an angle θ with the length. What are the tensile and shearing stresses on this plane?**

**(a)For what angle is the tensile stress maximum?**

**(b)For what angle is the shearing stress maximum?**

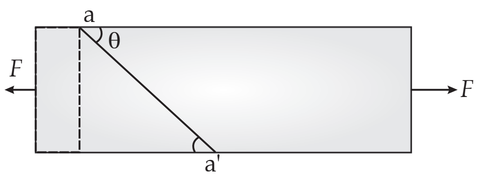
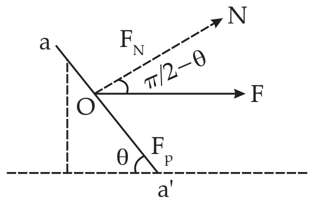



**Fig 9.5**

**[NCERT Exemp. Q. 9.24, Page 70]**

**Ans.** Let the cross sectional area of the bar be A.




Consider the equilibrium of the plane *aa*'. A force F must be acting on this plane making an angle $\frac{\pi}{2} - \theta$ with the normal ON. Resolving F into components, along the plane (FP) and normal to the plane

$$F_p = F\cos\theta$$

$$F_n = F\sin\theta$$

Let the area of the aa' be A'. Then

$$\frac{A}{A'} = \sin\theta$$

$$\therefore A' = \frac{A}{\sin\theta} \quad (1)$$

$$\text{Tensile stress} = \frac{\text{Normal force}}{\text{Area}}$$

$$= \frac{F\sin\theta}{A'} = \frac{F\sin\theta}{A/\sin\theta} = \frac{F\sin^2\theta}{A} \qquad \text{(from (1))}$$

$$\text{shearing stress} = \frac{\text{Parallel Force}}{\text{Area}} = \frac{F\cos\theta}{A/\sin\theta}$$

$$= \frac{F}{A}\cos\theta.\sin\theta = \frac{F}{2A}.2\cos\theta\sin\theta$$

$$= \frac{F}{2A}.\sin 2\theta$$

(a) Maximum Tensile stress, $\sin^2\theta = 1$

or $\sin\theta = 1$

$$\theta = \frac{\pi}{2}$$

(b) Maximum shearing stress, $\sin 2\theta = 1 = \sin\frac{\pi}{2}$

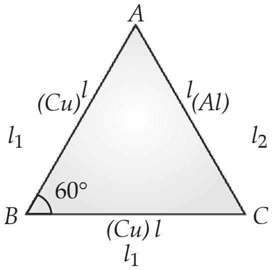
$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$

**Q. 2. An equilateral triangle ABC is formed by two Cu rods AB and BC and one A*l* rod. It is heated in such a way that temperature of each rod increases by ΔT. Find change in the angle ABC. [Coeff. of linear expansion for Cu is $\alpha_1$, coeff. of linear expansion for A*l* is $\alpha_2$]**

**[NCERT Exemp. Q. 9.27, Page 71]**

**Ans.** Due to temperature increase, length of each side will change, so, the angle corresponding to any vertex also changes. Let us consider the following diagram.




$$Let\, l_1 = AB, l_2 = AC, l_3 = BC$$

$$\cos\theta = \frac{l_3^2 + l_1^2 - l_2^2}{2l_3 l_1}$$

or $2l_3l_1\cos\theta = l_3^2 + l_1^2 - l_2^2$

Differentiating the following equation

$$2(l_3dl_1 + l_1dl_3)\cos\theta - 2l_1l_3\sin\theta d\theta$$

$$= 2l_3dl_3 + 2l_1dl_1 - 2l_2\,dl_2$$

$$[\because \Delta t = \text{change in temperature}]$$

Now,

$$dl_1 = l_1\alpha_1\,\Delta t$$

$$dl_2 = l_2\alpha_1\,\Delta t$$

$$dl_3 = l_3\alpha_2\,\Delta t$$

and $l_1 = l_2 = l_3 = l$

$$(l^2\alpha_1\Delta t + l^2\alpha_1\Delta t)\cos\theta + l^2\sin\theta d\theta$$

$$= l^2\,\alpha_1\Delta t + l^2\,\alpha_1\Delta t - l^2\,\alpha_2\Delta t$$

$$\sin\theta d\theta = 2\alpha_1\Delta t(1-\cos\theta) - \alpha_2\,\Delta t$$

Putting θ = 60°

$d\theta \times \sin 60° = 2\alpha_1\,\Delta t\,(1 - \cos 60°) - \alpha_2\,\Delta t)$

$$d\theta \times \frac{\sqrt{3}}{2} = 2\alpha_1\,\Delta t \times \frac{1}{2} - \alpha_2\Delta t$$

$$d\theta \times \frac{\sqrt{3}}{2} = (\alpha_1 - \alpha_2)\Delta t$$

$$\text{change in angle}, d\theta = \frac{(\alpha_1 - \alpha_2)\Delta t}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2(\alpha_1 - \alpha_2)\Delta t}{\sqrt{3}}$$

**Q. 3. A stone of mass m is tied to an elastic string of negligible mass and spring constant k. The unstretched length of the string is L and has negligible mass. The other end of the string is fixed to a nail at a point p. Initially the stone is at the same level as the point P. The stone is dropped vertically from point P.**

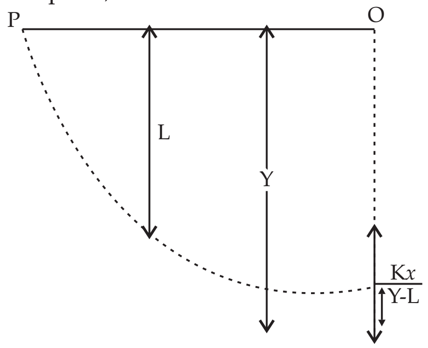
**(a) Find the distance y from the top when the mass comes to rest for an instant, for the first time.**

**(b) What is the maximum velocity attained by the stone in this drop?**

**(c) What shall be the nature of the motion after the stone has reached its lowest point?**

**[NCERT Exemp. Q. 9.29, Page 71]**

**Ans.** Considering the diagram below, stone is dropped from point, P.




**(a)** Till the stone drops through a length L it will be in free fall. After that the elasticity of the string will force it to a SHM. Let the stone come to rest instantaneously at Y.

The loss in potential energy of the stone is the potential energy stored in the stretched string.

$$mgy = \frac{1}{2}k(y-L)^2$$

Or, $mgy = \frac{1}{2}ky^2 - kyL + \frac{1}{2}kL^2$

or $\frac{1}{2}ky^2 - (kL+mg)y + \frac{1}{2}kL^2 = 0$

$$y = \frac{(kL+mg) \pm \sqrt{(kL+mg)^2 - k^2L^2}}{k}$$

$$= \frac{(kL+mg) \pm \sqrt{2mgkL + m^2g^2}}{k}$$

Retaining, the positive sign.

We get, $y = \frac{(kL+mg) + \sqrt{2mgkL + m^2g^2}}{k}$

**(b)** In Simple Harmonic Motion (SHM), the maximum velocity is attained when the body passes, through the "equilibrium position" i.e. when the instantaneous acceleration is zero, i.e., $mg - kx = 0$ where x is the extension from L:

or $mg = kx$

Let the velocity be $v$. Then,

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = mg(L+x)$$

(*By law of conservation of energy*)

$$\frac{1}{2}mv^2 = mg(L+x) - \frac{1}{2}kx^2$$

Now $mg = kx$

or $x = \frac{mg}{k}$

Therefore, $\frac{1}{2}mv^2 = mg\left(L + \frac{mg}{k}\right) - \frac{1}{2}k\frac{m^2g^2}{k^2}$

$$\frac{1}{2}mv^2 = mgL + \frac{m^2g^2}{k} - \frac{1}{2}\frac{m^2g^2}{k}$$

$$\frac{1}{2}mv^2 = mgL + \frac{1}{2}\frac{m^2g^2}{k}$$

$\therefore$ $v^2 = 2gL + mg^2/k$

$$v = (2gL + mg^2/k)^{1/2}$$

**(c)** Consider the particle at an instantaneous position $y$ (stone is at lowest position). Then Equation of motion of stone is

$$\frac{md^2y}{dt^2} = mg - k(y-L)$$

or $\frac{d^2y}{dt^2} + \frac{k}{m}(y-L) - g = 0$

Make a transformation of variables: $z = \frac{k}{m}(y-L) - g$

So, $\frac{d^2z}{dt^2} + \frac{kz}{m} = 0$

It is second order differential equation representing SHM.

Comparing with $\frac{d^2z}{dt^2} + \omega^2 z = 0$

Angular frequency of harmonic motion, $\omega = \sqrt{\frac{k}{m}}$

Solution of above equation z = A cos (ωt + φ)

or $y = \left(L + \frac{m}{k}g\right) + A'\cos(\omega t + \phi)$

Thus, stone will perform SHM with angular frequency ω about the point, $y_0 = L + \frac{mg}{k}$

# TOPIC-2
## Modulus of Elasticity

## Quick Review

➢ **Modulus of elasticity or coefficient of elasticity (E)** of a body is defined as the ratio of stress to the corresponding strain produced, within the elastic limit, *i.e.,*

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Modulus of elasticity is of three types :

**(a) Young's modulus of elasticity (Y)** is defined as the ratio of normal stress to the longitudinal strain, within the elastic limit, *i.e.,*

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

$$= \frac{F/A}{\Delta l / l} = \frac{F}{A} \times \frac{l}{\Delta l}$$

Y is the property of solid material only. Y increases on mixing the impurity in the solid and decreases on increasing the temperature of the solid body.

**(b) Bulk modulus of elasticity (K)** is first defined by Maxwell. It is defined as the ratio of normal stress to the volumetric strain, within the elastic limit, *i.e.*,

$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

$$= -P\frac{V}{\Delta V}$$

K is the property for solids, liquids and gases.

For gases, bulk modulus is of two types :

**(i) Isothermal Bulk modulus of elasticity** ($K_t$) which is equal to the pressure of gas (P).

**(ii) Adiabatic Bulk modulus of elasticity** ($K_a$) which is equal to $\gamma$ P, where $\gamma = \frac{C_p}{C_v}$.

K is maximum for solids, less for liquids and least for gases.

- **Compressibility** is defined as the reciprocal of bulk modulus of elasticity. The pressure necessary to stop volume expansion of a piece of metal is $P = K\gamma\Delta T$, where K is bulk modulus of elasticity of metal, $\gamma$ is the coefficient of volume expansion ($\gamma = 3\alpha$).
- **Modulus of Rigidity** ($\eta$) is defined as the ratio of tangential stress to the shearing strain, within the elastic limit, *i.e.*,

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta}$$

$\eta$ is the characteristic of solid material only as the liquids and gases do not have fixed shape. $\eta$ for liquid is zero.

- **Poisson's ratio** ($\sigma$). It is defined as the ratio of lateral strain to the longitudinal strain, *i.e.*,

$$\sigma = \frac{\text{Lateral strain } (\beta)}{\text{Longitudinal strain } (\alpha)}$$

$$= \frac{-\Delta D / D}{\Delta l / l} = -\frac{-\Delta D . l}{D . \Delta l}$$

Numerically value of $\sigma$ lies between –1 and + $\frac{1}{2}$ but practical value of $\sigma$ lies between 0 and + $\frac{1}{2}$.

For rubber, $\sigma = \frac{1}{2}$

## Know the Terms

- **Ductile materials** are those materials which show large plastic range beyond the elastic limit, and which can be drawn into sheets and springs. Examples are copper, silver, iron, aluminium etc.
- **Brittle materials** are those materials which show very small range beyond the elastic limit. For such materials the breaking point lies close to the elastic limit. Examples are cast iron, glass etc.
- **Elastomers** are those materials for which stress and strain variation is not in straight line, within the elastic limit, and strain produced is much larger than the stress applied. Such materials have no plastic range. The breaking point lies very close to elastic limit. Example is rubber.

  The material which can be greatly stretched are called elastomers.
- **Elastic after effect :** The time delay in regaining the original configuration by the elastic body after the removal of a deforming force is called elastic after effect. It is least for quartz or phosphor bronze and maximum for glass. Elastic after effect is a temporary absence of elastic properties. A temporary loss of elastic properties due to continuous use of a body for long time is called **elastic fatigue.**

## Know the Formulae

- Young's modulus $$Y = \frac{Fl}{\pi r^2 \Delta l}$$
- Bulk modulus $$K = -\frac{FV}{A\Delta V} = -P\frac{V}{\Delta V}$$

- Modulus of Rigidity $\sigma = \frac{FL}{A\Delta L}$
- Poisson's Ratio $\sigma = \frac{-\Delta D.l}{D.\Delta l}$
- **Relation Among Various Elastic Constants :**

**(i)** Relation between σ, α and β,

$$\sigma = \frac{\beta}{\alpha}$$

where $\beta = dD/D$

and $\alpha = dl/l$

**(ii)** Relation between Y and α, $Y = \frac{1}{\alpha}$ when stress = 1 unit

**(iii)** Relation between η, α and β, $\eta = \frac{1}{2(\alpha+\beta)}$

**(iv)** Relation between Y, K and σ, $Y = 3K(1 - 2\sigma)$

**(v)** Relation between Y, η and σ, $Y = 2\eta(1 + \sigma)$

**(vi)** Relation between K, η and σ, $\sigma = \frac{3K - 2\eta}{2\eta + 6K}$

**(vii) Relation between Y, K and η,** $\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$

- Elastic potential energy in stretched wire, $U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume of wire}$
- Elastic potential energy per unit volume of wire,

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} Y \times (\text{Strain})^2$$

- Interatomic force constant, ($k$) $k = r_0 \times Y$

Here, $r_0$ = interatomic distance

- Energy stored per unit volume of a strained body, $u = \frac{1}{2} Y \times (\text{Strain})^2$
- Work done in a stretching wire $W = \frac{1}{2} \times \text{Load} \times \text{Extension.}$

## Know the Links

- *www.learncbse.in*
- *www.vedantu.com*
- *www.toppers.com*

## MCQ/Fillups/True or False (1 mark each)

### (A) Multiple Choice Questions

**Q. 1. Modulus of rigidity of ideal liquids is**

**(a) infinity.**

**(b) zero**

**(c) unity.**

**(d) some finite small non-constant value.**

**[NCERT Exemp. Q. 9.1, Page 65]**

**Ans. Correct option:** (b)

*Explanation:* The frictional force (viscous force) cannot exist in case of ideal liquid and since they cannot sustain shearing stress or tangential force are zero, so there is no Modulus at rigidity *i.e.,*

$$\eta = 0$$

**Q. 2. The temperature of a wire is doubled. The Young's modulus of elasticity**

**(a) will also double.**

**(b) will become four times.**

**(c) will remain same.**

**(d) will decrease.**

**[NCERT Exemp. Q. 9.3, Page 65]**

**Ans. Correct option:** (d)

*Explanation:* Young's Modulus

$$Y = \frac{\text{Strass}}{\text{Strain}} = \frac{(F/A)}{(\Delta L/L)} = \frac{FL}{A \times \Delta L}$$

$$= \frac{FL}{A \times L\alpha\Delta T} = \frac{F}{A\alpha\Delta T} \qquad [\because \Delta L = L\,\alpha\,AT)$$

$$\Rightarrow Y \propto \frac{1}{\Delta T}$$

When temperature increases $\Delta T$, hence Y decreases.

**Q. 3. A rigid bar of mass M is supported symmetrically by three wires each of length l. Those at each end are of copper and the middle one is of iron. The ratio of their diameters, if each is to have the same tension, is equal to**

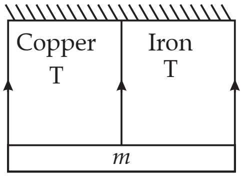
**(a)** $Y_{copper}/Y_{iron}$ **(b)** $\sqrt{\frac{Y_{iron}}{Y_{copper}}}$

**(c)** $\frac{Y^2_{iron}}{Y^2_{copper}}$ **(d)** $\frac{Y_{iron}}{Y_{copper}}$

**[NCERT Exemp. Q. 9.6, Page 66]**

**Ans. Correct option:** (b)

*Explanation:*




Let T be tension in each wire. As the bar is supported symmetrically by three wires,

$\therefore$ extension in each wire is same. As $Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$,

If D is the diameter of the wire, then

$$Y = \frac{\frac{F}{\pi\left(\frac{D}{2}\right)^2}}{\frac{\Delta L}{L}} = \frac{4FL}{\pi D^2 \Delta L}$$

As per the conditions of the problem, same for each wire.

$$\therefore\ Y \propto \frac{1}{D^2} \text{ or } D \propto \sqrt{\frac{1}{Y}}$$

F (tension),Length L, and extension $\Delta$ L is

$$\therefore\ \frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}}$$

**Q. 4. Consider two cylindrical rods of identical dimensions, one of rubber and the other of steel. Both the rods are fixed rigidly at one end to the roof. A mass M is attached to each of the free ends at the centre of the rods.**

**(a) Both the rods will elongate but there shall be no perceptible change in shape.**

**(b) The steel rod will elongate and change shape but the rubber rod will only elongate.**

**(c) The steel rod will elongate without any perceptible change in shape, but the rubber road will elongate and the shape of the bottom edge will change to an ellipse.**

**(d) The steel rod will elongate, without any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre.**

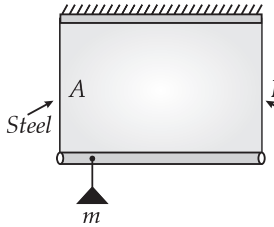
**[NCERT Exemp. Q. 9.8, Page 67]**

**Ans. Correct option:** (d)

*Explanation:* If mass M is attached at the centre of rubber and steel rod, then both rods will be elongated. But to different elastic properties of material the steel rod will elongate without making any change in shape, but the rubber rod will elongate with the shape at the bottom edge tapered to a tip at the centre.

**Q. 5. A rod length l and negligible mass is suspended at its two ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths (as shown in figure). The cross-sectional areas of wires A and B are 1.0 mm² and 2.0 mm², respectively.**

**($Y_{Al} = 70 \times 10^9$ Nm$^{-2}$ and $Y_{steel} = 200 \times 10^9$ Nm$^{-2}$)**




**(a) Mass *m* should be suspended close to wire A to have equal stresses in both the wires.**

**(b) Mass *m* should be suspended close to B to have equal stresses in both the wires.**

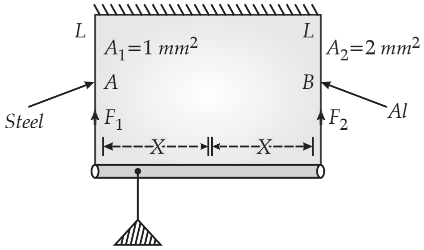
**(c) Mass *m* should be suspended at the middle of the wires to have equal stresses in both the wires.**

**(d) Mass *m* should be suspended close to wire A to have equal strain in both wires.**

**[NCERT Exemp. Q. 9.11, Page 68]**

**Ans. Correct option:** (b) and (d)

*Explanation:*




Let the mass *m* is suspended at a distance *x* from left end to develop equal stress in wire

Stress steel $S_1 = \frac{F_1}{A_1} = \frac{F_1}{10^{-6}}$;

Stress for aluminium

$$S_2 = \frac{F_2}{A_2} = \frac{F_2}{2\times10^{-6}}$$

$$F_1 x = F_2(l - x)$$

or $\frac{F_1}{F_2} = \frac{l-x}{x}$ ...(i)

For equal stress

$S_1 = S_2$

or $\frac{F_1}{10^{-6}} = \frac{F_2}{2\times10^{-6}}$

or $\frac{F_1}{F_2} = \frac{1}{2} = \frac{l-x}{x}$ By (i)

or $l = \frac{3}{2}x$ or $x = \frac{2}{3}l$

or $l = \frac{3}{2}x$ or $x = \frac{2}{3}l$

$\therefore$ mass should be suspended close to wire B (Al wire)

$strain = \frac{stress}{y}$

$\therefore$ for equal strain

$$\frac{S_1}{r_s} = \frac{S_2}{r_{Al}}$$

or $\dfrac{\frac{F_1}{10^{-6}}}{Y_s} = \dfrac{\frac{F_1}{(2\times10^{-6})}}{Y_{Al}}$

or $\frac{F_1}{F_2} = \frac{Y_s}{2Y_{Al}}$

$= \frac{200\times10^9}{2\times70\times10^9} = \frac{10}{7}$

or $\frac{l-x}{x} = \frac{10}{7}$ by (i)

or $l = \frac{17}{7}x$

or $x = \frac{7}{17}l$

**Q. 6. For an ideal liquid**

**(a) the bulk modulus is infinite.**

**(b) the bulk modulus is zero.**

**(c) the shear modulus is infinite.**

**(d) the shear modulus is zero.**

**[NCERT Exemp. Q. 9.12, Page 69]**

**Ans. Correct option:** (a) and (d)

*Explanation:*

$$B = \frac{\text{Volume stress}}{\text{Volume strain}}$$

$\therefore$ ideal liquid is incompressible, so $\Delta V = 0$. It means volume strain is 0

$\therefore B = \infty$

Shear modulus, $\eta = \frac{\text{shear stress}}{\text{shear strain}}$

A liquid cannot sustain tangential force. It may contain tangential viscous drag. So $\eta = 0$

**Q. 7. A copper and a steel wire of the same diameter are connected end to end. A deforming force F is applied to this composite wire which causes a total elongation of 1 cm. The two wires will have**

**(a) the same stress.**

**(b) different stress.**

**(c) the same strain.**

**(d) different strain.**

**[NCERT Exemp. Q. 9.13, Page 69]**

**Ans. Correct option:** (a) and (d)

*Explanation:*

$$\text{Stress} = \frac{F}{A}, \text{Strain} = \frac{\text{stress}}{Y}$$

For same stress, strain $\propto \frac{1}{Y}$

So, two wires will have different strain due to different value of Y.

## (B) True or False

**Q. 8. Read the following two statements below carefully and state, with reasons, if it is true or false:**

**(a) The Young's modulus of rubber is greater than that of steel.**

**(b) The stretching of a coil is determined by its shear modulus. [NCERT Exemp. Q. 9.4, Page 248]**

**Ans. (a) Incorrect.** This is because if steel and rubber wires of same length and area of cross-section are subjected to same deforming force, then the extension produced in steel is less than the extension produced in rubber, so $Y_s > Y_r$. In other words, for producing same strain in steel and rubber, more stress is required in case of steel.

**(b) Correct.** The reason is that when a coil spring is stretched, there is neither a change in the length of the coil (*i.e.*, length of the wire forming the coil spring) nor a change in its volume. Since the change takes place in the shape of the coil spring, its stretching is determined by its shear modulus.

# Very Short Answer Type Questions **(1 mark each)**

**Q. 1. The Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress?**

**[NCERT Exemp. Q. 9.14, Page 69]**

**Ans.** Young's modulus,

$$Y = \frac{\text{stress}}{\text{Longitudinal strain}}$$

$Y \propto$ stress (for same longitudinal strain)

$$\therefore \frac{Y_{steel}}{Y_{rubber}} = \frac{stress_{steel}}{stress_{rubber}} \quad (A)$$

(Given: $Y_{steel} > Y_{rubber}$),

$$\therefore \frac{y_{steel}}{y_{rubber}} > 1$$

From $eq^n$ (A) $\frac{strees_{steel}}{stress_{rubber}} > 1$ or $stress_{steel} > stress_{rubber}$

**Q. 2. Is stress a vector quantity?**

**[NCERT Exemp. Q. 9.14, Page 69]**

**Ans.** No,

$$Stress = \frac{Magnitude\ of\ restoring\ force\ by\ solid}{area\ of\ cross\text{-}section}$$

Therefore, stress is tensor quantity

**Q. 3. Identical springs of steel and copper are equally stretched. On which, more work will have to be done?**

**[NCERT Exemp. Q. 9.15, Page 69]**

**Ans.** Work done in stretching a wire, $W = \frac{1}{2} F x \Delta l$.

[F = applied force, $\Delta l$ = extension in wire]
Springs are equally stretched, therefore, for same force (F).
$W \propto \Delta l$ ...(i)
Y(Young's modulus)

$$Y = \frac{F}{A} \times \frac{l}{\Delta l} \ or\ \Delta l = \frac{F}{A} \times \frac{l}{Y}$$

Both springs are identical,

$$\Delta l \propto \frac{1}{Y} \quad ...(ii)$$

From (i) & (ii),

$$W \propto \frac{1}{Y} \text{ or } \frac{W_{steel}}{W_{copper}} = \frac{Y_{copper}}{Y_{steel}} < 1 \ (\because Y_{steel} > Y_{copper})$$

or $W_{steel} < W_{copper}$, Hence, more work will be done for stretching copper spring.

**Q. 4. What is the Young's modulus for a perfect rigid body?**

**[NCERT Exemp. Q. 9.17, Page 69]**

**Ans.** Young's Modulus, $Y = \frac{F}{A} \times \frac{l}{\Delta l}$

A rigid body cannot be deshaped by applying any deforming force, $\therefore \Delta l = 0$.

or $$Y = \frac{F}{A} \times \frac{l}{0} = \infty$$

$\therefore$ For perfect rigid body, Young's modulus is infinity

**Q. 5. What is the Bulk modulus for a perfect rigid body?**

**[NCERT Exemp. Q. 9.18, Page 69]**

**Ans.** Bulk modulus, $K = \frac{p}{\frac{\Delta V}{V}} = \frac{pV}{\Delta V}$

As perfect rigid body does not change it's shape even after infinite force. Hence $\Delta V = 0$

or $$k = \frac{PV}{0} = \infty$$

Therefore, Bulk modulus for a perfect rigid body is infinity.

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by *l*, Another wire of the same material of length 2L and radius 2r, is pulled by a force 2f. Find the increase in length of this wire.**

**[NCERT Exemp. Q. 9.19, Page 69]**

**Ans.** Young's Modulus $= \frac{f}{A} \times \frac{L}{l}$, where $l$ is the increase in length of wire I

For I wire, $Y = \frac{f}{\pi r^2} \times \frac{L}{l}$ (i)

For II wire, the increase in length be $l'$.

Then $$Y = \frac{2f}{4\pi r^2} \times \frac{2L}{l'}$$

$$Y = \frac{f}{\pi r^2} \times \frac{L}{l'} \quad (ii)$$

From $eq^n(s)$ (i) and (ii)

$$\frac{f}{\pi r^2} \times \frac{L}{l} = \frac{f}{\pi r^2} \times \frac{L}{l'}$$

$$\therefore \quad l = l'$$

**Q. 2. A steel rod ($Y = 2.0 \times 10^{11}\ Nm^{-2}$, and $\alpha = 10^{-5}\ °C^{-1}$) of length 1 m and area of cross-section 1 $cm^2$ is heated from 0°C to 200°C without being allowed to extend or bend. What is the tension produced in the rod?**

**[NCERT Exemp. Q. 9.20, Page 70]**

**Ans.** Here, the equation of thermal expansion for linear expansion will be applied because of increase in temperature of the rod, length, increase.

$\Delta T = 200°C - 0°C = 200°C$

$\alpha = 10^{-5}\ °C^{-1}$, $l = 1$ m

Area of cross – section, $A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$

$$\therefore \frac{\Delta l}{l} = \alpha \Delta T = 10^{-5} \times 200 = 2 \times 10^{-3}$$

Tension produced in steel rod $= YA\alpha\Delta T$
$= 2.0 \times 10^{11} \times 1 \times 10^{-4} \times 2 \times 10^{-3}$
$= 4 \times 10^4$ N.

**Q. 3. To what depth must a rubber ball be taken in deep sea so that its volume is decreased by 0.1%. (The**

**bulk modulus of rubber is $9.8 \times 10^8$ Nm$^{-2}$, and the density of sea water is $10^3$ kgm$^{-3}$.)**

**[NCERT Exemp. Q. 9.21, Page 70]**

**Ans.** Bulk Modulus, K = $9.8 \times 10^8$ N/m$^2$

Density of sea water ($\rho$) = $10^3$ kg/m$^3$

Volume decrease (Percentage),

$$\left(\frac{\Delta V}{V}\times 100\right)=0.1$$

$$\frac{\Delta V}{V}=\frac{0.1}{100}$$

$$=\frac{1}{1000}=1\times 10^{-3}$$

Let rubber ball be taken up to depth h

$\therefore$ change in pressure, $P=h\rho g$

Bulk Modulus,

$$B=\left|\frac{P}{\Delta V/V}\right|=\frac{h\rho g}{\Delta V/V}$$

$$\text{or, } h=\frac{B\times(\Delta V/V)}{\rho g}$$

$$=\frac{9.8\times 10^8\times 1\times 10^{-3}}{10^3\times 9.8}$$

$$h=100 \text{ m}$$

**Q. 4. A truck is pulling a car out of a ditch by means of a steel cable that is 9.1 m long and has a radius of 5 mm. When the car just begins to move, the tension in the cable is 800 N. How much has the cable stretched? (Young's modulus for steel is $2 \times 10^{11}$ Nm$^{-2}$.)**

**[NCERT Exemp. Q. 9.22, Page 70]**

**Ans. Given:**

Steel cable's length, l = 9.1 m

Radius, $r$ = 5mm = $5 \times 10^{-3}$ m.

Tension in cable, F = 800 N

Young's modulus = $2 \times 10^{11}$ N/m$^2$

Young's modulus,

$$Y=\frac{F}{A}\times\frac{l}{\Delta l}$$

$$\text{or } \Delta l=\frac{F}{\pi r^2}\times\frac{l}{Y}$$

$$=\frac{800\times 9.1}{3.14\times(5\times 10^{-3})^2\times 2\times 10^{11}}$$

$$=4.64\times 10^{-4}\text{ m}$$

**Q. 5. A steel wire of length 4.7 m and cross-section 3.0 $\times 10^5$ m$^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-section 4.0 $\times 10^{-5}$ m$^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper ?**

**[NCERT Ex. Q. 9.1, Page 247]**

**Ans.** Using the relation,

$$Y=\frac{Fl}{A\Delta l},$$

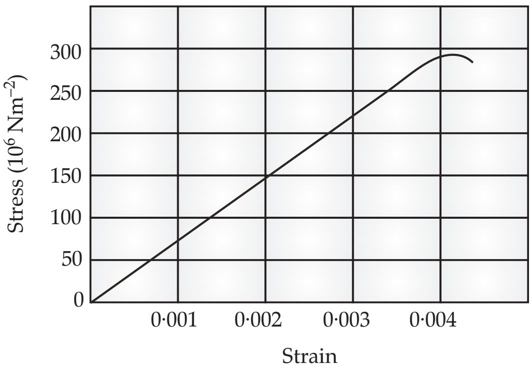
$$Y_S=\frac{Fl}{A\Delta l}=\frac{F\times 4.7}{(3\times 10^{-5})\Delta l}$$

and $$Y_C=\frac{Fl}{A\Delta l}=\frac{F\times 3.5}{(4\times 10^{-5})\Delta l}$$

then $$\frac{Y_S}{Y_C}=\frac{F\times 4.7}{(3\times 10^{-5})\Delta l}\times\frac{(4\times 10^{-5})\Delta l}{F\times 3.5}$$

$$=1.85$$

**Q. 6. Figure shows the strain-stress curve for a given material. What are (a) Young's modulus, and (b) approximate yield strength for this material ?**




**[NCERT Ex. Q. 9.2, Page 247]**

**Ans.** Put the given graph for a stress of $150 \times 10^6$ Nm$^{-2}$, the strain is 0.002.

**(a)** From the formula Young's modulus of the material (Y) is given by

$$Y=\frac{150\times 10^6}{0.002}=\frac{150\times 10^6}{0.002}=\frac{150\times 10^6}{2\times 10^{-3}}$$

$$= 75 \times 10^9 \text{ Nm}^{-2} = 7.5 \times 10^{10} \text{ Nm}^{-2}.$$

**(b)** Yield strength of a material is defined as the maximum stress it can sustain without crossing the elastic limit.

$\therefore$ From graph, the approximate yield strength of the given material

$= 300 \times 10^6$ Nm$^{-2}$ = $3 \times 10^8$ Nm$^{-2}$.

**Q. 7. The Marina Trench is located in the Pacific Ocean and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about $1.1 \times 10^8$pa. A steel ball to initial volume 0.32 m$^3$ is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?**

**Bulk Modulus for steel = $1.6 \times 10^{11}$ N/m$^2$**

**[NCERT Ad. Ex. Q. 9.21, Page 249]**

**Ans.** From question :

P = $1.1 \times 10^8$pa ,V = 0.32m$^3$, B = $1.6 \times 10^{11}$N/m$^2$

Change in volume,

$$\Delta V=\frac{PV}{B}$$

$$=\frac{1.1\times 10^8\times 0.32}{1.6\times 10^{11}}$$

$$\Delta V = 2.2 \times 10^{-4} \text{ m}^3$$

# Long Answer Type Questions (5 marks each)

**Q. 1. (a) A steel wire of mass $\mu$ per unit length with a circular cross section has a radius of 0.1 cm. The wire is of length 10 m when measured lying horizontal, and hangs from a hook on the wall. A mass of 25 kg is hung from the free end of the wire. Assuming the wire to be uniform and lateral strains << longitudinal strains, find the extension in the length of the wire. The density of steel is 7860 kg m$^{-3}$ (Young's modules Y = 2 × 10$^{11}$ Nm$^{-2}$).**

**(b) If the yield strength of steel is 2.5 × 10$^8$ Nm$^{-2}$. What is the maximum weight that can be hung at the lower end of the wire?**

**[NCERT Exemp. Q. 9.25, Page 70]**

**Ans.** Different forces are acting on the wire which is hanging, because different part of wire will be acted upon by different forces. Therefore,

Consider a small element $dx$ at a distance $x$ from the load ($x$ = 0). Let T($x$) and T ($x$ + $dx$) are tensions on the two cross sections a distance $dx$ apart.

Then

$$T(x+dx)+T(x)=dmg$$

$$=\mu g dx \text{ (where } \mu \text{ is the mass/length)}$$

$$[\because dm=\mu dx].$$

$$\frac{dT}{dx}dx=\mu g dx$$

or $dT=\mu g dx$ $\quad [\because dT=T(x+dx)-T(x)].$

or $T(x)=\mu g x+C$

At $x=0, T(0)=Mg$ or $C=Mg$

$\therefore T(x)=\mu g x+Mg$

Let the length dx at x increase by dr. then

$$\text{Young's Modulus, } Y=\frac{\text{stress}}{\text{strain}}$$

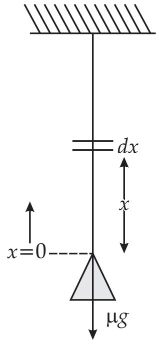
$$\frac{\frac{T(x)}{A}}{\frac{dr}{dx}}=Y$$

or $\dfrac{dr}{dx}=\dfrac{1}{YA}T(x)$

or $dr=\dfrac{1}{YA}\int_0^L(\mu g x+Mg)\,dx$

or $\dfrac{1}{YA}\left[\dfrac{\mu g x^2}{2}+Mgx\right]_0^L$

or $\dfrac{1}{YA}\left[\dfrac{mgL^2}{2}+MgL\right]$ $\quad [\because \mu L=m]$




($m$ is the mass of the wire)

$A=\pi\times(10^{-3})^2\,\text{m}^2,$

$Y=200\times10^9\,\text{Nm}^{-2}$

$m=\pi\times(10^{-3})^2\times10\times7860$ kg

$$\therefore r=\frac{1}{2\times10^{11}\times\pi\times10^{-6}}\left[\frac{\pi\times786\times10^{-3}\times10\times10}{2}+25\times10\times10\right]$$

$$=\left[196.5\times10^{-6}+3.98\times10^{-3}\right]\approx4\times10^{-3}\,\text{m}$$

**(b)** Clearly, the maximum tension would be at $x$=L

$T=\mu gL+Mg=(m+M)g \quad [\because m=\mu L]$

The yield force = (yield strength, $y$) area

= 250 × 10$^6$ × π × (10$^{-3}$)$^2$ = 250 × πN

At yield point, T=Yield Force

(M + M)g = 250 × π

m = π × (10$^{-3}$)$^2$ × 10 × 7860 << M,

∴ Mg ≈ 250 × π

Hence, $M=\dfrac{250\times\pi}{10}=25\times\pi\approx75$kg.

**Q. 2. A steel rod of length 2$l$, cross sectional area A and mass M is set rotating in a horizontal plane about an axis passing through the centre. If Y is the Young's modulus for steel, find the extension in the length of the rod. (Assume the rod is uniform.)**

**[NCERT Exemp. Q. 9.26, Page 71]**

**Ans.** Consider an element at $r$ of width dr. Let ($r$) and T ($r$ + $dr$) be the tensions at the two edges, respectively.

Net centrifugal force in element = $\omega^2 r dm$ [ω = angular velocity of rod]

= $\omega^2 r\,\mu\,dr$ $\quad$ [∴$\mu$ = mass/length]

$$-T(r+dr)+T(r)=\mu\omega^2 r dr$$

$$-\frac{dT}{dr}dr=\mu\omega^2 r dr$$

$-dT=\mu\omega^2 r dr$

∴ Tension and centrifugal force are opposite.

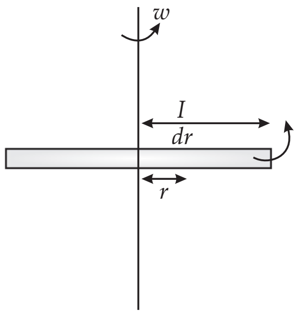
$$\therefore -\int_{T=0}^{T} dt = \int_{r=l}^{r=r} \mu\omega^2 r dr$$

$$T = \mu\omega^2 \left.\frac{r^2}{2}\right|_r^l + c$$

$$T = \frac{\mu\omega^2}{2}(l^2 - r^2) + c$$

At $r = l$, $T = 0$

$$T(r) = \frac{\mu w^2}{2}\left(l^2 - r^2\right)$$




Let the increase in length of the element $dr$ be $d\delta$ therefore, Young's Modulus,

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{T(r)}{A}}{\frac{d\delta}{dr}}$$

$$Y = \frac{\left(\mu\omega^2/2\right)\left(l^2 - r^2\right)/A}{\frac{d(\delta)}{dr}}$$

$$\therefore \frac{d\delta}{dr} = \frac{1}{YA}\frac{\mu\omega^2}{2}(l^2 - r^2)$$

$$\therefore d\delta = \frac{1}{YA}\frac{\mu\omega^2}{2}(l^2 - r^2)dr$$

Change in length, $\delta = \frac{1}{YA}\frac{\mu\omega^2}{2}\int_0^l (l^2 - r^2)dr$

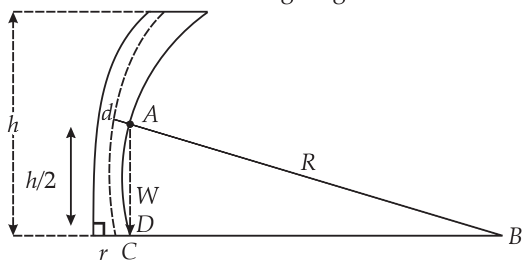
$$= \frac{1}{YA}\frac{\mu\omega^2}{2}\left[l^3 - \frac{l^3}{3}\right] = \frac{1}{3YA}\mu\omega^2 l^3$$

The total change in length is $2\delta = \frac{2}{3YA}\mu\omega^2 l^3$

**Q. 3. In nature, the failure of structural members usually results from large torque because of twisting or bending rather than due to tensile or compressive strains. This process of structural breakdown is called buckling and in cases of tall cylindrical structures like trees, the torque is caused by its own weight bending the structure. Thus the vertical through the centre of gravity does not fall within the base. The elastic torque caused because of this bending about the central axis of the tree is given by $\frac{Y\pi r^4}{4R}$. $Y$ is the Young's modulus, r is the radius of the trunk and R is the radius of curvature of the bent surface along the height of the tree containing the centre of gravity (the neutral surface). Estimate the critical height of tree for a given radius of the trunk.**

**[NCERT Exemp. Q. 9.28, Page 71]**

**Ans.** Let us consider the following diagram.




From question,

Torque on trunk of tree $= \frac{Y\pi r^4}{4R}$

[r = radius of tree
R = radius of curvature of bent surface]

When the tree is about to buckle,

$$Wd \quad \frac{Y \quad r}{}$$

If R > > $h$, then the centre of gravity is at a height $l = \frac{1}{2}h$ from the ground.

From $\Delta ABC$

$$R^2 = (R - d)^2 + \left(\frac{1}{2}h\right)^2$$

If $d$ << R

$$R^2 = R^2 - 2Rd + \frac{1}{4}h^2$$

$$\therefore d = \frac{h^2}{8R}$$

If $\varpi_0$ is the weight/volume

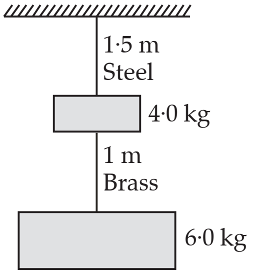
$$\frac{Y\pi r^4}{4R} = W_0(\pi r^2 h)\frac{h^2}{8R}$$

[$\because$ Torque is due to weight]

$$\text{or } h = \left(\frac{2Y}{W_0}\right)^{1/3} r^{2/3}$$

Thus, critical height, $h = \left(\frac{2Y}{W_0}\right)^{1/3} r^{2/3}$

**Q. 4. Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is $2.0 \times 10^{11}$ Pa. Compute the elongations of steel and brass wires. (1 Pa = 1 $Nm^{-2}$).**




**[NCERT Exemp. Q. 9.5, Page 248]**

**Ans.** For steel wire,

Total force, $F_1 = 4 + 6 = 10$ kgf
$= 10 \times 9.8$ N
$l_1 = 1.5$ m, $\Delta l_1 = ?$
$2r_1$ = Diameter of wire = 0.25 cm.

$$r_1 = \frac{0.25 \times 10^{-2}}{2} \text{ m}$$

$= 0.125 \times 10^{-2}$ m

$\therefore$ Area $A_1 = \pi r_1^2$.

Let $Y_1$ be the Young's modulus of steel wire,

Then $$Y_1 = \frac{F_1/A_1}{\Delta l_1/l_1}$$

or $$\Delta l_1 = \frac{F_1 l_1}{A_1 Y_1}$$

$$= \frac{(10 \times 9.8)(1.5) \times 7}{22 \times (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}}$$

$= 1.5 \times 10^{-4}$ m.

In case of brass wire

$F_2 = 6.0$ kgf $= 6 \times 9.8$ N
$Y_2 = 0.91 \times 10^{11}$ pa
$2r_2 = 0.25$ cm

$$\therefore \quad r_2 = \frac{0.25}{2}$$

$= 0.125 \times 10^{-2}$ m

$\therefore$ $l_2 = 1.0$ m, $\Delta l_2 = ?$

$$\therefore \quad \Delta l_2 = \frac{F_2 l_2}{A_2 Y_2}$$

$$= \frac{(6 \times 9.8) \times 1.0 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times 0.91 \times 10^{11}}$$

$= 1.09 \times 10^{-4}$ m.

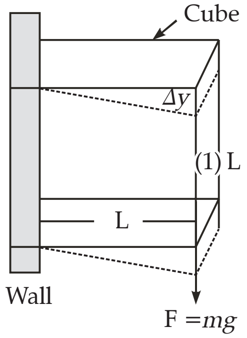
**Q. 5. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 G Pa. What is the vertical deflection of this face ?**

**[NCERT Ex. Q. 9.6, Page 248]**

**Ans. Given :** Side of cube, L = 10 cm = 0.1 m

$\therefore$ A = area of the face (1)
$= L \times L = L^2$
$= (0.1)^2$
$= 0.01$ m$^2$.

Mass attached to Face (1),
M = 100 kg




When F be the force on face (1) due to this mass. Then

F = Mg = 100 × 9.8 N.

$\therefore$ Shear stress on the face is given

$$= \frac{F}{A} = \frac{100 \times 9.8}{0.01} \text{ Nm}^{-2}$$

$= 9.8 \times 10^4$ Nm$^{-2}$

Shear modulus of aluminium,

$\eta = 25$ GPa $= 25 \times 10^9$ Nm$^{-2}$

Using the relation,

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} \quad ...(1)$$

Suppose Δy = Vertical displacement of the face

$\therefore$ Shearing strain,

$$\frac{\Delta y}{L} = \frac{\text{Shearing stress}}{\eta}$$

$$= \frac{(F/(A))}{\eta}$$

or $$\Delta y = \frac{\text{Shearing stress}}{\eta} \times L$$

$$= \frac{9.8 \times 10^4 \times 0.1}{25 \times 10^9}$$

or $\Delta y = 0.0392 \times 10^{-5}$ m

or $= 3.92 \times 10^{-7}$ m.

$\approx 4 \times 10^{-7}$ m

**Q. 6. Four identical hollow cylindrical columns of steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform. Calculate the compressional strain of each column. The Young's modulus of steel is 2.0 × 10$^{11}$ Pa ?** **[NCERT Ex. Q. 9.7, Page 248]**

**Ans.** Cross-sectional area of a column

$= \pi (r_2^2 - r_1^2)$
$= \pi (0.6^2 - 0.3^2)$
$= 0.27\pi$ m$^2$

Force on one column,

$$F = \frac{50000 \times 9.8}{4}$$

$$Y = \frac{F/A}{\Delta l/l}$$

Compressive strain

$$= \frac{\Delta l}{l} = \frac{F}{AY}$$

$$= \frac{50000 \times 9.8}{4(0.27\pi)(2 \times 10^{11})}$$

$$= 7.21 \times 10^{-7}.$$

**Q. 7. A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain ? (Y for copper = $1.1 \times 10^{11}$ $Nm^{-2}$.)**

**[NCERT Ex. Q. 9.8, Page 248]**

**Ans. Given :** $Y = 1.1 \times 10^{11}\ Nm^{-2}$

A = Area of cross-section

= 15.2 mm × 19.1 mm

$= 15.2 \times 10^{-3}$ m $\times 19.1 \times 10^{-3}$

Force, F = 44500 N.

Resulting Strain = Longitudinal Strain = ?

$$\therefore \quad Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{or} \quad \text{Strain} = \frac{\text{Stress}}{Y} = \frac{F}{AY}$$

or Longitudinal Strain

$$= \frac{44500}{15.2 \times 19.1 \times 10^{-6} \times 1.1 \times 10^{11}}$$

$$= 3.65 \times 10^{-3}.$$

**Q. 8. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension. Given Young modulus of copper and iron are $110 \times 10^9$ $Nm^{-2}$ and $190 \times 10^9$ $Nm^{-2}$**

**[NCERT Ex. Q. 9.10, Page 248]**

**Ans.** Suppose $Y_1$ and $Y_2$ be the Young's modulus of copper and iron wires respectively

$$Y_1 = 110 \times 10^9\ Nm^{-2}$$

$$Y_2 = 190 \times 10^9\ Nm^{-2}$$

Also suppose $a_1$ and $a_2$ be the areas of cross-section of copper and iron wires respectively. If $d_1$ and $d_2$ be their respective diameters.

Then $A_1 = \pi d_1^2/4$

and $A_2 = \pi d_2^2/4.$

$$\therefore \quad \frac{A_1}{A_2} = \frac{d_1^2}{d_2^2} = \left(\frac{d_1}{d_2}\right)^2$$

L = 2 m.

Suppose $\Delta l$ be the extension produced in each wire.

Suppose F = Tension produced in each wire

∴ Using relation,

$$Y = \frac{\text{Stress}}{\text{Strain}},$$

we get, Strain for copper wire $= \frac{F/A_1}{Y_1}$

and Strain for iron wire $= \frac{F/A_1}{Y_2}$

Since, the bar is supported symmetrically,

∴ The two strains are equal

$$\therefore \quad \frac{F}{A_1Y_1} = \frac{F}{A_2Y_2}$$

$$\text{or} \quad A_1Y_1 = A_2Y_2$$

$$\frac{A_1}{A_2} = \frac{Y_2}{Y_1}$$

$$\frac{\pi d_1^2/4}{\pi d_2^2/4} = \frac{Y_2}{Y_1}$$

$$\frac{d_1}{d_2} = \sqrt{\frac{Y_1}{Y_2}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}}$$

$$= \sqrt{\frac{19}{11}} = \sqrt{1.73}$$

It means, $\frac{d_1}{d_2} = 1.31$

or $d_1 : d_2 = 1.31 : 1.$

**Q. 9. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 $cm^2$. Calculate the elongation of wire when the mass is at the lowest point of its path. $Y_{steel} = 2 \times 10^{11} N/m^2$**

**[NCERT Ex. Q. 9.10, Page 248]**

**Ans.** Given: $m$ = 14.5 kg, $l$ = r = 1m

$\omega$ = 2 rps = 2 × 2 × rad/s

$A = 0.065 \times 10^{-4}\ m^2$

Tension in wire at lowest position on vertical circle = F = $mg + mr\omega^2$

$$= 14.5 \times 9.8 + 14.5 \times 1 \times 4 \times \frac{22}{7} \times \frac{22}{7} \times 4$$

= 142.1 + 2291.6

= 2433.7 N

$$Y = \frac{Fl}{A\Delta l}$$

$$\text{or} \quad \Delta l = \frac{Fl}{AY}$$

$$= \frac{2433.7 \times 1}{0.065 \times 10^{-4} \times 2 \times 10^{11}}$$

$$= 1.87 \times 10^{-3}\ \text{m}$$

$$\Delta l = 1.87\ mm$$

**Q. 10. Compute the bulk modulus of water from the following data : Initial volume = 100 litre, pressure increase = 100 atmosphere, final volume =100.5 litre (1 atmosphere = $1.013 \times 10^5$ pa). Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.**

**[NCERT Ex. Q. 9.10, Page 248]**

**Ans.** Bulk Modulus, $B = \frac{-P}{\frac{\Delta v}{v}}$

$$= P\frac{v}{\Delta v}$$

$$= \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$$

or $B_w = 2.026 \times 10^9 \, Pa$

Bulk Modulus of air, $B_A = 1.0 \times 10^5$ Pa

$$\therefore \frac{\text{Bulk Modulus of water, } B_w}{\text{Bulk Modulus of air, } B_A} = \frac{2.026 \times 10^9}{1.0 \times 10^5} = 2.026 \times 10^4$$

This ratio is too large because gases are more compressible than those of liquids. Interatomic forces are more strong in liquids than gases.

**Q. 11. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03 × 10³ kg m⁻³ ? Compressibility of water is 45.8 × 10⁻¹¹ Pa⁻¹ and 1 atm = 1.03 × 10⁵ Pa.**

**[NCERT Ex. Q. 9.13, Page 248]**

**Ans.** Compressibility $= \frac{1}{K} = (45.8 \times 10^{-11}) \text{ Pa}^{-1}$

Pressure, **P** = 80 atm

$= 80 \times 1.013 \times 10^5$ Pa

Let change in volume,

$$\Delta V = \frac{M}{\rho} - \frac{M}{\rho_d} = M\left(\frac{1}{\rho} - \frac{1}{\rho_d}\right)$$

∴ Volumetric strain

$$= \frac{\Delta V}{V}$$

$$= M\left[\frac{1}{\rho} - \frac{1}{\rho_d}\right]\frac{\rho}{M} = 1 - \frac{\rho}{\rho_d}$$

where ρ is density of water at surface and $\rho_d$ is the density of water at depth.

$$\frac{\Delta V}{V} = \frac{1}{K}$$

$$1 - \left(\frac{1.03 \times 10^3}{\rho_d}\right) = 80 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11}$$

$$\rho_d = 1.034 \times 10^3 \text{ kgm}^{-3}.$$

**Q. 12. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.**

**[NCERT Ex. Q. 9.14, Page 248]**

**Ans.** Given : P = 10 atm

$= 10 \times 1.013 \times 10^5$ Pa

we know that,

K for glass slab $= 37 \times 10^9 \text{ Nm}^{-2}$.

Fractional change in volume of glass slab

$$= \frac{\Delta V}{V} = ?$$

From the relation

$$K = \frac{P}{\frac{\Delta V}{V}},$$

$$\frac{\Delta V}{V} = \frac{P}{K} = \frac{10 \times 1.013 \times 10^5}{37 \times 10^9}$$

$$= \frac{1.013}{37 \times 10^9} = 0.0274 \times 10^{-3}$$

$$= 2.74 \times 10^{-5}$$

$$\approx 0.003\%.$$

**Q. 13. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7 × 10⁶ Pa. K for copper = 140 × 10⁹ Pa.**

**[NCERT Ex. Q. 9.15, Page 248]**

**Ans. Given :** **L** = 10 cm = 0.1 m

K = bulk modulus of Cu

$= 140 \times 10^9$ Pa

$P = 7 \times 10^6$ Pa

ΔV = Volume contraction of solid copper cube = ?

∴ $V = L^3 = (0.1)^3 = 0.001 \text{ m}^3$.

Using formula, $K = -\frac{-\Delta\rho}{\left(\frac{\Delta V}{V}\right)}$

we get $\Delta V = -\frac{PV}{K} = \frac{7 \times 10^6 \times 0.001}{140 \times 10^9} \text{ m}^3$

$$= -\frac{1}{20} \times 10^{-6} \text{ m}^3$$

$$= -0.05 \times 10^{-6} \text{ m}^3$$

$$= -5 \times 10^{-2} \text{ cm}^3$$

Here negative sign shows volume contraction.

**Q. 14. How much should the pressure on a litre of water be changed to compress it by 0.10%.**

**[NCERT Ex. Q. 9.16, Page 248]**

**Ans.** Here V = 1 litre

ΔV = – 0.10% of V

$$= -\frac{0.10}{100} \times 1 = -\frac{1}{1000} \text{ litre}$$

Suppose ΔP = change in pressure required for compression of 1 litre of water.

K = bulk modulus of water $= 2.2 \times 10^9 \text{ Nm}^{-2}$

From the relation

$$K = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

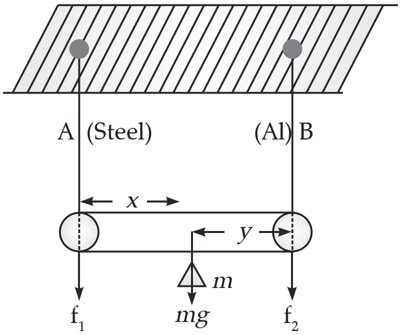
we get $\Delta P = -K.\frac{\Delta V}{V}$

or $\Delta P = 2.2 \times 10^9 \times \frac{\left(\frac{1}{1000}\right)}{1}$

or $\Delta P = \frac{2.2 \times 10^9}{1000}$

$$= 2.2 \times 10^6 \text{ Nm}^{-2}.$$

**Q. 15. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in figure. The cross-sectional areas of wires A and B are 1 mm$^2$ and 2 mm$^2$ respectively. At what point along the rod should a mass *m* be suspended in order to produce (a) equal stresses, and (b) equal strains in both steel and aluminium wires. Given Y for steel = $2 \times 10^{11}$ Pa and for Al = $7 \times 10^{10}$ Pa.**




**[NCERT Ad. Ex. Q. 9.18, Page 249]**

**Ans.** Suppose L = length of each of the two wires A and B made of steel and aluminium respectively.

Also suppose $A_1$, $A_2$ be the areas of cross-section of A and B respectively.

$\therefore$ $A_1 = 1 \text{ mm}^2 = (10^{-3})^2$

$= 10^{-6} \text{ m}^2$

$A_2 = 2 \text{ mm}^2 = 2 \times 10^{-6} \text{ m}^2$

$Y_1$ for Steel $= 2 \times 10^{11}$ Nm$^{-2}$

$Y_2$ for Aluminium $= 7 \times 10^{10}$ Nm$^{-2}$

Suppose $F_1$ and $F_2$ be the forces acting at their lower ends.

**(a)** $\therefore$ Stresses on A and B are $\frac{F_1}{A_1}$ and $\frac{F_2}{A_2}$ respectively.

If these stresses are equal, then

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

or $$\frac{F_1}{F_2} = \frac{A_1}{A_2} \quad ...(1)$$

Suppose $F_1$ and $F_2$ be produced by a weight mg suspended from the rods at a distance *x* and *y* from the two rods. Then we have

$$F_1 x = F_2 y$$

or $$\frac{F_1}{F_2} = \frac{y}{x} \quad ...(2)$$

$\therefore$ From equations (1) and (2),

$$\frac{y}{x} = \frac{A_1}{A_2}$$

or $$x = \frac{A_2}{A_1} y \quad ...(3)$$

Now $x + y = 1.05$ m (length of the rod)

$\therefore$ $y = 1.05 - x \quad ...(4)$

$\therefore$ From equations (3) and (4),

$$x = \frac{A_2}{A_1}(1.05 - x)$$

or $$A_1 x = 1.05 A_2 - x A_2$$

or $$x(A_1 + A_2) = 1.05 A_2$$

or $$x = \frac{1.05 A_2}{A_1 + A_2} = \frac{1.05 \times 2 \times 10^{-6}}{(1+2) \times 10^{-6}}$$

$$= \frac{1.05 \times 2}{3} = 0.70 \text{ m}$$

or $y = 1.05 - 0.70 = 0.35$ m

So, the mass *m* must be suspended at a distance of 0.70 m from A (steel wire) or 0.35 m from B.

**(b)** From the relation,

$$Y = \frac{\text{Stress}}{\text{Strain}},$$

or $$\text{Strain} = \frac{\text{Stress}}{Y}$$

for wire A, $$\text{Strain} = \frac{F_1/A_1}{Y_1}$$

and for wire B, $$\text{Strain} = \frac{F_2/A_2}{Y_2}$$

As the strains are equal.

Hence $$\frac{F_1}{A_1 Y_1} = \frac{F_2}{A_2 Y_2}$$

or $$\frac{F_2}{F_1} = \frac{A_2 Y_2}{A_1 Y_1} \quad (5)$$

Also from equation (2), we get

$$\frac{F_2}{F_1} = \frac{x}{y} \quad (6)$$

$\therefore$ From equation (5) and (6),

$$\frac{x}{y} = \frac{A_2 Y_2}{A_1 Y_1}$$

or $$x = \frac{A_2 Y_2}{A_1 Y_1}(1.05 - x) \; [\because y = 1.05 - x]$$

by using eqn. (4),

or $$A_1 Y_1 x = A_2 Y_2 \times 1.05 - A_2 Y_2 x$$

or $$x(A_1 Y_1 + A_2 Y_2) = 1.05 A_2 Y_2$$

$\therefore$ $$x = \frac{1.05 A_2 Y_2}{A_1 Y_1 + A_2 Y_2} \quad ...(7)$$

Putting the values of $Y_1$, $Y_2$ and $A_1$, $A_2$ from above in equation (7),

$$x = \frac{1.05 \times 2 \times 10^{-6} \times 7 \times 10^{10}}{10^{-6} \times 2 \times 10^{11} + 2 \times 10^{-6} \times 7 \times 10^{10}}$$

$$x = \frac{1.05 \times 14 \times 10^4}{2 \times 10^5 + 1.4 \times 10^5} \text{ m}$$

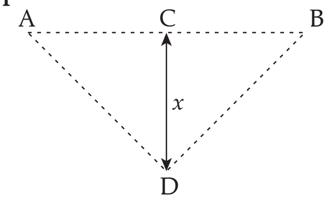
$$x = \frac{1.05 \times 14 \times 10^4}{3.4 \times 10^5} \text{ m}$$

$x = \frac{1.05 \times 14}{34}$ m = 0.43 m

$\therefore\ y = 1.05 - x = 1.05 - 0.43$
$= 0.62$ m.

It means that the mass $m$ must be suspended at a distance of 0.43 m from wire A.

**Q. 16. A mild steel wire of length 1 m and cross-sectional area 0.5 × 10$^{-2}$ cm$^2$ is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid point of the wire. Calculate the depression at the mid point.**




**[NCERT Ad. Ex. Q. 9.19, Page 249]**

**Ans.** From the above figure,

Let $x$ be the depression at the mid point i.e. $CD = x$ in. fig.,

$AC = CB = l = 0.5$ m;

$m = 100$ g $= 0.100$ Kg

$AD = BD = \sqrt{l^2 + x^2}$

Increase in length, $\Delta l$ = AD + DB – AB = 2AD – AB

$$\Delta l = 2\sqrt{l^2 + x^2} - 2l$$

$$\Delta l = 2l\left(1 - \frac{x^2}{l^2}\right)^{\frac{1}{2}} - 2l$$

$$\Delta l = 2l\left(1 + \frac{x^2}{2l^2}\right) - 2l = \frac{x^2}{l}$$

$$\therefore \quad \sin\theta = \frac{\Delta l}{2l} = \frac{x^2}{2l^2} = \frac{\text{increase in length}}{\text{orginal length}}$$

If T is the tension in the wire, then

$2T\cos\theta = mg$

or, $T = \frac{mg}{2\cos\theta}$

Here,

$$\cos\theta = \frac{x}{(l^2 + x^2)^{1/2}} = \frac{x}{l\left(1 + \frac{x^2}{l^2}\right)^{1/2}} = \frac{x}{l\left(1 + \frac{1}{2}\frac{x^2}{l^2}\right)}$$

As, $x << l$, so $1 >> \frac{x^2}{2l^2}$ and $1 + \frac{x^2}{2l^2} = 1$

$$\therefore \quad \cos\theta = \frac{x}{l}$$

$$\text{Hence, } T = \frac{mg}{2\left(\frac{x}{l}\right)} = \frac{mgl}{2x}$$

$$\text{Stress} = \frac{T}{A} = \frac{mgl}{2Ax}$$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{mgl}{2Ax} \times \frac{2l^2}{x^2} = \frac{mgl^3}{Ax^3}$$

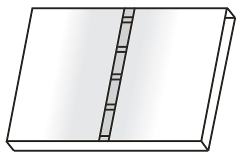
$$x = l\left[\frac{mg}{YA}\right]^{1/3} = 0.5\left[\frac{0.1 \times 10}{20 \times 10^{11} \times 0.5 \times 10^{-6}}\right]^{1/3}$$
$$= 0.01074\,\text{m}$$

**Q. 17. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9 × 10$^7$ Pa ? Assume that each rivet is to carry one quarter of the load.**

**[NCERT Ad. Ex. Q. 9.20, Page 249]**

**Ans.** If the riveted strip is subjected to a stretching load W, the tensile force (*i.e.*, tension) in each strip (equal to W) provides the shearing force on the four rivets. Since the load is shared uniformly, it means each rivet is under a shearing force equal to $\frac{W}{4}$.

Maximum shearing stress on each rivet
$= 6.9 \times 10^7$ Pa



Suppose A = area of each rivet on which the shearing force acts.

$\therefore$ Shearing stress on each rivet

$$= \frac{\text{Shearing force}}{\text{area}}$$

$$= \frac{W/4}{A} = \frac{W}{4A}$$

When $W_{max.}$ be the maximum permissible tension or load (*i.e.*, force) exerted by the riveted strip, then

$$\frac{W_{max.}}{4A} = 6.9 \times 10^7.$$

$\Rightarrow W_{max.} = 4A \times 6.9 \times 10^7$ ...(1)

**Given :** Diameter of each rivet,

D = 6 mm
$= 6 \times 10^{-3}$ m

$$\therefore \quad A = \frac{\pi D^2}{4}$$

$$= \frac{3.142 \times (6 \times 10^{-3})^2}{4} \quad ...(2)$$

$\therefore$ From equations (1) and (2),

$$W_{max} = 4 \times \frac{3.142 \times (6 \times 10^{-3})^2}{4} \times 6.9 \times 10^7$$

$= 7804.73$ N
$= 7.8 \times 10^3$ N.

**TIPS... & TRICKS...**

- *Understand stress and strain and their types.*
- *Behaviour of all substances between elastic and plastic.*
- *Learn about elastic limit.*
- *Longitudinal and shearing strain properties of solid and volumetric strain property of liquid and gas.*
- *Study and understands about Young's Modulus, Bulk Modulus and Modulus of rigidity.*
- *Learn formula relation between young's Modulus (Y) Bulk Modulus (B), Modulus of rigidity (η) and Poisson ratio (σ).*
- *Know that the Young Modulus (Y), Modulus of rigidity (η) and Poisson ratio (σ) for solid Material. But Bulk Modulus (B) and compressibility for liquid and gases.*

## Some Commonly Made Errors

- Sometimes students make mistake by considering stress a vector quantity.
- The concept of hydraulic compression is not clearly understand by students.

### EXPERT ADVICE

- *Hooke's law is valid only in the linear part of stress-strain curve.*
- *The young's modulus and shear modulus are relevant only for solids while bulk modulus is relevant for*
- *Remember, stress is not a vector quantity. Since, unlike a force, the stress cannot be assigned a specific direction.*

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CHAPTER

# 10 MECHANICAL PROPERTIES OF FLUIDS

**Chapter Objective**

**This chapter will help you understand :**

- *Pressure due to Fluid Column. Pascal's law and its applications (hydraulic lift, hydraulic brakes), effect of gravity on fluid pressure.*
- *Surface energy and surface tension, Angle of contact, Excess pressure across a curved surface, Application of surface Tension (ideas of drops' bubbles and capillary rise).*
- *Viscosity, Stocks' law, Terminal velocity, Streamline and turbulent flow, Critical velocity, Bernoulli's Theorem and its applications.*

## TOPIC-1
## Fluids at Rest



### Quick Review

- **Pressure :**

**(i)** Pressure is defined as the thrust acting per unit area of the surface in contact with liquid, *i.e.*,

$$P = \frac{\text{Thrust(F)}}{\text{Area(A)}} = \frac{F}{A} = h\rho g$$

**(ii)** Liquid pressure is independent of shape of the liquid surface as well as area of the liquid surface, but depends upon height of liquid column.

**(iii)** Total pressure at a depth $h$ below liquid surface is $P = h\rho g + P_0$, where $P_0$ is the atmospheric pressure.

**(iv)** S.I. unit of pressure is $Nm^{-2}$ or pascal (denoted by Pa) and its dimensional formula is $[ML^{-1}T^{-2}]$.

**(v)** Pressure is a scalar quantity because a liquid at rest exerts equal pressure in all directions at all points in the same horizontal plane.

- **Pascal's Law :** It **s**tates that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same. Pascal's law also states that the increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of liquid provided the gravity effect is neglected.

- **Atmospheric pressure :**

**(i)** It is defined as the pressure exerted by atmosphere.

**(ii)** At S.T.P., the value of atmospheric pressure is $1.01 \times 10^5$ $Nm^{-2}$ or $1.01 \times 10^6$ dyne/cm$^2$.

- **Archimedes' principle :** It states that when a body is immersed partly or wholly in a liquid at rest, it loses some of its weight, which is equal to the weight of the liquid displaced by the immersed part of the body.

Observed weight of body = True weight – Weight of liquid displaced.

If $W$ is the observed weight of body of density ρ when immersed in a liquid of density σ, then

$$\begin{aligned} w &= Mg - mg \\ &= Ah\rho g - Ah\sigma g \\ &= Ahg(\rho - \sigma) \\ &= Ahg\rho\left(1 - \frac{\sigma}{\rho}\right) = W\left(1 - \frac{\sigma}{\rho}\right) \end{aligned}$$

∴ True weight, $$W = \frac{\text{apparent weight}}{(1-\sigma/\rho)}$$

- **Laws of floatation :** It states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to or greater than the weight of the body.
  - **(a)** When true weight of the body $W > w$ (weight of the liquid displaced), the body will sink to the bottom of the liquid. It will be so when the density of solid body (ρ) is greater than the density of liquid (σ), *i.e.* $\rho > \sigma$.
  - **(b)** When $W < w$, the body will rise above the surface of liquid to such an extent that the weight of the liquid displaced by immersed part of the body (*i.e.* upward thrust) becomes equal to the weight of the body. The body then will float. In this case the density of solid body is less than the density of liquid, *i.e.*, $\rho < \sigma$.
  - **(c)** When $W = w$, the body is at rest anywhere in the liquid. The body will float with its whole volume just immersed in the liquid. In this case the density of body is equal to density of liquid, *i.e.*, $\rho = \sigma$.

  There will be equilibrium of floating body when
  - **(i)** Weight of liquid displaced by the immersed part of body is equal to the weight of the body.
  - **(ii)** The centre of gravity of the body and the centre of buoyancy lie along the same vertical line.
  - **(iii)** If the centre of gravity of the body lies vertically below the meta centre then body is in stable equilibrium.

  The body will be in unstable equilibrium if centre of gravity lies vertically above the meta centre.

## Know the Terms

- **Fluid** is the name given to a substance which begins to flow when external force is applied on it.
- **Thrust :** The total normal force exerted by liquid at rest on a given surface in contact with it is known as thrust of liquid on that surface. It is due to collision of molecules of liquid while moving at random, with the walls of the container and rebounding from them.
- **Density** of a substance is defined as the mass per unit volume of substance.
- **Relative density** is defined as the ratio of its density of the substance to the density of water at 4°C.
- **Buoyancy** is the upward force acting on the body immersed in a fluid.
- **Inter-molecular forces** is the forces between the molecules of substances.
- **Adhesive force :** is the force of attraction acting between molecules of two different materials. For example, the force acting between the molecules of water and glass.
- **Cohesive force :** is the force of attraction acting between molecules of the same material. For example, the force acting between the molecules of water or mercury etc.
- **Metacentre :** is a point where the vertical line passing through the centre of Buoyancy intersects the central line.
- **Inter-molecular Binding Energy** of a liquid is the minimum energy required to separate two molecules of a liquid from each others influence.

## Know the Formulae

- Pressure $= \frac{F}{A} = h\rho g$

  $h$ = height, ρ = Density of liquid, $g$ = Acceleration due to gravity.
- Gauge pressure = Total pressure – Atmospheric pressure
- Hydraulic lift $= \frac{F_1}{A_1} = \frac{F_2}{A_2}$

  $F_1$, $F_2$ = Forces on pistons of area of cross – sections $A_1$, $A_2$
- Density $= \frac{\text{Mass}}{\text{Volume}}$, Relative density $= \frac{\text{Density of substance}}{\text{Density of water at 4°C}}$
- **Archimedes' Principle :**

  Loss of weight of body in liquid = Weight of liquid displaced = Volume × Density of liquid × $g$
- **Law of floatation :**

  A body will float if, weight of body = Weight of liquid displaced.

## Know the Links

- *www.vedantu.com*
- *www.learncbse.in*
- *www.toppers.com*

## MCQ/Fillups/True or False

**(1 mark each)**

**(A) Multiple Choice Questions**

**Q. 1. Pressure is a scalar quantity, because**

**(a) it is the ratio of force to area and both force and area are vectors.**

**(b) it is the ratio of the magnitude of the force to area.**

**(c) it is the ratio of the component of the force normal to the area.**

**(d) it does not depend on the size of the area chosen.**

**[NCERT Exemp. Q. 10.7, Page 74]**

**Ans. Correct option:** (b) and (c)

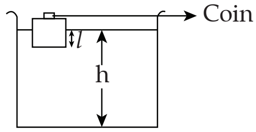
*Explanation:*

**(b)** It is a scalar quantity which requires only its magnitude for its complete description.

**(c)** $P = \frac{F}{A} = \frac{\text{Normal force}}{\text{Area}}$

**Q. 2. A wooden block with a coin placed on its top, floats to water as shown in figure.**

**The distance $l$ and $h$ are shown in the figure. After some time the coin falls into the water. Then**




**(a) $l$ decreases.** **(b) $h$ decreases.**

**(c) $l$ increases.** **(d) $h$ increases.**

**[NCERT Exemp. Q. 10.8, Page 74]**

**Ans. Correct option:** (a) and (b)

*Explanation:*

**(a)** When coin falls into water, block rises up $\therefore l$ decreases

**(b)** When coin is in water, volume of water displaced by coin is equal to the volume of coin $V_1$ (say). When coin is at the top of wooden block, it displaces volume of water ($V_2$), Which is more than $V_1$, because weight of coin = weight of volume of water displaced by coin

or $\rho_c V_1 g = \rho_l V_2 g \Rightarrow \frac{V_2}{V_1} = \frac{\rho_c}{\rho_l} \therefore \rho_c > \rho_l$

$\therefore V_2 > V_1$

Hence $h$ decreases.

## Very Short Answer Type Questions

**(1 mark each)**

**Q. 1. Iceberg floats in water with part of it submerged. What is the fraction of the volume of iceberg submerged if the density of ice is $P_i$ = 0.917 g cm$^{-3}$ ?**

**[NCERT Exemp. Q. 10.13, Page 74]**

**Ans.** Let the volume of iceberg be $V_i$ and volume of water displaced by iceberg be $V_w$ in floating condition. Weight of iceberg = Weight of water displaced by submerged ice.

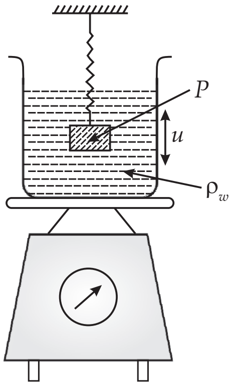
$\rho_w V_w g = \rho_i V_i g$ [$\because$ W = $mg$ = V$\rho g$]

$$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{0.917}{1} = 0.917.$$

**Q. 2. A vessel filled with water is kept on a weighing pan and the scale adjusted to zero. A block of mass $M$ and density $\rho$ is suspended by a massless spring of spring constant $k$. This block is submerged inside the water in the vessel. What is the reading of the scale?**

**[NCERT Exemp. Q. 10.14, Page 74]**

**Ans.** Considering the diagram below. Beaker is placed on weighing pan which is filled with water and then scale is adjusted to zero. When block is submerged into water the buoyant force acts on block by water.




By Newton's III law, this buoyant force acts as reaction force, so block will apply force downward due to which reading on scale increases.

This is equal to buoyant force = $V\rho_w g$

$\therefore$ Reading of Weighing scale $= \dfrac{M}{\rho} \cdot \rho_w g$

$= \dfrac{\rho_w}{\rho} \cdot Mg = \rho_w V_g$

**Q. 3. Explain why**

**(a) The blood pressure in human is greater at the feet than at the brain. Why ?**

**(b) Atmospheric pressure at a height of about 6 km decreases to nearly half its value at the sea level, though the 'height' of the atmosphere is more than 100 km. Why ?**

**(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area, and force is a vector. Why ?**

**[NCERT Ex. Q. 10.1, Page 272]**

**Ans. (a)** The height of the blood column in the human body is more at the feet than at the brain. That is why, the blood exerts more pressure at the feet than at the brain ($\because$ Pressure = hρg).

**(b)** The density of air is maximum near the surface of earth and decreases rapidly with height and at a height of about 6 km, it decreases to nearby half its value at the sea level. Beyond 6 km height the density of air decreases very slowly with height. Due to this reason the atmospheric pressure at the height of about 6 km decreases to nearly half of its value at sea level.

**(c)** Since, due to applied force on liquid, the pressure is transmitted equally in all directions inside the liquid. That is why there is no fixed direction for pressure due to liquid. Hence hydrostatic pressure is scalar quantity.

**Q. 4. Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases ? If so, why do the vessels filled with water to that same height give different readings on weighing scale ?**

**[NCERT Ex. Q. 10.23, Page 274]**

**Ans. (a)** Pressure on the basis of the two vessels is same because the vessels have been filled to common height.

Since having same area of bases of both vessels face same force.

**(b)** Vessels of different shape may have different non-zero vertical components so the vessels of different shape filled to the same height give different readings.

## Short Answer Type Questions

**(2 or 3 marks each)**

**Q. 1. A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the relative density of spirit ?**

**[NCERT Ex. Q. 10.9, Page 275]**

**Ans.** Since the mercury columns in the two arms are at level

$$h_1\rho_1 g = h_2\rho_2 g$$

or $$\rho_2 = \frac{h_1\rho_1}{h_2}$$

Hence here, $h_1 = 10$ cm, $h_2 = 12.5$ cm

and $\rho_1 = 1$ g cm$^{-3}$.

$$\rho_2 = \frac{10 \times 1}{12.5} = 0.8 \text{ g cm}^{-3}.$$

Relative density of spirit = 0.8

**Q. 2. In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms ? (Relative density of mercury = 13.6.)**

**[NCERT Ex. Q. 10.10, Page 273]**

**Ans.** Here pressure exerted by (10 + 15 = 25 cm) of water column = pressure exerted by $h$ cm of mercury column + pressure exerted by (12.5 + 15 = 27.5 cm) of spirit column.

*i.e.,* $$25 \times 1 \times g = h \times 13.6 \times g + 27.5 \times 0.8 \times g$$ (Using P = $h\rho g$)

*i.e.,* $$25 = 13.6\,h + 22$$

$$h = \frac{25 - 22}{13.6} = 0.22 \text{ cm.}$$

**Q. 3. A 50 kg of girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor ?**

**[NCERT Ex. Q. 10.5, Page 272]**

**Ans.** $$P = \frac{\text{Force}}{\text{Area}} = \frac{mg}{\pi(D/2)^2}$$

$$= \frac{4mg}{\pi D^2}$$

$$= \frac{4 \times 50 \times 9.8}{\left(\frac{22}{7}\right) \times \left(10^{-2}\right)^2}$$

$$= 6.2 \times 10^6 \text{ Pa.}$$

**Q. 4. Torricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m$^{-3}$. Determine the height of the wine column for normal atmospheric pressure.**

**[NCERT Ex. Q. 10.6, Page 273]**

**Ans.** $$P = 0.76 \times (13.6 \times 10^3) \times 9.8$$
$$= h \times 984 \times 9.8$$

or $$h = \frac{0.76\times13.6\times10^3\times9.8}{984\times9.8}$$
$$= 10.5 \text{ m}$$

**Q. 5. A vertical off shore structure is built to withstand a maximum stress of $10^9$ Pa. Is the structure suitable for putting up on top of an oil well in Bombay High? Take the depth of the sea to be roughly 3 km, and ignore the ocean currents.**

**[NCERT Ex. Q. 10.7, Page 273]**

**Ans. Given :** Maximum stress = $10^9$ Pa

$h = 3 \text{ km} = 3 \times 10^3 \text{ m}$

$P_{water} = 10^3 \text{ kg/m}^3$, $g = 9.8 \text{ m/s}^2$.

This structure will be suitable, provided the pressure exerted by sea water is less than maximum stress if can bear.

Pressure due to sea water, P = hρg

$= 3 \times 10^3 \times 10^3 \times 9.8$

$= 2.94 \times 10^7$ Pa

The pressure of sea water is less than the maximum stress of $10^9$Pa, the structure will be suitable for putting upon tap of oil well.

**Q. 6. A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm². What maximum pressure would the smaller piston have to bear ?**

**[NCERT Ex. Q. 10.8, Page 273]**

**Ans. Given :** The maximum force which the bigger piston can bear,

F = 3000 kgf

= 3000 × 9.8 N

= 29400 N

Area of piston, A = 425 cm²

$= 425 \times 10^{-4} \text{ m}^2$.

When P = maximum pressure on the bigger piston

Then $$P = \frac{F}{A} = \frac{29400}{425\times10^{-4}}$$

$= 6.92 \times 10^5$ Pa

As the liquid transmits pressure equally in all directions, hence the maximum pressure the smaller piston can bear is $6.92 \times 10^5$ Pa.

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. (a) Pressure decreases as one ascends the atmosphere. If the density of air is ρ, what is the change in pressure dp over a differential height dh?**

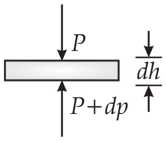
**(b) Considering the pressure p to be proportional to the density, find the pressure p at a height h if the pressure on the surface of the earth is $p_0$.**

**(c) If $p_0 = 1.03 \times 10^5 \text{ Nm}^{-2}$, $\rho_0 = 1.29 \text{ kgm}^{-3}$ and $g = 9.8 \text{ ms}^{-2}$, at what height will the pressure drop to (1/10) the value at the surface of the earth?**

**(d) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model.**

**[NCERT Exemp. Q. 10.21, Page 75]**

**Ans. (a)** Consider a horizontal parcel of air with cross section A and height *dh*. Let the pressure on the top surface and bottom surface be *p* and *p* + *dp*. If the parcel is in equilibrium, then the net upward force must be balanced by the weight.




i.e. $(p + dp) - pA = -\rho g A dh$ [such that, Weight = Density × Volume × g]

or $dp = -\rho g dh$ [here, ρ = density of air]

(–) ve sign indicates pressure decreases with height.

**(b)** Let the density of air on the earth's surface be $\rho_0$, then

$$\frac{p}{p_0} = \frac{\rho}{\rho_0}$$

or $$\rho = \frac{\rho_0}{p_0}p$$

$$\therefore dp = -\frac{\rho_0 g}{p_0}p\,dh \quad \text{[such that, } dp = -\rho g dh]$$

or $$\frac{dp}{p} = -\frac{\rho_0 g}{p_0}dh$$

or $$\int_{p_0}^{p}\frac{dp}{p} = -\frac{\rho_0 g}{p_0}\int_0^h dh \quad \begin{matrix}[\text{at } h = 0, r = p_0] \\ [\text{at } h = h, p = p]\end{matrix}$$

or $$ln\frac{p}{p_0} = -\frac{\rho_0 g}{p_0}h$$

Taking antilog,

or $$p = p_0 \exp\left(-\frac{\rho_0 g}{p_0}h\right)$$

**(c)** $$ln\frac{p}{p_o} = -\frac{\rho_o gh}{\rho_o}$$

$$ln\frac{1}{10} = -\frac{\rho_0 g}{p_0}h_0$$

$$\therefore h_o = -\frac{p_o}{\rho_0 g}\,ln\frac{1}{10} = \frac{-p_o}{\rho_0 g}ln(10)^{-1} = \frac{p_o}{\rho_0 g}\text{In}\,10$$

$$= \frac{p_0}{\rho_0 g}\times2.303$$

$$= \frac{1.013\times10^5}{1.29\times9.8}\times2.303 = 0.18\times10^5\text{m} = 18\times10^3\text{m}$$

**(d)** The assumption $p \propto \rho$ is valid only for the isothermal case which is only valid for small distances.

Temperature remains constant only near the surface of the earth, not at greater heights

**Q. 2. During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein ?**

**Densities of some common fluids at STP**

| Fluid | $\rho$ ($kgm^{-3}$) |
|---|---|
| Water | $1.00 \times 10^3$ |
| Sea water | $1.03 \times 10^3$ |
| Mercury | $1.36 \times 10^3$ |
| Ethyl alcohol | $0.806 \times 10^3$ |
| Whole Blood | $1.06 \times 10^3$ |
| Air | 1.29 |
| Oxygen | 1.43 |
| Hydrogen | $9.0 \times 10^{-2}$ |
| Interstellar space | $\approx 10^{-22}$ |

**[NCERT Ad. Ex. Q. 10.24, Page 275]**

**Ans. Given :** Gauge pressure,

$$P = 2000 \text{ Pa.}$$

Density of whole blood,

$$\rho = 1.06 \times 10^3 \text{ kg m}^{-3}$$
$$g = 9.8 \text{ ms}^{-2}$$

Suppose $h$ = height at which blood container must be placed

From the relation,

$$P = h\rho g,$$

or,
$$h = \frac{P}{\rho g}$$
$$= \frac{2000}{1.06 \times 10^3 \times 9.8} \text{ m}$$

or $h = 0.1925$ m

or $h \approx 0.2$ m

**Q. 3. A tank with a square base of area 1.0 $m^2$ is divided by a vertical partition in the middle. The bottom of the partition has small-hinged door of area 20 $cm^2$. The tank is filled with water in one compartment, and an acid (of relative density = 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep one door closed.**

**[NCERT Ad. Ex. Q. 10.21, Page 274]**

**Ans.** Water compartment,

$$P = h\rho g$$
$$= 4 \times 1.0 \times 10^3 \times 9.8$$
$$= 39.2 \times 10^3 \text{ Pa}$$

Acid compartment,

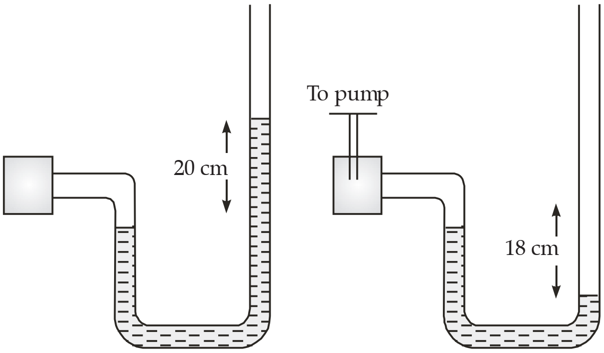
$$P' = h\rho' g$$
$$= 4 \times 1.7 \times 10^3 \times 9.8$$
$$= 66.64 \times 10^3 \text{ Pa}$$
$$P' - P = 66.64 \times 10^3 - 39.2 \times 10^3$$
$$= 27.44 \times 10^3 \text{ Pa}$$
$$A = 20 \text{ cm}^2$$
$$= 20 \times 10^{-4} \text{ m}^2.$$

Area, using pressure,

$$P = \frac{F}{A}$$
$$F = PA$$

or
$$F = (20 \times 10^{-4}) \times (27.44 \times 10^3)$$
$$= 54.88 \text{ N}$$

**Q. 4. A manometer reads the pressure of a gas in an enclosure as shown in figure (i). When a pump removes some of the gas, the manometer reads as in Figure (ii). The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.**




(i) (ii)

**(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (i) and (ii) in units of cm of mercury.**

**(b) How would the levels change in case (ii) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer ? (Ignore the small change in volume of the gas.)**

**[NCERT Ad. Ex. Q. 10.22, Page 274]**

**Ans. Given :** Atmospheric pressure,

$$P_0 = 76 \text{ cm of Hg.}$$

**(a)** In figure (i) pressure head,

$$h_1 = + 20 \text{ cm of Hg.}$$

$\therefore$ Absolute pressure (P) of the gas is greater than the $P_0$, *i.e.*,

Absolute pressure = Atmospheric pressure + Gauge pressure

$$P = P_0 + h_1\rho g$$
$$= 76 \text{ cm of Hg} + 20 \text{ cm of Hg}$$
$$= 96 \text{ cm of Hg.}$$

Gauge pressure is the difference between the absolute pressure and the atmospheric pressure. It means,

$$\text{Gauge pressure} = P - P_0$$
$$= 96 \text{ cm of Hg} - 76 \text{ cm of Hg}$$
$$= 20 \text{ cm of Hg}$$

In figure (ii), pressure head,

$$h_2 = -18 \text{ cm of Hg.}$$

$\therefore$ The absolute pressure of the gas is lesser than the atmospheric pressure is given by

$$P = P_0 + h_2\rho g$$

= 76 cm of Hg + (–18 cm) of Hg
= 58 cm of Hg

Gauge pressure = Absolute pressure – Atmospheric pressure
= 58 cm of Hg – 76 cm of Hg
= – 18 cm of Hg.

It means, Gauge pressure is simply equal to $h$ cm of Hg.

**(b) Given :** 13.6 cm of water added in the right limb is equivalent to $\frac{13.6}{13.6}$ = 1 cm of Hg column *i.e.*, $h$ = 1 cm of Hg column, which can be calculated as follows

$h_w$ = 13.6 cm of water

Suppose $h_m$ = height of Hg column equivalent to 13.6 cm of water, thus equilibrium.

$$h_m \rho_m g = h_w \rho_w g.$$

or

$$h_m = h_w \frac{\rho_w}{\rho_m} = \frac{h_w}{\left(\frac{\rho_m}{\rho_w}\right)}$$

$$= \frac{13.6}{13.6} = 1 \text{ cm of Hg}$$

The mercury will rise in the left limb such that the difference in the height of Hg column in the two limbs

= 20 cm – 1 cm
= 19 cm of Hg column.

**Q. 5. (a) It is known that density ρ of air decreases with height $y$ as $\rho = \rho_0 e^{-y/y_0}$ where $\rho_0$= 1.25kgm$^{-3}$ is the density at sea level, and $y_0$ is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains constant (isothermal conditions). Also assume that the value of $g$ remains constant.**

**(b) A large He balloon of volume 1425 m$^3$ is used to lift a payload of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise?**

**[Take $y_0$= 8000 m and $\rho_{He}$ = 0.18 $kg$ m$^{-3}$].**

**[NCERT Ad. Ex. Q. 10.31, Page 275]**

**Ans. (a)** Volume of the balloon, V = 1425 m$^3$

Mass of the payload, $m$ = 400 kg

Acceleration due to gravity, g = 9.8 m /s$^2$

$y_0$ = 8000 m

$\rho_{He}$= 0.18 kg m$^{-3}$

$\rho_0$ = 1.25 kg/m$^3$

Density of the balloon = ρ

Height to which the balloon rises = $y$

Density (ρ) of air decreases with height ($y$) as:

$$\rho = \rho_0 e^{-y/y_0}$$

$$\frac{\rho}{\rho_0} = e^{-y/y_0} \qquad \ldots\text{(i)}$$

This density variation is called the law of atmospherics.

It can be inferred from equation ($i$) that the rate of decrease of density with height is directly proportional to ρ, i.e., $\frac{d\rho}{dy} \propto \rho$

$$\frac{d\rho}{dy} = -k\rho$$

$$\frac{d\rho}{\rho} = -kdy$$

Where, $k$ is the constant of proportionality

Height changes from 0 to $y$, while density changes from $\rho_0$ to ρ. Integrating the sides between these limits, we get:

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_0^y kdy$$

$$[\log_e \rho]_{\rho_0}^{\rho_1} = -ky$$

$$\log_e \frac{\rho_1}{\rho_0} = -ky$$

$$\frac{\rho_1}{\rho_0} = e^{-ky} \qquad \ldots\text{(ii)}$$

Comparing equations (i) and (ii), we get

$$y_0 = \frac{1}{k}$$

$$k = \frac{1}{y_0} \qquad \ldots\text{(iii)}$$

From equations (i) and( iii) we get :

$\rho = \rho_0 e^{-y/y_0}$

**(b)** Density $\rho = \frac{\text{Mass}}{\text{Volume}}$

$$= \frac{\text{Mass of the payload} + \text{Mass of helium}}{\text{Volume}}$$

$$= \frac{m + V\rho_{He}}{V}$$

$$= \frac{400 + 1425 \times 0.18}{1425}$$

=0.46 kg /m$^3$

From equations(ii) and (iii),we can obtain:

$\rho = \rho_0 e^{-y/y_0}$

$$\log_e \frac{\rho}{\rho_0} = -\frac{y}{y_0}$$

$$\therefore y = -8000 \times \log_e \frac{0.46}{1.25}$$

= −8000× −1

= 8000 m = 8 km

Hence, the balloon will rise to a height of 8 km.

# TOPIC-2
## Surface Energy and Surface Tension

## Quick Review

- **Surface Tension :** It is the property of the liquid by virtue of which the free surface of the liquid at rest tends to have the minimum surface area and as such it behaves as if covered with a stretched membrane.
  - **(a)** Quantitatively, surface tension of a liquid is measured as the force acting per unit length of a line imagined to be drawn tangentially any where on the free surface of the liquid at rest. It acts at right angles to this line on both the sides and along the tangent to the liquid surface, *i.e.*, $S = F/l$.
  - **(b)** Surface tension of a liquid is also defined as the amount of work done in increasing the free surface of liquid at rest by unity at constant temperature, *i.e.*, $S = W/A$.

    or $W = S \times A$ = Surface tension × Area of liquid surface formed.
  - **(c)** Surface tension is a molecular phenomenon and it arises due to electromagnetic forces. The explanation of surface tension was first given by Laplace.
  - **(d)** S.I. units of surface tension is $Nm^{-1}$ or $Jm^{-2}$ and C.G.S. unit is dyne $cm^{-1}$ or erg-$cm^{-2}$.
  - **(e)** Dimensional formula of surface tension = $[ML^0T^{-2}]$
  - **(f)** Surface tension is a scalar quantity as it has no specific direction for a given liquid.
  - **(g)** Surface tension does not depend upon the area of the free surface of liquid at rest.
- **Surface Energy : It** is defined as the amount of work done against the force of surface tension in forming the liquid surface of a given area at a constant temperature, *i.e.*,

  Surface energy = Work done = S.T. × Surface area of liquid.

  The S.I. unit of surface energy is Joule and C.G.S. unit is erg.
  - **(a)** If small drops combine together to form a big drop, the surface area decreases, so surface energy decreases. Therefore the energy is released. If this energy is taken by drop, the temperature of drop increases.
  - **(b)** If a big drop is split into number of smaller drops, the surface area of drops increases. Hence, surface energy increases. So energy is spent.

## Know the Terms

- **Molecular range** is the maximum distance upto which a molecule can exert some measurable attraction on other molecules.
- **Sphere of influence** is an imaginary sphere drawn with a molecule as centre & molecular range as radius.
- **Surface film** is the top most layer of liquid at rest with thickness equal to the molecular range.
- **Angle of contact** between a liquid and solid in contact is defined as the angle enclosed between the tangents to the liquid surface at the point of contact and the solid surface inside the liquid.
- **Capillary tube** is a tube with a fine and uniform bore throughout its length.
- **Capillarity** is the phenomenon of rise or fall of liquid in a capillary tube.

## Know the Formulae

- **Surface tension,** $S = F/l$
- **Surface energy,** $E$ = Work done
- **Work done,** $W = S \times$ Increase in area
- **Excess of pressure inside the liquid drop is** $P = P_i - P_o = \frac{2S}{r}$
- **Excess of pressure inside the soap bubble is** $P = P_i - P_o = \frac{4S}{r}$

- **Total pressure in the air bubble at a depth h below the surface of liquid of density ρ is**

$$P = P_o + h\rho g + \frac{2S}{r}$$

- **Ascent/Decent Formula :**

$$h = \frac{2S\cos\theta}{r\rho g}$$

where,

$r$ = radius of capillary tube
$\rho$ = density
S = Surface tension
$\theta$ = angle of contact

## Know the Links

- *www.britannica.com*
- *www.khanacademy.org*
- *www.vedantu.com*
- *www.examfear.com*

## MCQ/Fillups/True or False

**(1 mark each)**

**(A) Multiple Choice Questions**

**Q. 1. The angle of contact at the interface of water-glass is 0°, Ethylalcohol-glass is 0°, Mercury-glass is 140° and methyliodide glass is 30°. A glass capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is**

**(a) water**
**(b) ethylalcohol**
**(c) mercury**
**(d) methyliodide.**

**[NCERT Exemp. Q. 10.5, Page 73]**

**Ans. Correct option:** (c)

***Explanation:*** According to the question, the observed meniscus of liquid in a capillary tube is of convex upward which is only possible when angle of contact is obtuse. It is so when one end of glass capillary tube is immersed in a trough of mercury.

**Q. 2. For a surface molecule**

**(a) the net force on it is zero.**
**(b) there is a net downward force.**
**(c) the potential energy is less than that of a molecule inside.**
**(d) the potential energy is more than that of a molecule inside.**

**[NCERT Exemp. Q. 10.6, Page 73]**

**Ans. Correct option:** (b) and (d)

***Explanation:***

**(b)** Surface molecule experiences a net downward force because on the above side of this molecule their is no liquid molecule.

**(d)** So, the potential energy is more than that of a molecule inside.

## Very Short Answer Type Questions

**(1 mark each)**

**Q. 1. Is surface tension a vector?**

**[NCERT Exemp. Q. 10.12, Page 74]**

**Ans.** Surface tension is a scalar quantity

$$\text{i.e. Surface Tension} = \frac{\text{Work done}}{\text{Surface Area}}$$

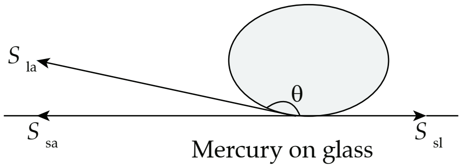
Work done and surface area both are scalar quantities, hence surface tension is also a scalar quantity.

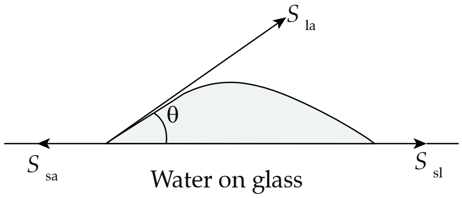
**Q. 2. Explain why**

**(a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.**
**(b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)**
**(c) Surface tension of a liquid is independent of the area of the surface**
**(d) Water with detergent dissolved in it should have small angles of contact.**
**(e) A drop of liquid under no external forces is always spherical in shape**

**[NCERT Ex. Q. 10.28, Page 272]**

**Ans. (a)** The angle between the tangent to the liquid surface at the point of contact and the surface inside the liquid is called the angle of contact (θ), as shown in the given figure.




$S_{la}$, $S_{sa}$, and $S_{sl}$ are the respective interfacial tensions between the liquid-air, solid-air, and solid-liquid interfaces. At the line of contact, the surface forces between the three media must be in equilibrium, i.e.,

$$\cos\theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

The angle of contact θ, is obtuse if $S_{sa} < S_{la}$ (as in the case of mercury on glass). This angle is acute if $S_{sa} < S_{la}$ (as in the case of water on glass).

(b) Mercury molecules (which make an obtuse angle with glass) have a strong force of attraction between themselves and a weak force of attraction toward solids. Hence, they tend to form drops.
On the other hand, water molecules make acute angles with glass. They have a weak force of attraction between themselves and a strong force of attraction toward solids. Hence, they tend to spread out.

**(c)** Surface tension is the force acting per unit length at the interface between the plane of a liquid and any other surface. This force is independent of the area of the liquid surface. Hence, surface tension is also independent of the area of the liquid surface.

(d) Water with detergent dissolved in it has small angles of contact (θ). This is because for a small θ, there is a fast capillary rise of the detergent in the cloth. The capillary rise of a liquid is directly proportional to the cosine of the angle of contact (θ). If θ is small, then cos θ will be large and the rise of the detergent water in the cloth will be fast.

**(e)** A liquid tends to acquire the minimum surface area because of the presence of surface tension. The surface area of a sphere is minimum for a given volume. Hence, under no external forces, liquid drops always take spherical shape.

## Short Answer Type Questions

**(2 or 3 marks each)**

**Q. 1. The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius $r = 2.5 \times 10^{-5}$ m. The surface tension of sap is T = $7.28 \times 10^{-2}$ Nm$^{-1}$ and the angle of contact is 0°. Does surface tension alone account for the supply of water to the top of all trees?**

**[NCERT Ad. Exemp. Q. 10.16, Page 75]**

**Ans. Given :**

Surface Tension, $T = 7.28 \times 10^{-2}$ N/m

Angle of contact(θ) = 0°

Radius ($r$) = $2.5 \times 10^{-5}$ m

The height to which the sap will rise is

$$h = \frac{2T\cos 0^\circ}{\rho g r} = \frac{2 \times 7.28 \times 10^{-2} \times 1}{10^{-3} \times 9.8 \times 2.5 \times 10^{-5}} \simeq 0.6 \text{ m}$$

[Here, ρ = density].

This is the maximum height to which the sap can rise due to surface tension. Since many trees have heights much more than this, capillary action alone cannot account for the rise of water in all trees.

**Q. 2. Mercury has an angle of contact equal to 140° with soda-lime glass. A narrow tube of radius 1.00 mm made of glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube, relative to the liquid surface outside ? (Surface tension of mercury at the temperature of the experiment is 0.465 Nm$^{-1}$, Density of mercury = $13.6 \times 10^3$ kg m$^{-3}$)**

**[NCERT Ad. Ex. Q. 10.29, Page 275]**

**Ans.** Using the relation,

$$h = \frac{2T\cos\theta}{r\rho g}$$

we get

$$h = \frac{2 \times 0.465 \times \cos 140^\circ}{1 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

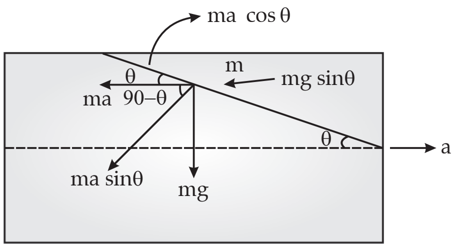
$$= -5.34 \times 10^{-3} \text{ m}$$

Negative sign shows that the mercury level is depressed in the tube.

**Q. 3. The free surface of oil in a tanker, at rest, is horizontal. If the tanker starts accelerating the free surface will be titled by a angle θ. If the acceleration is a ms$^{-2}$, what will be the slope of the free surface?**

**[NCERT Exemp. Q. 10.17, Page 75]**

**Ans.** Let us consider the following diagram.




If the tanker accelerates in the positive direction with acceleration $a$. Consider a parcel, the fluid of mass 8m. Forces which are acting on particle w.r.t. the tanker. Now, balancing forces (in equilibrium) along inclined direction component of weight = component of pseudo force [ma = pseudo force]

$$mg\sin\theta = ma\cos\theta$$

$$\text{or } g\sin\theta = a\cos\theta$$

$$\text{or } \frac{\sin\theta}{\cos\theta} = \frac{a}{g}$$

$$\therefore \tan\theta = \frac{a}{g}$$

This is required slope.

**Q. 4. Two mercury droplets of radii 0.1 cm and 0.2 cm collapse into one single drop. What amount of energy is released? The surface tension of mercury T = 435.5 × $10^{-3}$ $Nm^{-1}$**

**[NCERT Exemp. Q. 10.18, Page 75]**

**Ans.** Radii of mercury droplets, $r_1 = 0.1\text{cm} = 1 \times 10^{-3}\text{m}$

$r_2 = 0.2\text{cm} = 2 \times 10^{-3}\text{m}$

Surface Tension, T = 435.5 ×$10^{-3}$ N/m

Let $V_1$ and $V_2$ be the volume of the droplets and $V$ of the resulting drop, and R be the radius of big drop formed by collapsing.

Then $V = V_1 + V_2$ or $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3$

or $R^3 = r_1^3 + r_2^3 = (0.001 + 0.008)\ \text{cm}^3 = 0.009\ \text{cm}^3$

$\therefore R = 0.21$ cm

Change in surface area, $\Delta A = 4\pi[R^2 - (r_1^2 + r_2^2)]$

$\therefore$ Energy released, $\Delta U = T\Delta A = 4\pi T\left[R^2 - \left(r_1^2 + r_2^2\right)\right]$

$= 4 \times 3.14 \times 435.5 \times 10^{-3}$

$[(0.21)^2 \times 10^{-4} - (1 \times 10^{-6} + 4 \times 10^{-6})]$

$= 435.5 \times 4 \times 3.14\ [4.41 - 5] \times 10^{-6} \times 10^{-3}$

$= -32.23 \times 10^{-7}$ J

$\therefore$ $= 3.22 \times 10^{-6}$ J energy will be absorbed.

**Q. 5. If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of the droplets. Let a drop of radius R, break into N small droplets each of radius r. Estimate the drop in temperature.**

**[NCERT Exemp. Q. 10.19, Page 75]**

**Ans.** When a big drop having radius $R$ breaks into $N$ droplets each of radius $r$, the volume remains constant.

$\therefore$ Volume of big drop = N × volume of each small drop

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3$$

$$\text{or} \quad R^3 = Nr^3 \text{ or } N = \frac{R^3}{r^3}$$

$$\text{Change in surface area} = 4\pi R^2 - N4\pi r^2 = 4\pi\left(R^2 - Nr^2\right)$$

$$\text{Energy released} = T \times \Delta A = T \times 4\pi\left(R^2 - Nr^2\right)$$

Released energy lowers the temperature by $\Delta\theta$, then

$$\text{Energy released} = ms\Delta\theta$$

$$T \times 4\pi\left(R^2 - Nr^2\right) = \left(\frac{4}{3} \times R^3 \times \rho\right)s\Delta\theta$$

$$\begin{bmatrix} s = \text{specific heat of liquid} \\ \rho = \text{density} \end{bmatrix}$$

$$\text{or} \quad \Delta\theta = \frac{T \times 4\pi\left(R^2 - Nr^2\right)}{\frac{4}{3}\pi R^3 \rho \times s} = \frac{3T}{\rho S}\left[\frac{R^2}{R^3} - \frac{Nr^2}{R^3}\right]$$

$$\Delta\theta = \frac{3T}{\rho s}\left[\frac{1}{R} - \frac{1}{r}\right]. \qquad \because \left(N = \frac{R^3}{r^3}\right)$$

**Q. 6. The surface tension and vapour pressure of water at $20^\circ C$ is $7.28 \times 10^{-2}\ Nm^{-1}$ and $2.33 \times 10^3$ Pa , respectively. What is the radius of the smallest spherical water droplet which can form without evaporating at 20°C ?**

**[NCERT Exemp. Q. 10.20, Page 75]**

**Ans.** Surface Tension of water, $T = 7.28 \times 10^{-2}$ N/m.

Vapour pressure, $P = 2.33 \times 10^3$Pa.

The drop will evaporate if the water pressure is more than the vapour pressure. Let a water droplet of radius $R$ can be formed.

Vapour pressure = Excess pressure in drop

$$\therefore p = \frac{2T}{r} = 2.33 \times 10^3\ Pa$$

$$\therefore r = \frac{2T}{p} = \frac{2\left(7.28 \times 10^{-2}\right)}{2.33 \times 10^3}$$

$$= 6.25 \times 10^{-5}\ \text{m}$$

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. Surface tension is exhibited by liquids due to force of attraction between molecules of the liquid. The surface tension decrease with increase in temperature and vanishes at boiling point. Given that the latent heat of vaporisation for water $L_v$ = 540 kcal $kg^{-1}$, the mechanical equivalent of heat J = 4.2 J $cal^{-1}$, density of water $\rho_w = 10^3\ kgm^{-3}$, Avagadro's No $N_A = 6.0 \times 10^{26}$ K $mole^{-1}$ and the molecular weight of water $M_A$ = 18 kg for 1 mole.**

**(a) estimate the energy required for one molecule of water to evaporate.**

**(b) show that the inter-molecular distance for water is $d = \left[\frac{M_A}{N_A} \times \frac{1}{\rho_w}\right]^{1/3}$ and find its value.**

**(c) 1 g of water in the vapour state at 1 atm occupies 1601 $cm^3$. Estimate the intermolecular distance at boiling point, in the vapour state.**

**(d) During vaporisation a molecule overcomes a force $F$, assumed constant, to go from an inter-molecular distance $d$ to $d'$. Estimate the value of $F$, Where d=3.1 x $10^{-10}$m .**

**(e) Calculate $F/d$ which is a measure of the surface tension. [NCERT Exemp. Q. 10.22, Page 75]**

**Ans.** Given, $L_v$ = 540 kcal/kg

= 540 × $10^3$× 4.2 J/kg

**(a)** Such that, 1 kg of water requires energy for evaporation = $L_v$ k cal

∴ $M_A$ kg of water requires $M_A L_v$ k cal

Since there are $N_A$ molecules in $M_A$ kg of water the energy required for 1 molecule to evaporate is

$$u = \frac{M_A L_v}{N_A} J$$

$$\left[\text{Here, } N_A = 6\times10^{26} = \text{Avogadro Number}\right]$$

$$= \frac{18 \times 540 \times 4.2 \times 10^3}{6 \times 10^{26}}$$

$$= 90 \times 18 \times 4.2\times10^{-23} \text{ J}$$

$$= 6.8 \times 10^{-20} \text{ J}$$

**(b)** Let us consider the water molecules to be points at a distance d from each other.

$$\text{Volume of } N_A \text{ molecules} = \frac{M_A}{\rho_w} l$$

Thus, the volume around one molecule is $\frac{M_A}{N_A \rho_w} l$

The volume around one molecule is, $d^3 = (M_A / N_A \rho_w)$

$$\therefore d = \left(\frac{M_A}{N_A \rho_w}\right)^{1/3} = \left(\frac{18}{6\times10^{26}\times10^3}\right)^{1/3}$$

$$= \left(30\times10^{-30}\right)^{1/3} \text{ m} \simeq 3.1\times10^{-10} \text{m}$$

**(c)** 1 kg of vapour occupies = 1601 × $10^{-3}$ $m^3$

∴ 18 kg of vapour occupies = 18 × 1601 × $10^{-3}$ $m^3$

or 6 × $10^{26}$ molecules occupies = 18 × 1601 × $10^{-3}$ $m^3$

$$\therefore 1 \text{ molecule occupies} = \frac{18\times1601\times10^{-3}}{6\times10^{26}} m^3$$

If $d_1$ is the inter molecular distance. then

$d_1^3$ = (3 × 1601 ×$10^{-29}$)$m^3$

∴ $d_1$ = $(30 \times 1601)^{1/3}$ × $10^{-10}$ m

= 36.3 ×$10^{-10}$ m

**(d)** To change the distance form $d$ to $d_1$,

Work done = $F(d_1 - d)$

Work done = energy required to evaporate 1 molecule

$$\text{or } F = \frac{6.8\times10^{-20}}{d_1 - d}$$

$$= \frac{6.8\times10^{-20}}{(36.3-3.1)\times10^{-10}}$$

$$= 0.2048\times10^{-10} \text{ N}$$

**(e)** $$\text{Surface Tension} = \frac{F}{d}$$

$$= \frac{0.2048\times10^{-10}}{3.1\times10^{-10}}$$

$$= 6.6\times10^{-2} \text{ Nm}^{-1}$$

**Q. 2. A hot air balloon is a sphere of radius 8 m. The air inside is at a temperature of 60°C. How large a mass can the balloon lift when the outside temperature is 20°C ? (Assume air is an ideal gas, R = 8.314 J $mol^{-1}$ $K^{-1}$, 1 atm. = 1.013 × $10^3$ Pa; the membrane tension is 5 $Nm^{-1}$.)**

**[NCERT Exemp. Q. 10.23, Page 76]**

**Ans.** Let the pressure inside the balloon be $P_i$ and the outside pressure be $P_0$

$$P_i - P_o = \frac{2T}{r}$$

$$[T = \text{Surface Tension}, \ r = \text{radius of balloon}]$$

Considering the air to be an ideal gas

$$P_i V = n_i R T_i$$

where, $V$ is the volume of the air inside the balloon, $n_i$ is the number of moles inside and $T_i$ is the temperature inside, and $P_0 V = n_0 R T_0$

where $V$ is the volume of the air displaced and $n_0$ is the number of moles displaced and $T_0$ is the temperature outside.

$$\text{So, } n_i = \frac{P_i V}{R T_i} = \frac{M_i}{M_A}$$

where $M_i$ is the mass of the air inside and $M_A$ is the molar mass of the air and $n_0 = \frac{P_0 V}{R T_0} = \frac{M_0}{M_A}$

where $M_0$ is the mass of air outside that has been displaced. If W is the load it can raise, then

$$W + M_i g = M_0 g$$

$$\text{or } W = M_0 g - M_i g$$

In atmospheric air, 21% $O_2$ and 79% $N_2$ is present.

∴Molar mass of air

$M_A$ = 0.21 × 32 + 0.79 × 28 = 28.84 g

∴Weight raised by balloon

$$\text{or } W = \frac{M_A V}{R}\left(\frac{P_0}{T_0} - \frac{P_i}{T_i}\right)g \qquad \text{here, } P_0 = P_i$$

$$= \frac{0.02884\times\frac{4}{3}\pi\times8^3}{8.314}\times1.013\times10^5\left(\frac{1}{293}-\frac{1}{333}\right)\times9.8 \text{ N}$$

$$= 3044.2 \text{ N}$$

$$\text{Therefore, Mass lifted by balloon} = \frac{W}{g}$$

$$= \frac{3044.2 \text{ N}}{9.8 \text{ m/s}^2}$$

$$= 310.6 \text{ kg}$$

**Q. 3. An U-shaped wire is dipped in a soap solution and removed. The thin soap film formed between the wire and a light slider supports a weight of 1.5 × $10^{-2}$ N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film ?**

**[NCERT Ex. Q. 10.17, Page 273]**

**Ans.** It is quite clear that a soap film has two free surfaces, total length of the film to be supported,

$l = 2 \times 30$ cm

⇒ $l = 60$ cm = 0.60 m

Suppose T = Surface tension of the film

When F = Total force on the slide due to surface tension.

Then $F = T \times 2l = T \times 0.6$ N,

$W = 1.5 \times 10^{-2}$ N.

In equilibrium position, the force F on the slider due to surface tension must be balanced by the weight (W) supported by the slider.

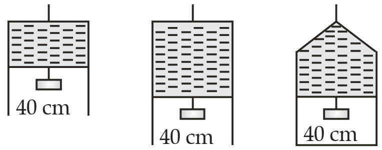
*i.e.*, $F = W = mg$

or $T \times 0.6 = 1.5 \times 10^{-2}$

$$T = \frac{1.5\times10^{-2}}{0.6}$$

$= 2.5 \times 10^{-2}$ Nm$^{-1}$.

**Q. 4. Figure (a) shows a thin liquid film supporting a small weight = 4.5 × $10^{-2}$ N. What is the weight supported by a film of the same liquid at the same temperature in figure (b) and (c) ? Explain your answer physically.**




**(a) (b) (c)**

**[NCERT Ex. Q. 10.18, Page 273]**

**Ans. Given :** Length of the film supporting the weight

$l = 40$ cm = 0.4 m.

Total weight supported (*i.e.*, force)

$w = 4.5 \times 10^{-2}$ N.

T = surface tension = ?

The film has two free surfaces, so total length,

$L = 2l = 2 \times 0.4$ m.

∴ By formula $T = \frac{\text{Force}}{\text{Length}} = \frac{4.5\times10^{-2}}{2\times0.4}$

$= 5.625 \times 10^{-2}$ Nm$^{-1}$.

As the liquid is same for all the cases (a), (b) and (c) and temperature is also same, therefore surface tension for cases (b) and (c) will also be same, *i.e.*, $5.625 \times 10^{-2}$ Nm$^{-1}$.

In Figure (b) and (c), the length of the film supporting the weight is also the same as that of (a), hence the total weight supported in each case is $4.5 \times 10^{-2}$ N.

**Q. 5. What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature ? Surface tension of mercury at that temperature (20°C) is 4.65 × $10^{-1}$ Nm$^{-1}$. The atmospheric pressure is 1.01 × $10^5$ Pa. Also give the excess pressure inside the drop.**

**[NCERT Exemp. Q. 10.19, Page 274]**

**Ans.** Here, $S = 4.65 \times 10^{-1}$ Nm$^{-1}$

R = 3 mm

$= 3 \times 10^{-3}$ m

$P_{outside} = 1.01 \times 10^5$ Pa.

Using the relation,

Excess pressure $= \frac{2S}{r}$,

we get,

$$P_{outside} - P_{inside} = \frac{2\times4.65\times10^{-1}}{3\times10^{-3}}$$

= 310 Pa

Total pressure $= 1.01 \times 10^5 + 310$

$= 1.013 \times 10^5$ Pa

**Q. 6. What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temp. (20° C) is 2.50 × $10^{-2}$ Nm$^{-1}$? If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble ? (1 atmospheric pressure is 1.0 × $10^5$ Pa.)** **[NCERT Ex. Q. 10.20, Page 274]**

**Ans.** Soap bubble

In this case, excess pressure

$$= \frac{4S}{R} = \frac{4\times2.50\times10^{-2}}{5\times10^{-3}}$$

= 20 Pa

Air bubble

Here, excess pressure

$$= \frac{2S}{R} = \frac{2\times2.50\times10^{-2}}{5\times10^{-3}}$$

= 10 Pa

$P_{outside}$ = Pressure depth of 40 cm

= Atm. pressure + column pressure

$= (1.01 \times 10^5) + 0.4 \times 1.2 \times 10^3 \times 9.8$

$= 1.0570 \times 10^5$ Pa

Density of soap solution

ρ = R.D. × 1000 = 1.20 × 1000

= 1200 g m$^{-3}$

Total pressure inside the air bubble

= Atmospheric pressure + pressure due to 40 cm soap solution + Excess pressure

$= 1.01 \times 10^5 + h\rho g + p'$

$= 1.01 \times 10^5 + 0.40 \times 1200 \times 9.8 + 10$

= 101000 + 4704 + 10 = 105714 Pa

**Q. 7. Two narrow bores of diameters 3.0 mm and 6.00 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube ? (Surface tension of water at the temperature of the experiment is 7.3 × $10^{-2}$ N-m$^{-1}$. Take the angle of contact to be zero and density of water to be 1.0 × $10^3$ kg m$^{-3}$. (g = 9.8 ms$^{-2}$)**

**[NCERT Ex. Q. 10.30, Page 275]**

**Ans.** For 3.00 mm tube, $h = \frac{2S\cos\theta}{r\rho g}$

$= \frac{2 \times 7.3 \times 10^{-3} \times \cos 0^\circ}{(1.5 \times 10^{-3})(1 \times 10^3)9.8}$

$= 9.932 \times 10^{-3}$ m.

For 6.00 mm tube, $h' = \frac{2S\cos\theta}{r'\rho g}$

$= \frac{2 \times 7.3 \times 10^{-2} \times \cos 0^\circ}{(2 \times 10^{-3})(1 \times 10^3)9.8}$

$= 4.966 \times 10^{-3}$ m

Difference of levels $= h - h'$

$= (9.932 - 4.966)10^{-3}$

$= 4.966 \times 10^{-3}$ m

# TOPIC-3
## Viscosity and Bernoulli's theorem

### Quick Review

- **Bernoulli's theorem :** Bernoulli's theorem state that the total energy (pressure energy, P.E. and K.E.) of an incompressible non-viscous liquid in steady flow remain constant throughout the flow of the liquid

  $P + \rho g h + \frac{1}{2}\rho v^2 =$ constant.

- **Torricelli's theorem :** According to this theorem, velocity of efflux *i.e.*, the velocity with which the liquid flows out of an orifice is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid.

### Know the Terms

- **Viscosity** is the property of liquid due to which a backward dragging force called viscous force act tangentially on the layer of the liquid in motion.
- **Terminal velocity** is the maximum constant velocity acquired by the body while falling freely in a viscous medium.
- **Streamline flow of a liquid** is that flow in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point.
- **Streamline** is the actual path followed by the procession of particles in a steady flow which may be straight or curved such that tangent to it at any point indicates the direction of flow of liquid at that point.
- **Tube of flow** is the bundle of streamlines having the same velocity of the liquid particle over any cross-section perpendicular to the direction of flow of liquid.
- **Laminar flow** is a flow in which the liquid moves in layers.
- **Turbulent flow** is a flow when a liquid moves with a velocity greater than its critical velocity, the motion of particles of liquid becomes disorderly or irregular.
- **Critical velocity** is that velocity of liquid flow, upto which its flow is streamlined and above which its flow becomes turbulent.
- **Reynold number** is a pure number which determines the nature of flow of liquid through a pipe.
- **Venturimeter** is a device used for measuring the speed of incompressible liquid and rate of flow of liquid through pipes.

### Know the Formulae

- **Newton's viscous dragging force :** $F = \pm \eta A \frac{dv}{dx}$

  $\eta$ = coeff. of viscosity, A = area of layer of liquid, $dv/dx$ = velocity gradient.

- **Poiseuille's Formula :** $V = \frac{\pi p r^4}{8\eta l}$

  $p$ = Pressure difference across length $l$ of horizontal tube of radius $r$ & $V$ = volume.

- **Stoke's Law :** $F = 6\pi\eta r v$

➢ **Terminal velocity :**

$$v = \frac{2r^2(\rho-\sigma)g}{9\eta}$$

*where,*

$\rho$ = density of spherical body
$\sigma$ = density of medium
$r$ = radius of spherical body
$\eta$ = coeff. of viscosity

➢ **Reynold's Number :**

$$R_N = \frac{\rho D v}{\eta}$$

Where, symbols have their usual meaning

➢ **Bernoulli's Theorem :**

**(a)** $\frac{P}{\rho} + gh + \frac{1}{2}v^2$ = a constant

$\frac{P}{\rho}$ = pressure energy per unit mass

$gh$ = P. E. per unit mass

$\frac{1}{2}v^2$ = K.E. per unit mass

➢ **Venturimeter :** Volume of liquid flowing per second

$$V = a_1a_2\sqrt{\frac{2\rho_m gh}{\rho\left(a_1^2 - a_2^2\right)}}$$

$a_1$, $a_2$ = area of cross-section of bigger & smaller tube of venturimeter.
$h$ = difference of pressure head at the two tube of venturimeter. $\rho_m$ - density of liquid in manometer

➢ **Velocity of efflux :** $v = \sqrt{2gh}$

## Know the Links

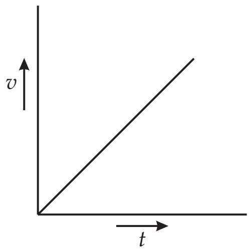
☞ *www.vedantu.com*
☞ *www.learncbse.in*
☞ *www.britannica.com*

## MCQ/Fillups/True or False

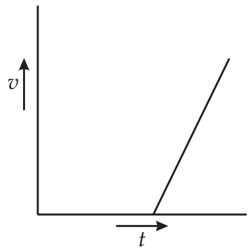
**(1 mark each)**

**(A) Multiple Choice Questions**

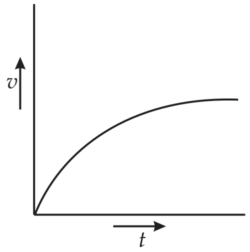
**Q. 1. A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero initial velocity. From the plot shown in figure. Indicate the one that represents the velocity ($v$) of the pebble as a function of time ($t$).**



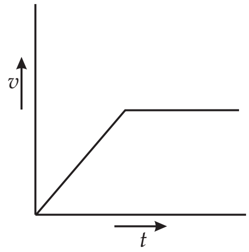

**(a)**




**(b)**




**(c)**



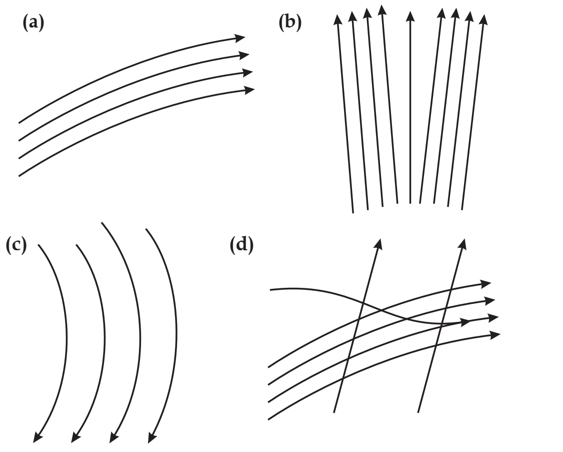

**(d)**

**[NCERT Exemp. Q. 10.1, Page 72]**

**Ans. Correct option:** (c)

***Explanation:*** As the pebble acquires terminal velocity after some time.

**Q. 2. Which of the following diagrams does not represent a streamline flow?**




**[NCERT Exemp. Q. 10.2, Page 73]**

**Ans.** **Correct option:** (d)

*Explanation:* Lines of flow do not intersect each other

**Q. 3. Along a streamline**

**(a) the velocity of a fluid particle remains constant.**
**(b) the velocity of all fluid particles crossing a given position is constant.**
**(c) the velocity of all fluid particles at a given instant is constant.**
**(d) the speed of a fluid particle remains constant.**

**[NCERT Exemp. Q. 10.3, Page 73]**

**Ans.** **Correct option:** (b)

*Explanation:* Along a streamline flow, the velocity of every fluid particle while crossing a given position is the same.

**Q. 4. An ideal fluid flows through a pipe of circular cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in two pipes is**

**(a) 9 : 4**
**(b) 3 : 2**
**(c)** $\sqrt{3}:\sqrt{2}$
**(d)** $\sqrt{2}:\sqrt{3}$

**[NCERT Exemp. Q. 10.4, Page 73]**

**Ans.** **Correct option:** (a)

*Explanation:* According to equation of continuity

$$a_1 v_1 = a_2 v_2$$

$$\text{or } \frac{v_1}{v_2} = \frac{a_2}{a_1} \text{ or } \frac{\pi \frac{d_2^2}{4}}{\pi \frac{d_1^2}{4}} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{3.75}{2.50}\right)^2 = \frac{9}{4}$$

**Q. 5. With increase in temperature, the viscosity of**

**(a) gases decreases.**
**(b) liquids increases.**
**(c) gases increases.**
**(d) liquids decreases.**

**[NCERT Exemp. Q. 10.9, Page 74]**

**Ans.** **Correct option:** (c) and (d)

*Explanation:* Coefficient of viscosity (η) for gases

$$\eta \propto \sqrt{T}$$

η for liquids,

$$\eta \propto \frac{1}{\sqrt{T}}$$

**Q. 6. Streamline flow is more likely for liquids with**

(a) **high density.**
**(b) high viscosity.**
**(c) low density.**
**(d) low viscosity.**

**[NCERT Exemp. Q. 10.10, Page 74]**

**Ans.** **Correct option:** (b) and (c)

*Explanation:* It is because critical velocity ($v_c$) $\propto \frac{\eta}{\rho}$

**(B) Fill in the blanks :**

**Q. 7. Fill in the blanks using the words from the list appended with each statements :**

**(a) Surface tension of liquids generally _____ with temperature. (increases / decreases)**
**(b) Viscosity of gases ______ with temperature, whereas viscosity of liquids ______ with temperature. (increases / decreases)**
**(c) For solids with elastic modulus of rigidity, the shearing force is proportional to ______ while for fluids it is proportional to ______ (shear strain / rate of shear strain)**
**(d) For a fluid in steady flow, the increases in flow speed at a construction follows form _____ while the decrease of pressure there follows from _____. (conservation of mass / Bernoulli's principle)**
**(e) For the model of a plane in a wind tunnel, turbulence occurs at a ______ speed for turbulence for an actual plane. (greater / smaller)**

**[NCERT Ex. Q. 10.3, Page 272]**

**Ans.**

**(a)** decreases
**(b)** increases, decreases
**(c)** shear strain, rate of shear strain
**(d)** conservation of mass, Bernoulli's principle
**(e)** greater

## Very Short Answer Type Questions **(1 mark each)**

**Q. 1. Is viscosity a vector?**

**[NCERT Exemp. Q. 10.11, Page 74]**

**Ans.** Viscosity is a scalar quantity because it is a property of liquid having no direction.

**Q. 2. Explain why :**

**(a) To keep a piece of paper horizontal, you should blow over, not under it.**

**(b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers.**

**(c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.**

**(d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.**

**(e) A spinning cricket ball in air does not follow a parabolic trajectory.**

**[NCERT Ex. Q. 10.4, Page 272]**

**Ans. (a)** When we blow air over a piece of paper, velocity of air above the paper becomes more than that below it. Since, K.E. of air above the paper increases, so in accordance with Bernoulli's theorem ($P + \frac{1}{2}\rho v^2$ = constant), its pressure energy and hence its pressure decreases. Because of greater value of pressure below the piece of paper = atmospheric pressure, it remains horizontal and does not fall. While we blow air under the paper, the pressure on the lower side decreases. The atmospheric pressure above the paper will therefore bend the paper downwards. So the paper will not remain horizontal.

**(b)** This can be cleared from the equation of continuity, *i.e.*, $a_1v_1 = a_2v_2$. We try to close a water tap with our fingers, the area of cross-section of the outlet of water jet is reduced considerably as the openings between our fingers provide constriction (*i.e.* regions of smaller area). Hence, velocity of water increases greatly and fast jets of water come through the openings between our fingers.

**(c)** From Bernoulli's theorem, we know that

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant} \quad ...(i)$$

Here the size of the needle controls the velocity of flow and the thumb pressure controls pressure.

Now P occurs with power 1 and velocity ($v$) occurring with power 2 in equation (i), therefore, the velocity has more influence. That is why the needle of syringe has a better control over the flow rate.

**(d)** If a fluid is flowing out of a small hole in a vessel, it acquires a large velocity and hence possesses large momentum. Since no external force is acting on the system, a backward velocity must be attained by the vessel (according to the law of conservation of momentum). As a result of it, an impulse (backward thrust) is experienced by the vessel.

**(e) This is because of Magnus effect :** Let a ball moving to the right be given a spin at the top of the ball. The velocity of air at the top is higher than the velocity of air below the ball. So according to Bernoulli's theorem, the pressure above the ball is less than the pressure below the ball. Thus there is a net upward force on the spinning ball, so the ball follows a curved path. This effect is known as Magnus effect.

**Q. 3. Can Bernoulli's equation be used to describe the flow of water through a rapid flow in a river ? Explain. [NCERT Ex. Q. 10.11, Page 273]**

**Ans.** No, Bernoulli's equation cannot be used to describe the rapid flow of water in a river (*i.e.*, turbulent flow). This equation can be used for stream-line flow.

**Q. 4. Does it matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation ?**

**[NCERT Ex. Q. 10.12, Page 273]**

**Ans.** No, it does not matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation, provided the atmospheric pressure at the two places of consideration are significantly different.

## Short Answer Type Questions

**(2 or 3 marks each)**

**Q. 1. A cubical block of density ρ is floating on the surface of water. Out of its height $L$, fraction x is submerged in water. The vessel is in an elevator accelerating upward with acceleration a. What is the fraction immersed?**

**[NCERT Exemp. Q. 10.15, Page 75]**

**Ans. (i)** When cubical block submerged into water –

By principle of floatation –

$V\rho g = V' \rho_w g$

$$\begin{bmatrix} V = \text{volume of water displaced by block} \\ V' = \text{volume of block inside water} \\ = \text{area of base of block} \times \text{height} \end{bmatrix}$$

$V' = L^2x$

$V$ = volume of block = $L^3$, $\rho_B$ = Density of block

$\therefore L^3\rho_B = L^2 x\rho_w$

$$\Rightarrow \frac{\rho_B}{\rho_w} = \frac{x}{L} \text{ or } x = \frac{\rho_B}{\rho_w}L \quad ...(i)$$

**(ii)** When immersed block is in lift [moving in upward direction]

Then, net acceleration = $g + a$

Weight of block = $m(g + a) = V\rho_B(g + a)$

$= L^3\rho_B(g + a)$

Let $x_1$ be the part of block submerged into water in moving lift.

Weight of block = Buoyant force

$$\frac{\rho_B}{\rho_w} = \frac{x_1}{L}$$

$$\text{or } x_1 = L \cdot \frac{\rho_B}{\rho_w} \qquad \text{...(ii)}$$

From (i) & (ii), we conclude that it is independent of acceleration of lift.

**Q. 2. In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy.**

**(a) What is the largest average velocity of blood flow in an artery of diameter $2 \times 10^{-3}$ m if the flow must remain laminar ?**

**(b) Do the dissipative forces become more important as the fluid velocity increases ? Discuss qualitatively.**

**[NCERT Ad. Ex. Q. 10.25, Page 275]**

**Ans. (a)** $d = 2 \times 10^{-3}$ m

$\eta = 2.08 \times 10^{-3}$ Pa

$p = 1.06 \times 10^3$ kg/m$^3$

$N_R = 2000$

The largest average velocity of blood is given as :

$$V_{arg} = \frac{N_R \eta}{pd}$$

$$= \frac{2000 \times 2.08 \times 10^{-3}}{1.06 \times 10^3 \times 2 \times 10^{-3}}$$

$= 1.966$ m/s.

**(b)** Yes, the dissipative forces become more important as the fluid velocity increases.

From Newton's law of viscous drag we know that

$$F = -\eta A \frac{dv}{dx}$$

Clearly as $v$ increases, velocity gradient increases and hence, viscous drag, *i.e.*, dissipative force also increases because of turbulence.

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. (a) What is the largest average velocity of blood flow in an artery of radius $2 \times 10^{-3}$ m if the flow must remain laminar ?**

**(b) What is the corresponding flow rate ? (Take viscosity of blood to be $2.084 \times 10^3$ Pa).**

**[NCERT Ad. Ex. Q. 10.26, Page 275]**

**Ans. Given :** Density of whole blood

$\rho = 1.06 \times 10^3$ kg m$^{-3}$.

Diameter of artery, $D = 2 \times 2 \times 10^{-3}$ m

$= 4 \times 10^{-3}$ m

$\eta$ of blood $= 2.084 \times 10^{-3}$ Pa

For flow of the blood to remain laminar,

$N_R = 2000$.

**(a)** Suppose $v$ = average velocity of blood flow in an artery = ?

From the relation,

$$N_R = \frac{\rho v D}{\eta},$$

$$\text{or} \quad v = \frac{N_R \eta}{\rho D}$$

$$= \frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^3 \times 4 \times 10^{-3}}$$

or $v = 0.983$ ms$^{-1}$ $= 0.98$ ms$^{-1}$.

**(b)** Flow rate, $R = \pi r^2 V$

$= 3.14 \times (2 \times 10^{-3})^2 \times 0.983$

$= 1.235 \times 10^{-5}$ m$^3$s$^{-1}$

**Q. 2. A plane is in level flight at constant speed and each of its wings has an area of 25 m$^2$. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. [Take air density = 1 kgm$^{-3}$, g = 9.8 m/s$^2$].**

**[NCERT Ad. Ex. Q. 10.27, Page 275]**

**Ans.** Given: $v_1 = 180$ km/h $= 50$ m/s

$v_2 = 234$ km/h $= 65$ m/s

$A = 2 \times 25 = 50$ m$^2$, $\rho = 1$ kg/m$^3$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$= \frac{1}{2} \times 1\left(65^2 - 50^2\right)$$

$= 862.5$

Upward force $= (P_1 - P_2)A$

$= 862.5 \times 50$

$= 43125 N$

From question –

plane is in level flight, $mg = (P_1 - P_2)$ A

$$m = \frac{(P_1 - P_2)A}{g}$$

$$= \frac{43125}{9.8}$$

$m = 4.4 \times 10^3$ kg

**Q. 3. In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius $2.0 \times 10^{-5}$ m and density $1.2 \times 10^3$ kg m$^{-3}$ ? Take the viscosity of air at the temperature of the experiment to be $1.8 \times 10^{-5}$ Pa. How much is the viscous force on the drop at that speed ? Neglect buoyancy of the drop due to air.**

**[NCERT Ad. Ex. Q. 10.28, Page 275]**

**Ans.** Here, $\rho = 1.2 \times 10^3$ kg m$^{-3}$

$\eta = 1.8 \times 10^{-5}$ Nm$^{-2}$

$r = 2 \times 10^{-5}$ m

Using one relation of terminal velocity

$$v = \frac{2r^2\rho g}{9\eta}$$

$$= \frac{2\left(2\times10^{-5}\right)^2\left(1.2\times10^3\right)9.8}{9\left(1.8\times10^{-5}\right)}$$

$$= 5.81 \times 10^{-2}\ \text{ms}^{-1}$$

$$= 5.81\ \text{cms}^{-1}$$

Viscous force

$$F = 6\pi\eta r v$$

$$= 6 \times \frac{22}{7} \times 1.8 \times 10^{-5} \times 2 \times 10^{-5} \times 5.81 \times 10^{-2}$$

$$= 3.94 \times 10^{-10}\ \text{N}.$$

**Q. 4. Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is $4.0 \times 10^{-3}$ kgs$^{-1}$. What is the pressure difference between the two ends of the tube ? (Density of glycerine = $1.3 \times 10^3$ kgm$^{-3}$ and viscosity of glycerine = 0.83 Pa). [You may also like to check if the assumption of laminar flow in the tube is correct].**

**[NCERT Ex. Q. 10.13, Page 273]**

**Ans. Given :** Radius, $r = 1.0\ \text{cm} = 10^{-2}\ \text{m}$

Length of tube, $l = 1.5$ m

Density of glycerine,

$$\rho = 1.3 \times 10^3\ \text{kg m}^{-3}.$$

Mass of glycerine flowing per sec,

$$M = 4 \times 10^{-3}\ \text{kg s}^{-1}$$

Viscosity of glycerine,

$$\eta = 0.83\ \text{Pa}.$$
$$= 0.83\ \text{Nm}^{-2}\ \text{s}.$$

Suppose P = pressure difference between two ends of the tube

and V = Volume of glycerine flowing per sec.

$$V = \frac{M}{\rho} = \frac{4\times10^{-3}\text{kgs}^{-1}}{1.3\times10^{3}\text{kgm}^{-3}}$$

$$= \frac{4}{1.3} \times 10^{-6}$$

$$= 3.08\times10^{-6}\ \text{m}^3\ \text{s}^{-1}$$

Now from Poiseuille's formula, we have

$$V = \frac{\pi p r^4}{8\eta l}$$

or $$p = \frac{8V\eta l}{\pi r^4}$$

or $$p = \frac{8\times0.83\times1.5\times\left(\frac{4}{1.3}\times10^{-6}\right)}{3.142\times\left(10^{-2}\right)^4}$$

$$= 9.7537 \times 10^2\ \text{Pa}$$
$$= 9.8 \times 10^2\ \text{Pa}$$

Reynold's number, $R = \dfrac{4pV}{\pi d\eta}$

$$= \frac{4\times1.3\times10^3\times3.08\times10^{-6}}{\pi(0.02)\times0.83} = 0.3$$

Hence, the flow is laminar.

**Q. 5. In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surface of the wing are 70 ms$^{-1}$ and 63 ms$^{-1}$ respectively. What is the lift on the wing if its area is 2.5 m$^2$. (Take the density of air to be 1.3 kg m$^{-3}$.)**

**[NCERT Ex. Q. 10.14, Page 273]**

**Ans.** Here $\rho = 1.3\ \text{kg m}^{-3}$, $a = 2.5\ \text{m}^2$,

$v_1 = 70\ \text{ms}^{-1}$ and $v_2 = 63\ \text{ms}^{-1}$

Using Bernoulli's equation,

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2$$

$$(P_2 - P_1) = \frac{1}{2}(v_1^2 - v_2^2)\rho$$

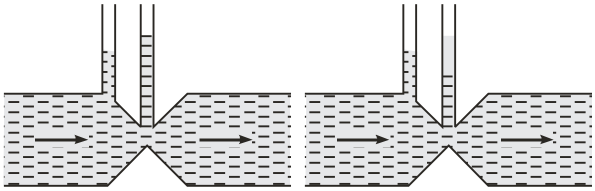
$$= \frac{1}{2}[(70)^2 - (63)^2] \times 1.3$$

$$= 605.2\ \text{Nm}^{-2}.$$

Therefore lift on the wing

$$F = (P_2 - P_1) \times a$$
$$= 605.2 \times 2.5$$
$$= 1513\ \text{N}$$

**Q. 6. Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect ? Why ?**



(a) (b)

**[NCERT Ex. Q. 10.15, Page 273]**

**Ans.** Figure (a) is incorrect. According to the equation of continuity $a$V = constant, where area of flow is more. So, the velocity of liquid flow at a constriction of tube is more than other portion.

According to Bernoulli's theorem

$$P = \frac{1}{2}\rho v^2 = \text{constant}$$

**Q. 7. The cylindrical tube of a spray pump has a cross-section of 8.0 cm$^2$ one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m-min$^{-1}$, what is the speed of ejection of the liquid through the holes ?**

**[NCERT Ex. Q. 10.16, Page 273]**

**Ans.** Here, $a_1 = 8\ \text{cm}^2 = 8 \times 10^{-4}\ \text{m}^2$

$v_1 = 1.5\ \text{m min}^{-1}$
$= 0.025\ \text{ms}^{-1}$

Diameter of one hole,

$$d = 1\ \text{mm} = 10^{-3}\ \text{m}$$

$$\text{Area of hole} = \frac{\pi d^2}{4} = \frac{\pi \times (10^{-3})^2}{4}$$

$$= \frac{\pi}{4} \times 10^{-6}\ \text{m}^2$$

Therefore total cross-section of 40 holes

$$= \frac{\pi}{4} \times 10^{-6} \times 40\ \text{m}^2$$

If $v_2$ is the speed of ejection of the liquid through the hole

$$a_1 v_1 = a_2 v_2$$

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{8 \times 10^{-4} \times 0.025}{\frac{\pi}{4} \times 10^{-6} \times 40}$$

$$= 0.637\ \text{m/s}$$

### TIPS... & TRICKS...

- *Understand about Pascal's law and its applications.*
- *Study about pressure and atmospheric pressure and their units.*
- *Learn and understand Archimedes principle and laws of flotation.*
- *Study intra-molecular forces as cohesive and adhesive Forces.*
- *Understand about surface tension and surface energy.*
- *Study and understand about capillary and its applications.*
- *Study streamline and turbulence of liquid.*
- *Understand critical and terminal velocity.*
- *Study Bernoulli's theorem and its applications.*
- *Study principle of continuity and Stoke's law.*

## Some Commonly Made Errors

- Students get confused in the concept of capillary action.
- Sometime, students get confused whether the flow is laminar or streamline.
- The concept of gauge pressure is not clear to most of the students.

### EXPERT ADVICE

- *Students should give focus to the learning of Bernoulli's principle.*
- *Remember that the gauge pressure is the difference of the actual pressure and atmospheric pressure.*
- *If the Reynold's number is less than 1000 it is laminar or streamline flow. If $R_e$ > 2000 than it is turbulent.*

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**Visit :** *https://youtu.be/jp0pAdVKhbI*

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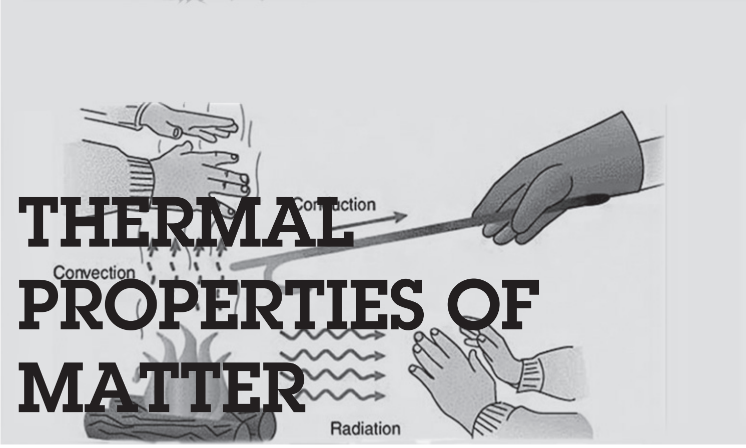
CHAPTER 11

# THERMAL PROPERTIES OF MATTER




## Chapter Objective

**This chapter will help you understand :**

- *Heat, Temperature, Thermal expansion, Thermal expansion of solids, liquids and gases, Anomalous expansion of water, Specific heat capacity : $C_P$ . $C_V$ – Calorimetry, change of state.*
- *Heat transfer-conduction, convection and radiation, Thermal conductivity, Qualitative idea of Black Body radiation, Wien's displacement law, Stefan's law, Green house effect.*

## TOPIC-1
## Thermal Expansion and Heat Capacities



### Quick Review

- **Four Scales of Temperature :**

| S. No. | Scale | Ice point | Steam point | No. of divisions | Smallest division |
|---|---|---|---|---|---|
| 1. | Centigrade scale | 0°C | 100°C | 100 | 1°C |
| 2. | Fahrenheit scale | 32°F | 212°F | 180 | 1°F |
| 3. | Reaumur scale | 0°R | 80°R | 80 | 1°R |
| 4. | Themodynamic scale of Absolute Kelvin scale | 273 K | 373 K | 100 | 1 K |

- **Thermal Expansion of solids. :** It is the phenomenon of expansion of solids on heating. It is of three types :
  - **(a) Linear Expansion :** It is the increase in length of a solid on heating. $\alpha$ is called coefficient of linear expansion.
  - **(b) Area Expansion :** It is the increase in surface area of a solid on heating. $\beta$ is called coefficient of area expansion.
  - **(c) Volume Expansion :** It is the increase in volume of a solid on heating. $\gamma$ is called coefficient of volume expansion.
- **Expansion of liquids :**
  - **(a)** Coefficient of real expansion of a liquid is defined as the real increase in volume of the liquid per unit original volume per °C rise in temperature. If $\gamma_r$ is the coefficient of real expansion of a liquid, then

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

  - **(b)** Coefficient of apparent expansion of a liquid is defined as the apparent increase in volume per unit original volume per °C rise in temperature. If $\gamma_a$ is the coefficient of apparent expansion of a liquid, then

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

## Know the Terms

- **Heat** is a form of energy, which produces the sensation of warmth. The thermal energy in matter is present in the form of translational, rotational and vibrational energy of its atoms/molecules.
- **Temperature** of a body is a measure of degree of hotness/coldness of the body. This macroscopic property determines the direction of flow of heat, when the given body is placed in contact with some other body.
- Anomalous expansion of water is the volume of water decreases with increase in temperature from 0°C to 4°C.
- **Specific heat of a substance** is the amount of heat required to raise the temperature of unit mass of substance through unit degree.
- **Molar specific heat of a substance** is the amount of heat required to raise the temperature of 1 gm mole of substance through 1°C.
- **Heat capacity or thermal capacity** of a body is the amount of heat required to raise the temperature of whole body through 1°C or 1 K.
- **Water equivalent** is the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature & represented by W.
- **Change of state** is the conversion of one of the states of matter to another.
- **Latent heat** of a substance is the amount of heat required to change the state of unit mass of the substance at constant temperature (Q = ML). Its units are cal/g or joule/kg and its dimensions are $[L^2T^{-2}]$.

## Know the Formulae

- **Temperature**

  **(a)** Relation between °C, °F and °R is

$$\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4}$$

  **(b)** T K = ($t$°C + 273) or $t$°C = (T K – 273)

  **(c)** Temperature. diff. of 1°C = Temp. diff. of 1 K.

  **(d)** Normal temp. of a person is 98.6°F or 37°C.

  **(e)** The temp. of – 40° is same in °C and °F.

- **Thermal expansion**

  **(a)** Coeff. of linear expansion $\alpha = \dfrac{\Delta L}{L\Delta T}$

  **(b)** Coeff. of Area expansion $\beta = \dfrac{\Delta S}{S(\Delta T)}$

  **(c)** Coeff. of volume expansion $\gamma = \dfrac{\Delta V}{V(\Delta T)}$

  **(d)** $\beta = 2\alpha$ ; $\gamma = 3\alpha$

  **(e)** In liquids. $\gamma_r = \gamma_a + \gamma_g$

  where,

  $\gamma_r$ = Coefficient of real expansion of liquid

  $\gamma_a$ = Coefficient of apparent expansion of liquid

  $\gamma_g$ = Coefficient of expansion of vessel

- **Specific heat.** $\Delta Q = ms\Delta T$
- **Molar specific heat** $C = M \times s$
- **Latent heat** $\Delta Q = ML$

  $L$ = Latent heat

- **Specific heat of gases** $C_p - C_V = R$, Here $R = \dfrac{PV}{T}$

## Know the Links

- *www.topper.com*
- *www.vedantu.com*
- *www.khanacademy.org*
- *www.learncbse.in*

## MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. A bimetallic strip is made of aluminium and steel ($\alpha_{AL} > \alpha_{steel}$). On heating, the strip will**

**(a) remain straight**

**(b) get twisted**

**(c) will bend with aluminium on concave side.**

**(d) will bend with steel on concave side.**

**[NCERT Exemp. Q. 11.1, Page 77]**

**Ans. Correct option:** (d)

*Explanation:* According to the question,

$\alpha_{AL} > \alpha_{steel}$

∴ Aluminium will expand more than that of steel strip. Due to it, this steel strip will bend on concave side

**Q. 2. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly**

**(a) its speed of rotation increases.**

**(b) its speed of rotation decreases.**

**(c) its speed of rotation remains same.**

**(d) its speed increases because its moment of inertia increases.**

**[NCERT Exemp. Q. 11.2, Page 77]**

**Ans. Correct option:** (b)

*Explanation:* Its M.I increases .

∴ Angular momentum, $L = I\omega$,

$\omega$ is angular speed which decreases to conserve L.

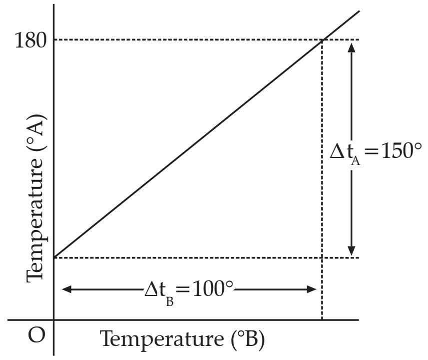
**Q. 3. The graph between two temperature scales A and B is shown in figure. Between upper fixed point and lower fixed point there are 150 equal division on scale A and 100 on scale B. The relationship for conversion between the two scales is given by**

(a) $\dfrac{t_A - 180}{100} = \dfrac{t_B}{150}$

(b) $\dfrac{t_A - 30}{150} = \dfrac{t_B}{100}$

(c) $\dfrac{t_B - 180}{150} = \dfrac{t_A}{100}$

(d) $\dfrac{t_B - 40}{100} = \dfrac{t_A}{180}$




**[NCERT Exemp. Q. 11.3, Page 77]**

**Ans. Correct option:** (b)

*Explanation:* For the scale A,

Lower fixed point = 30°A

Upper fixed point = 180°A

For the scale B,

Lower fixed point = 0°B

Upper fixed point = 100°B

∴ The relationship between the two scales A & B is given by

$$\frac{T_A - 30}{150} = \frac{T_B - 0}{100}$$

**Q. 4. An aluminium sphere is dipped into water. Which of the following is true?**

**(a) Buoyancy will be less in water at 0°C than that in water at 4°C.**

**(b) Buoyancy will be more in water at 0°C than that in water at 4°C.**

**(c) Buoyancy in water at 0°C will be same as that in water at 4°C.**

**(d) Buoyancy may be more or less in water at 4°C depending on the radius of the sphere.**

**[NCERT Exemp. Q. 11.4, Page 78]**

**Ans. Correct option:** (a)

*Explanation:* Buoyancy force at 0°C

$F_0{}^\circ C = VP_4{}^\circ C$

and Buoyancy at 4°C

$F_4{}^\circ C = VP_4{}^\circ C$

$$\Rightarrow \frac{F_4{}^\circ C}{F_0{}^\circ C} = \frac{R_4{}^\circ C}{P_0{}^\circ C} > 1 \Rightarrow F_4{}^\circ C > F_0{}^\circ C$$

**Q. 5. As the temperature is increased, the time period of a pendulum**

**(a) increases as its effective length increases even though its centre of mass still remains at the centre of the bob.**

**(b) decreases as its effective length increases even though its centre of mass still remains at the centre of the bob.**

**(c) increases as its effective length increases due to shifting of centre of mass below the centre of the bob.**

**(d) decreases as its effective length remains same but the centre of mass shifts above the centre of the bob. [NCERT Exemp. Q. 11.5, Page 78]**

**Ans. Correct option:** (a)

*Explanation:* Let $T = 2\pi\sqrt{\frac{L_0}{g}}$ at temperature $\theta_o$

and $T' = 2\pi\sqrt{\frac{L}{g}}$ at temperature θ

$$\therefore \frac{T'}{T} = \sqrt{\frac{L}{L_0}} = \sqrt{\frac{L_0(1+\alpha\Delta\theta)}{L_0}} = 1+\frac{1}{2}\alpha\Delta\theta > 1$$

$\therefore$ T′ > T

**Q. 6. The radius of a metal sphere at room temperature T is R , and the coefficient of linear expansion of the metal is α. The sphere is heated a little by a temperature ΔT so that its new temperature is T + ΔT. The increase in the volume of the sphere is approximately**

**(a) $2\pi R\alpha\Delta T$**

**(b) $\pi R^2\alpha\Delta T$**

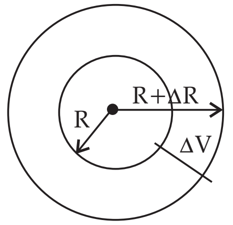
**(c) $4\pi R^3\alpha\Delta T/3$**

**(d) $4\pi R^3\alpha\Delta T$**

**[NCERT Exemp. Q. 11.7, Page 79]**

**Ans. Correct option:** (d)

*Explanation:* As the temperature increases radius of the sphere increases as shown. So, the volume of the sphere increases.




Original volume of the sphere,

$$V_0 = \frac{4}{3}\pi R^3$$

Coefficient of linear expansion = α

Hence, coefficient of volume expansion = 3α

$$\because \quad \gamma = \frac{\Delta V}{V\times\Delta T} \text{ and } \gamma = 3\alpha$$

$$\therefore \quad 3\alpha = \frac{V}{\left(\frac{4}{3}\pi R^3\right)\Delta T}$$

or $\Delta V = 4\pi R^3\alpha\Delta T$

## Very Short Answer Type Questions (1 mark each)

**Q. 1. Is the bulb of a thermometer made of diathermic or adiabatic wall?**

**[NCERT Exemp. Q. 11.13, Page 80]**

**Ans.** The bulb of a thermometer is made of diathermic because diathermic walls allow exchange of heat energy between two systems while adiabatic do not.

**Q. 2. A student records the initial length l, change in temperature ΔT and change in length Δl of a rod as follows :**

| S. No | L(m) | ΔT (°C) | Δl (m) |
|---|---|---|---|
| 1. | 2 | 10 | $4 \times 10^{-4}$ |
| 2. | 1 | 10 | $4 \times 10^{-4}$ |
| 3. | 2 | 20 | $2 \times 10^{-4}$ |
| 4. | 3 | 10 | $6 \times 10^{-4}$ |

**If the first observation is correct, what can you say about observations 2, 3 and 4 ?**

**[NCERT Exemp. Q. 11.14, Page 80]**

**Ans.**

$$\alpha = \frac{\Delta l}{l\Delta T}$$

From I observation, $\alpha = \frac{4\times10^{-4}}{2\times10} = 2\times10^{-5}\, C^{-1}$

For II, observation $\Delta l = \alpha l\Delta T$

$= 2\times10^{-5}\times1\times1$

$= 2\times10^{-4} \neq 4\times10^{-4}$ m

For III, observation $\Delta l = \alpha l\Delta T$

$= 2\times10^{-5}\times2\times20$

$= 8\times10^{-4}$ m $= 2\times10^{-4}$ m

For IV, observation $\Delta l = \alpha l\Delta T$

$= 2\times10^{-5}\times3\times10$

$= 6\times10^{-4}$ m $= 6\times10^{-4}$ m

Therefore, IV observation is correct and II, III are wrong.

**Q. 3. Calculate the temperature which has same numeral value on celsius and Fahrenheit scale.**

**[NCERT Exemp. Q. 11.16, Page 80]**

**Ans.** Let T be the value of temperature which has same numeral value on Celsius and Fahrenheit scale.

Now, $\frac{°F-32}{180}=\frac{°C}{100}$

Let F=C=T

$\frac{T-32}{180}=\frac{T}{100}$

or T=−40°C or −40°F

**Q. 4. Answer the following :**

**(a) The triple point of water is a standard fixed point in modern thermometer. Why ? What is wrong in taking melting point of ice and the boiling point of water as standard fixed points (as was originally done in Celsius scale) ?**

**(b) There were two fixed points in the original Celsius scale as mentioned above which were assigned the numbers 0°C and 100°C respectively. On the absolute scale one of the fixed points is the triple point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) Scale ?**

**(c) The absolute temperature (Kelvin scale) T is related to the temperature T on the Celsius scale by $T_K$ = T – 273.15. Why do we have 273.15 in this relation and not 273.16 ?**

**(d) What is the temperature of triple point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale ?**

**[NCERT Ex. Q. 11.4, Page 299]**

**Ans. (a)** This is because the triple point of water is unique condition of temperature and pressure, *i.e.*, it occurs only at one particular set of values of pressure and temperature. Hence, it is easily reproducible. Hence, it is used as a standard temperature in thermometry.

The melting point of ice and boiling point of water are pressure dependent which means they change with change in the value of pressure. They are also very sensitive to the dissolved impurities. Therefore these have been discarded as a reference point and triple of water has been taken as a fixed point.

(b) The 0 K or absolute zero itself is the other fixed point on the absolute scale of temperature. It corresponds to the temperature when the volume and pressure of a gas will become zero.

(c) Triple point is 0.01°C and not 0°C. Thus to make a celsius degree equal to Kelvin degree 273.15 is assigned instead of 273.16. Moreover ice point on Kelvin scale is 273.15 K and the corresponding is point on celsius scale is 0°C which is clear from the relation, *i.e.*, $T_K$ = T – 273.15. When we have 273.16 instead of 273.15 in the above relation, then the ice point on celsius scale will be 0 which is not the case.

(d) The unit interval size of Fahrenheit scale is 212 – 32 = 180 is given by

$$T = \frac{273.16}{100} \times 180$$

$$= 491.69$$

# Short Answer Type Questions **(2 or 3 marks each)**

**Q. 1. Find out the increase in moment of inertia I of a uniform rod (coefficient of linear expansion α) about its perpendicular bisector when its temperature is slightly increased by ΔT.**

**[NCERT Exemp. Q. 11.18, Page 81]**

**Ans.** M.I. about its axis along perpendicular bisector

$$=\frac{1}{12}ml^2$$

When temperature increased by ΔT, length of rod increases.

$$\Delta l = l\alpha\Delta T$$

$$\therefore \text{ New M.I., } I_1=\frac{M}{12}(l+\Delta l)^2=\frac{M}{12}(l^2+\Delta l^2+2l\Delta l)$$

Neglecting $(\Delta l)^2$ (very very small quantity) –

$$I_1=\frac{M}{12}(l^2+2l\Delta l)$$

$$=\frac{Ml^2}{12}+\frac{Ml\Delta l}{6}=I+\frac{Ml\Delta l}{6}.$$

Therefore, moment of inertia increase.

$$\Delta I = I_1 - I = \frac{Ml\Delta l}{6}$$

$$=2\left(\frac{Ml^2}{12}\right)\frac{\Delta l}{l}$$

$$\Delta I = 2I\alpha\Delta T.$$

**Q. 2. 100 g of water is supercooled to –10°C. At this point, due to some disturbance mechanized or otherwise some of it suddenly freezes to ice. What will be the temperature of the resultant mixture and how much mass would freeze?**
**[$S_w$ = 1 cal/g/°C and $L^w_{Fusion}$ = 80 cal / g]**

**[NCERT Exemp. Q. 11.20, Page 81]**

**Ans.** Given :

Mass of water = 100gm

Change in temperature = 0 – (–10) = 10°C

Specific heat of water $S_w$ = 1Cal /g/°c

Latent heat of fusion $L^w_{fusion}$ = 80 cal/g

Heat required by water from −10°C to 0°C.

$$Q = ms\Delta T$$

$$= 100 \times 1 \times 10$$

$$Q = 1000 \text{ cal}$$

Let ice melted be $m$ gm,

$$Q = mL$$

or $$m = \frac{Q}{L} = \frac{1000}{80} = 12.5 \text{ gm.}$$

**Q. 3. Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between $T_A$ and $T_B$?**

**[NCERT Ex. Q. 11.2, Page 299]**

**Ans. Given :**

Triple point of water = 200A = 350B = 273.16K

$$\therefore\ 1A = \frac{273.16}{200}K, \quad 1B = \frac{273.16}{350}K.$$

If $T_A$ and $T_B$ represent the triple point of water on scales A & B,

$$\frac{273.16}{200}T_A = \frac{273.16}{350}T_B$$

or $$\frac{T_A}{T_B} = \frac{200}{350} = \frac{4}{7}$$

$$\therefore\ T_A = \frac{4}{7}T_B$$

## Long Answer Type Questions (5 marks each)

**Q. 1. We would like to prepare a scale whose length does not change with temperature. It is proposed to prepare a unit scale of this type whose length remains, say 10 cm. We can use a bimetallic strip made of brass and iron each of different length whose length (both components) would change in such a way that difference between their lengths remain constant. If $\alpha_{iron} = 1.2 \times 10^{-5}$ / K and $\alpha_{brass} = 1.8 \times 10^{-5}$ / K, what should we take as length of each strip?**

**[NCERT Exemp. Q. 11.22, Page 82]**

**Ans. From question,**

$l_{iron} - l_{brass}$ = 10 cm = constant at all temperatures.

Let $l_o$ be length at temperature 0°C and length l after change in temperature.

$$l^o_{iron} - l^o_{brass} = 10 \text{ cm at all tempertature}$$

$$\therefore\ l^o_{iron}(1+\alpha_{iron}\Delta t) - l^o_{brass}(1+\alpha_{brass}\Delta t) = 10\ cm$$

$$l^o_{iron}\alpha_{iron} = l^o_{brass}\alpha_{brass}$$

$$\therefore\ \frac{l^o_{iron}}{l^o_{brass}} = \frac{1.8\times10^{-5}}{1.2\times10^{-5}} = \frac{1.8}{1.2} = \frac{3}{2}$$

Then, $l^o_{iron} - l^o_{brass} = 10$

$$3x - 2x = 10$$

$$x = 10$$

Length of iron rod = 3 × 10 = 30 cm

Length of brass rod = 2 × 10 = 20 cm

The difference between length is 10cm.

**Q. 2. We would like to make a vessel whose volume does not change with temperature (take a hint from the problem above). We can use brass and iron ($\beta_{vbrass} = 6 \times 10^{-5}$/K and $\beta_{viron} = 3.55 \times 10^{-5}$/K) to create a volume of 100 cc. How do you think you can achieve this ? [NCERT Exemp. Q. 11.23, Page 82]**

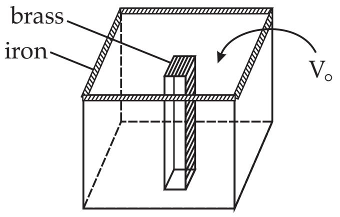
**Ans. From question,**

The difference in volume is constant.

Let $V_i$ and $V_b$ be the volume of iron and brass vessel at 0°C respectively, and $V_i'$ and $V_b'$ be the volume at 0°C.

Similarly, $\gamma_i$ & $\gamma_b$ be the volume expansion coefficients of iron & brass respectively.

Now,




$$V_i - V_b = 100\text{ cc} = V'_i - V'_b \qquad \text{(i)}$$

$$V'_i = V_i(1+\gamma_1\Delta\theta)$$

$$V'_b = V_b(1+\gamma_b\Delta\theta)$$

$$V'_i - V'_b = (V_i - V_b) + \Delta\theta(V_i\gamma_1 - V_b\gamma_b)$$

As, $V_i - V_b$ = constant

$\therefore$ $V_i\gamma_i = V_b\gamma_b$

$$\Rightarrow \frac{V_i}{V_b} = \frac{\gamma_b}{\gamma_i} = \frac{\frac{3}{2}\beta_b}{\frac{3}{2}\beta_i} = \frac{\beta_b}{\beta_i}$$

$$= \frac{6\times10^{-5}}{3.55\times10^{-5}} = \frac{6}{3.55}$$

$$\frac{V_i}{V_b} = \frac{6}{3.55} \qquad (ii)$$

From eqn (s), (i) & (ii)

$V_i$ = 244.9cc

$V_b$ = 144.9cc

**Q. 3. Calculate the stress developed inside a tooth cavity filled with copper when hot tea at temperature of 57°C is drunk. You can take body (tooth) temperature to be 37°C and $\alpha = 1.7 \times 10^{-5}$/°C, bulk modulus for copper = $140 \times 10^9$ N/m².**

**[NCERT Exemp. Q. 11.24, Page 82]**

**Ans. Given :**

Change in temperature

$$\Delta T = 57°\,C - 37°\,C = 20°\,C$$

Linear expansion, $\alpha = 1.7 \times 10^{-5}$° $C^{-1}$

Cubical expansion, $r = 3\alpha$

$$= 3 \times 1.7 \times 10^{-5}$$

$$= 5.1 \times 10^{-5}\ K^{-1}$$

Let V be the volume of cavity, due to increase in temperature ΔT, volume increased by ΔV,

$$\Delta V = \gamma V \Delta T$$

$$\text{or } \frac{\Delta V}{V} = \gamma \Delta T$$

Thermal stress produced = B × volumetric strain

$$= B \times \frac{\Delta V}{V}$$

$$= B \times \gamma \Delta T$$

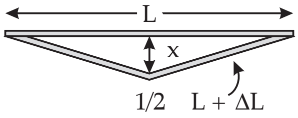
Thermal stress = $140 \times 10^9 \times 1.7 \times 10^{-5} \times 20$

$= 14280 \times 10^4$

$= 1.428 \times 10^8\ N/m^2$

Thermal stress is about $10^3$ times of atmospheric pressure i.e., $1.01 \times 10^5\ N/m^2$

**Q. 4. A rail track made of steel having length 10 m is clamped on a railway line at its two ends as shown in fig. On a summer day due to rise in temperature by 20° C, it is deformed as shown in figure. Find x (displacement of the centre) if $\alpha_{steel} = 1.2 \times 10^{-5}/°C$.**




**[NCERT Exemp. Q. 11.25, Page 82]**

**Ans.** Let us consider the diagram below,

Given: $\alpha = 1.2 \times 10^{-5}°C^{-1}$, L = 10m, ΔT = 20°C.

By pythagoras theorem,

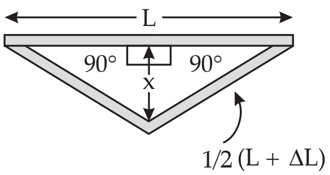
$$x^2 = \left(\frac{L}{2} + \frac{\Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2$$

$$x = \sqrt{\left(\frac{L}{2} + \frac{\Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2}$$

$$x = \frac{1}{2}\sqrt{\Delta L^2 + 2L\Delta L}$$

$\Delta L^2 << L$, ∴ neglecting $\Delta L^2$

$$x = \frac{1}{2} \times \sqrt{2L\Delta L}$$




But ΔL = LαΔ t

$$\therefore x = \frac{1}{2}\sqrt{2L \times L\alpha\Delta t}$$

$$= \frac{1}{2}L\sqrt{2\alpha\Delta t}$$

$$= \frac{10}{2} \times \sqrt{2 \times 1.2 \times 10^{-5} \times 20}$$

$$= 5 \times \sqrt{4 \times 1.2 \times 10^{-4}}$$

$$= 5 \times 2 \times 1.1 \times 10^{-2}$$

$$= 0.11 \text{ m}$$

$x = 11$ cm

**Q. 5. The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperature on the celsius and fahrenheit scales.**

**[NCERT Ex. Q. 11.1, Page 299]**

**Ans.** Relation between Kelvin and Celcius scale

$T_C = T_K - 273.15$

where $T_C$, $T_K$ are temperatures in Celsius and Kelvin respectively.

**For Neon :** $T_C = 24.57 - 273.15$

$= -248.58°C$

**For $CO_2$ :** $T_C = 216.55 - 273.15$

$= -56.60°C$

Relation between Kelvin and Fahrenheit

$$\frac{T_F - 32}{180} = \frac{T_K - 273.15}{100}$$

$$T_F = \frac{9}{5}(T_K - 273.15) + 32$$

**For Neon :** $T_F = \frac{9}{5}(24.57 - 273.15) + 32$

$= -415.44°F$

**For $CO_2$ :** $T_F = \frac{9}{5}(216.55 - 273.15) + 32$

$= -69.88°F$

**Q. 6. The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law!**
**$R = R_o [1 + \alpha(T - T_o)]$. The resistance is 101.6 W at the normal melting point of lead 600.5 K. What is the temperature when the resistance is 123.4 W?**

**[NCERT Ex. Q. 11.3, Page 299]**

**Ans.** $R_o = 101.6\ \Omega$, $T_o = 273.16$ K, $R_1 = 165.5\ \Omega$,

$T_1 = 600.5$ K, $R_2 = 123.4\ \Omega$,

As, $R = R_o [1 + \alpha(T - T_o)]$

$165.5 = 101.6 [1 + \alpha (600.5 - 273.16)]$

$$\frac{165.5}{101.6} = 1 + (\alpha \times 327.34)$$

$\alpha = 1.92 \times 10^{-3}\ K^{-1}$

Also, $R_1 = R_0 [1 + \alpha(T - T)]$

$123.4 = 101.6 [1 + \alpha (T_2 - 273.16)]$

$123.4 = 101.6 [1 + 1.92 \times 10^{-3} (T_2 - 273.16)]$

[from (1) putting α]

$$\therefore T_2 = \frac{0.214}{1.92 \times 10^{-3}} + 273.16$$

$T_2 = 384.83$ K

**Q. 7. Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:**

| Temperature | Pressure thermometer A | Pressure thermometer B |
|---|---|---|
| Triple-point of water | $1.250 \times 10^5$ Pa | $0.200 \times 10^5$ Pa |
| Normal melting point of sulphur | $1.797 \times 10^5$ Pa | $0.287 \times 10^5$ Pa |

**(a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?**

**(b) What do you think is the reason behind the slight difference in answers of thermometers A and B? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings?**

**[NCERT Ex. Q. 11.5, Page 300]**

**Ans. (a)** Triple point of water, $T = 273.16$ K

At this temperature, pressure in thermometer A, $P_A = 1.250 \times 10^5\ Pa$

Let $T_1$ be the normal melting point of sulphur.

At this temperature, pressure in thermometer, $P_1 = 1.797 \times 10^5\ Pa$

According to Charles' law, we have the relation:

$$\frac{P_A}{T} = \frac{P_1}{T_1}$$

$$\therefore T_1 = \frac{P_1 T}{P_A} = \frac{1.797 \times 10^5 \times 273.16}{1.25 \times 10^5}$$

$$= 392.69\ K$$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer A is 392.69 K.

At triple point 273.16 K, the pressure in thermometer B,

$P_B = 0.200 \times 10^5\ Pa$

At temperature $T_1$, the pressure in thermometer B,

$P_2 = 0.287 \times 10^5\ Pa$

According to Charles' law, we can write the relation:

$$\frac{P_B}{T} = \frac{P_2}{T_1}$$

$$\frac{0.200 \times 10^5}{273.16} = \frac{0.287 \times 10^5}{T_1}$$

$$\therefore\ T_1 = \frac{0.287 \times 10^5}{0.200 \times 10^5} \times 273.16 = 391.98\,K$$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer B is 391.98 K.

**(b)** The oxygen and hydrogen gas present in thermometers A and B respectively are not perfect ideal gases. Hence, there is a slight difference between the readings of thermometers A and B.

To reduce the discrepancy between the two readings, the experiment should be carried under low pressure conditions. At low pressure, these gases behave as perfect ideal gases.

**Q. 8. A steel tape 1m long is correctly calibrated for a temperature of 27.0°C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0°C. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0°C? Coefficient of linear expansion of steel = $1.20 \times 10^{-5}\,K^{-1}$.**

**[NCERT Ex. Q. 11.6, Page 300]**

**Ans.** Let length of the steel tape at temperature $T = 27°C$, $l = 1$ m = 100 cm

At temperature $T_1 = 45°C$, the length of the steel rod, $l_1 = 63$ cm

Coefficient of linear expansion of steel, $\alpha = 1.20 \times 10^{-5}\,K^{-1}$

Let $l_2$ be the actual length of the steel rod and $l'$ be the length of the steel tape at 45°C.

$$l' = l + \alpha l\,(T_1 - T)$$

$$\therefore l' = 100 + 1.20 \times 10^{-5} \times 100(45 - 27)$$

$$= 100.0216 \text{ cm}$$

Hence, the actual length of the steel rod measured by the steel tape at 45°C can be calculated as:

$$l_2 = \frac{100.0216}{100} \times 63 = 63.0136 \text{ cm.}$$

Therefore, the actual length of the rod at 45.0°C is 63.0136 cm. Its length at 27.0°C is 63.0 cm.

**Q. 9. A large steel wheel is to be fitted on a shaft of the same material. At 27°C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft ? Assume the coefficient of linear expansion of the steel to be constant over the required temperature range. $\alpha_{steel} = 1.20 \times 10^{-5}K^{-1}$.**

**[NCERT Ex. Q. 11.7, Page 300]**

**Ans.** Suppose $l_1$ and $l_2$ be the outer diameter dimensions of a steel at temperatures $T_1$ and $T_2$ respectively.

Given $\alpha_{steel} = 1.20 \times 10^{-5}K^{-1}$.

$l_1 = 8.70$ cm

$l_2 = 8.69$ cm

$T_1 = 27°C = 273 + 27 = 300K$

$T_2 = ?$

When the steel shaft is cooled, its linear dimension, *i.e.*, its diameter decreases according to the formula.

$l_2 = l_1[1 + \alpha(T_2 - T_1)]$ ...(i)

When the shaft is cooled to temperature $T_2$ such that $l_2 = 8.69$ cm, the wheel can slip on the shaft.

$\therefore$ By putting all values in eq. (i), we get

$8.69 = 8.70[1 + 1.20 \times 10^{-5}(T_2 - 300)]$

or $T_2 - 300 = \dfrac{8.69 - 8.70}{8.70 \times 1.20 \times 10^{-5}}$

$= -95.78$ K

or $T_2 = 300 - 95.78 = 204.22$ K

$= 204.22 - 273.15$

$= -68.93°C$

or $T_2 = -69°C$.

**Q. 10. A hole is drilled in a Cu sheet. The diameter of the hole is 4.24 cm at 27°C. What is the $\Delta d$ ($d$ = diameter) of the hole when the sheet is heated to 227°C ? (Coefficient of linear expansion of Cu = $1.7 \times 10^{-5}\,°C^{-1}$.) [NCERT Ex. Q. 11.8, Page 300]**

**Ans.** Using, $S_2 = S_1(1 + \beta\Delta T)$ when

$$S_1 = \frac{\pi d_1^2}{4}$$

$$= \frac{\pi}{4}(4.24)^2$$

$$= \pi \times 4.494 \text{ cm}^2$$

$$\Delta T = 227 - 27$$

$$= 200°C$$

$$\beta = 2\alpha$$

$$= 2 \times 1.7 \times 10^{-5} \text{ °C}^{-1}.$$

we get, $S_2 = (\pi \times 4.94)[1 + (2 \times 1.7 \times 10^{-5})(200)]$

$$= \pi \times 4.525 \text{ cm}^2$$

or $\frac{\pi d_2^2}{4} = \pi \times 4.525$

or $d_2 = \sqrt{4 \times 4.525} = 4.254$ cm

Change in diameter

$$= d_2 - d_1$$

$$= 4.254 - 4.24 = 0.014 \text{ cm}$$

**Q. 11. A brass wire 1.8 m long at 27°C is held tight with little tension between two rigid supports. If the wire is cooled to a temperature of – 39°C. What is the tension developed in the wire, if its diameter is 2.0 m ? (Co-efficient of linear expansion of brass = 2.0 × $10^{-5}$ $K^{-1}$; young's modulus of brass = 0.91 × $10^{11}$ Pa). [NCERT Ex. Q. 11.9, Page 300]**

**Ans.** Here, $l_1 = 1.8$ m,

$$\Delta T = (-39) - 27$$

$$= -66°C$$

$$\alpha = 2 \times 10^{-5} \text{ °C}^{-1}$$

$$l_2 = l_1(1 + \alpha\Delta T)$$

$$= 1.8[1 + (2 \times 10^{-5})(-66)]$$

$$= 1.798 \text{ m}$$

Change in length

$$= 1.8 - 1.798$$

$$= 0.002 \text{ m}$$

Here $a = \frac{\pi d^2}{4} = \frac{\pi}{4}(2 \times 10^{-3})^2$

$$= 3.14 \times 10^{-6} \text{ m}^2$$

$$Y = \frac{F}{A} \times \frac{l}{\Delta l}$$

or $F = \frac{YA\Delta l}{l}$

$$= (0.91 \times 10^{11})(3.14 \times 10^{-6})\left(\frac{0.002}{1.8}\right)$$

$$= 3.81 \times 10^2 \text{ N}$$

**Q. 12. A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter, what is the change in length of the combined rod at 250°C, if the original lengths are at 40.0°C ? Is there a 'thermal stress' developed at the junction ? The ends of the rod are free to expand. (Cofficient of linear expansion of brass = 2.0 × $10^{-5}$ $K^{-1}$; steel = 1.2 × $10^{-5}$ $K^{-1}$). [NCERT Ex. Q. 11.10, Page 300]**

**Ans.** Using $l_2 = l_1(1 + \alpha\Delta T)$,

$$l_2 = 50[1 + (2 \times 10^{-5})(250 - 40)]$$

$$= 50.21 \text{ cm}$$

Steel Rod $l_2' = l_1'(1 + \alpha\Delta T')$,

$$l_2' = 50[1 + (1.2 \times 10^{-5})(250 - 240)]$$

$$= 50.126 \text{ cm}$$

Combined length = 50.21 + 50.126

= 100.336

Change in combined length at 250°C

= 100.336 – 100

= 0.336 cm

**Q. 13. The coefficient of volume expansion of glycerine is 49 × $10^{-5}$ °$C^{-1}$. What is the fractional change in its density for a 30°C rise in temperature ? [NCERT Ex. Q. 11.11, Page 301]**

**Ans.** Here, $\Delta T = 30°C$

and $\gamma = 49 \times 10^{-5} \text{ °C}^{-1}$

$$V = V_0[1 + \gamma\Delta T),$$

we get

$$V = V_0[1 + (49 \times 10^{-5})(30)]$$

$$= 1.015 V_0$$

Initial density, $\rho_0 = \frac{m}{V_0}$

Final density, $\rho = \frac{m}{V} = \frac{m}{1.015V_0} = \frac{\rho_0}{1.015}$

$$= 0.986 \rho_0$$

Fractional change in density

$$= \frac{\rho_0 - \rho}{\rho_0} = \frac{\rho_o 0.986\rho_o}{\rho_o} = 0.014$$

$$= 0.014$$

**Q. 14. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 min, assuming 50% of power is used up heating the machine itself or lost to the surrounding ? (Specific heat of aluminium = 0.91 $Jg^{-1}$ $k^{-1}$.) [NCERT Ex. Q. 11.12, Page 301]**

**Ans.** Here $t = 2.5 \text{ min} = 2.5 \times 60\ s$

and $P = 10^4$ W

Energy used in drilling

$$Q' = P \times t$$

$$= 1.5 \times 10^6 \text{ J}$$

Useful energy $Q = Q' \times \frac{50}{100} = \frac{1.5 \times 10^6 \times 50}{100}$

$$= 7.5 \times 10^5 \text{ J}$$

Using $Q = mc\Delta T$,

$$7.5 \times 10^5 = 8 \times 0.91 \times 10^3 \Delta T$$

$$7.5 \times 10^5 = 7.28 \times 10^3 \Delta T$$

or $\Delta T = \frac{7.5 \times 10^5}{7.28 \times 10^3} = 103°C$

**Q. 15. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of Copper = 0.39 $Jg^{-1}$ $K^{-1}$, Fusion heat of Water = 335 J/g.) [NCERT Ex. Q. 11.13, Page 301]**

**Ans.** Here $M_1 = 2.5$ kg, T = 500°C

and $\mathbf{c} = 0.39 \times 10^3$ Jkg$^{-1}$ °k$^{-1}$

Using $Q = Mc\Delta t$,

Heat absorbed by Cu block,

$$Q = 2.5 \times 0.39 \times 10^3 \times 500 \text{ J}$$

Heat absorbed by the ice block to melt,

$$Q = ML$$
$$= M \times 3.35 \times 10^3 \text{ J}$$

Then

$$(3.35 \times 10^3) M = 2.5 \times 0.39 \times 10^3 \times 500$$

$$M = \frac{2.5 \times 0.39 \times 10^3 \times 500}{3.35 \times 10^3}$$

$$= 1.455 \text{ kg}$$

**Q. 16. In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in Cu calorimeter of water equivalent 0.025 kg containing 150 cm$^3$ of water at 27°C. The final temperature is 40°C. Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal ?**

**[NCERT Ex. Q. 11.14, Page 301]**

**Ans.** Here $T_1 = 150°C$,

$T_2 = 27°C$

and $\mathbf{T} = 40°C$

$M_1 = 0.20$ kg

and $W = 0.25$ kg

vol. of water in calorimeter

$= 150$ cm$^3$

$= 150 \times 10^{-6}$ m$^3$

Mass of water in calorimeter,

$$M_2 = (150 \times 10^{-6}) \times 10^3$$
$$= 150 \times 10^{-3} \text{ kg}$$

Using,

Heat gained = Heat Lost

$$(M_2 + W)\, c_2(T - T_2) = M_1 c_1 (T_1 - T)$$

$$(150 \times 10^{-3} + 0.025) \times (4.2 \times 10^{-3})(40 - 20)$$
$$= 0.20 c_1(150 - 40)$$

or $\mathbf{9.55 \times 10^{-3}} = 22c_1$

$$c_1 = 434 \text{ J kg}^{-1} \text{°C}^{-1}$$

**Q. 17. Given below are observations on molar specific heats at room temperature of some common gases.**

| Gas | Molar specific heat ($C_v$) (cal mol$^{-1}$) $K^{-1}$ |
|---|---|
| **Hydrogen** | **4.87** |
| **Nitrogen** | **4.97** |
| **Oxygen** | **5.02** |
| **Nitric oxide** | **4.99** |
| **Carbon monoxide** | **5.01** |
| **Chlorine** | **6.17** |

**The measured molar specific heats of these gases are markedly different from those for monatomic gases. Typically, molar specific heat of a monatomic gas is 2.92 cal/mol K. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?**

**[NCERT Ex. Q. 11.15, Page 301]**

**Ans.** The gases listed in the given table are diatomic. Besides the translational degree of freedom, they have other degrees of freedom (modes of motion). Heat must be supplied to increase the temperature of these gases. This increases the average energy of all the modes of motion. Hence, the molar specific heat of diatomic gases is more than that of monatomic gases.

If only rotational mode of motion is considered, then the molar specific heat of a diatomic gas $= \frac{5}{2}R$

$$= \frac{5}{2} \times 1.98 = 4.95 \,\text{cal mol}^{-1}\text{K}^{-1}$$

With the exception of chlorine, all the observations in the given table agree with $\left(\frac{5}{2}R\right)$.

This is because at room temperature, chlorine also has vibrational modes of motion besides rotational and translational modes of motion.

**Q. 18. A child running a temperature of 101 °*F* is given an antipyrine (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98 °*F* in 20 min, what is the average rate of extra evaporation caused, by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 *cal g*$^{-1}$.**

**[NCERT Ex. Q. 11.16, Page 301]**

**Ans.** Initial temperature of the body of the child, $T_1 = 101°F$

Final temperature of the body of the child, $T_2 = 98°F$

Change in temperature,

$$\Delta T = 101 - 98 = 3°F = 3 \times \frac{5}{9}$$

Time taken to reduce the temperature, $t = 20$ min

Mass of the child, m = 30 kg

Specific heat of the human body = Specific heat of water = c = 1000 cal / kg / °C

Latent heat of evaporation of water, $L = 580$ cal g$^{-1}$

The heat lost by the child is given as:

$$\Delta Q = mc\Delta T$$

$$= 30 \times 1000 \times (101 - 98) \times \frac{5}{9}$$

$$= 50000 \, cal$$

Let $m_1$ be the mass of the water evaporated from the child's body in 20 min.

Loss of heat through water is given by:

$$\Delta Q = m_1 L$$

$$\therefore m_1 = \frac{\Delta Q}{L}$$

$$= \frac{50000}{580} = 86.2g$$

$\therefore$ Average rate of extra evaporation caused by the drug $= \frac{m_1}{t}$

$$= \frac{86.2}{20} = 4.3g / \text{min}$$

# TOPIC-2
## Heat Transfer

## Quick Review

- **Thermal Conductivity :**

  **(i) Coefficient of Thermal Conductivity :** It is equal to rate of flow of heat per unit area per unit temperature gradient across the solid. It is represented by K & its value depends on nature of material of solid.

$$K = \frac{\Delta Q \Delta x}{\Delta T A}$$

$$K = \Delta Q, \text{ when } \left(\frac{\Delta x}{\Delta T}\right) = 1, A = 1$$

  **(ii) Thermal resistance** corresponds to electrical resistance (V/*i*) and is given by Temperature diff/rate of flow of heat *i.e.*,

$$R_{Th} = \frac{T_1 - T_2}{dQ/dt}$$

$$= \frac{x}{KA}$$

  where symbols have their usual meaning.

  Clearly, greater is the value of K, smaller is the thermal resistance.

- **Total emittance or emissive power** of a body at a certain temperature is the total amount of thermal energy emitted per unit time per unit area of the body for all possible wavelengths. It is represented by $e'$

$$e' = \int_0^\infty e_\lambda d\lambda$$

- **Emissivity** ($\varepsilon$) of a body at a given temperature is the ratio of emissive power of the body ($e$) to the emissive power of perfectly black body (E) at that temperature,

  *i.e.,* $$\varepsilon = \frac{e}{E}$$

  Similarly, we can define monochromatic absorptance or spectral absorptive power. Total absorptance or absorbing power

$$a = \int_0^\infty a_\lambda d\lambda$$

- **Kirchhoff's Law.** From this law, at a given temperature and for a given wavelength, the ratio of spectral emissive power ($e_\lambda$) to spectral absorptive power ($a_\lambda$) for all bodies is constant which is equal to spectral emissive power of a perfectly black body ($E_\lambda$) at the same temperature and for the same wavelength, *i.e.,* $\frac{e_\lambda}{a_\lambda} = E_\lambda$ clearly, $e_\lambda \propto a_\lambda$ it means good emitters are good absorbers. The law implies that at a particular temperature, a body can absorb only those wavelengths, which it is capable of emitting.

- **Wien's law :** From this law, the wavelength ($\lambda_m$) corresponding to which energy emitted/sec/area by a perfectly black body is maximum, is inversely proportional to the absolute temperature (T) of the black body.

$$\lambda_m \propto \frac{1}{T}$$

  or $$\lambda_m = \frac{b}{T}$$

where $b$ is a constant of proportionality and is known as Wien's constant $b = 2.898 \times 10^{-3}$ mK.

- **Newton's law of cooling.** According to this law, when difference in temperature of a liquid and its surroundings is small (~ 30°C), then the rate of loss of heat of the liquid is directly proportional to difference in temperatures of the liquid and the surroundings, *i.e.*,

$$-\frac{dQ}{dt} \propto (\theta - \theta_0)$$

or $$-\frac{dQ}{dt} = K(\theta - \theta_0)$$

where K is constant of proportionality.

- **Stefan's law :** From this law, the total energy emitted/sec/area (E) by a perfectly black body corresponding to all wavelengths is directly proportional to fourth power of the absolute temp. (T) of the body *i.e.*

$$E \propto T^4$$

or $$E = \sigma T^4$$

There σ is a constant of proportionality and is called *Stefan's constant*. Its value is

$$\sigma = 5.67 \times 10^{-8} \text{ watt m}^{-2}\text{K}^{-4}$$

If Q is the total amount of heat energy emitted by the black body, then by definition,

$$E = \frac{Q}{At}$$

$$\therefore \quad Q = At \times E = At(\sigma T^4)$$

If the body is not perfectly black and has an emissivity $e$, then $Q = eAt(\sigma T^4)$

- **Stefan Boltzman law :** From this law, the net amount of radiation emitted per second per unit area of a perfectly black body at temperature T is equal to difference in the amounts of radiation emitted/sec/area by the body and by the black body enclosure at $T_0$.

$$\Rightarrow \quad E' = E - E_0$$

As $$E = \sigma T^4$$

and $$E_0 = \sigma T_0^4$$

$$\therefore \quad E' = \sigma T^4 - \sigma T_0^4$$
$$= \sigma(T^4 - T_0^4)$$

Proceeding as above, total energy lost

$$Q' = E'At$$
$$= At\sigma(T^4 - T_0^4)$$

When the body and enclosure are not perfectly black and have emissivity ε, then

$$Q' = \varepsilon At\sigma(T^4 - T_0^4)$$

## Know the Terms

- **Conduction** is the mode of transfer of heat from one part of the body to another, from particle to particle in the direction of fall of temperature without any actual movement of heated particles.
- **Thermal convection** is the phenomenon of transfer of heat by actual mass motion of the medium. All liquids and gases are heated by convection.
- **Radiation** is the phenomenon of transfer of heat from source to the receiver without any actual movement of source or receiver and without heating the intervening medium. For example, heat comes to us from the sun through radiation.
- **Energy flux** is the energy flowing per second per unit area normal to surface. Its unit is watt/$m^2$.
- **Energy density** is the total energy per unit volume. Its unit is joule/$m^3$.
- **Steady-State** is the state of rod in which temperature of each part becomes constant & there is no further absorption of heat.
- **Variable state** is the state of rod in which temperature of every cross-section of the rod goes on increasing.
- **Monochromatic emittance or spectral emissive power** of a body corresponding to a particular wavelength λ at a particular temperature is the amount of radiant energy emitted per unit time per unit surface area of the body

within unit wavelength interval around λ. It is represented by $e_\lambda$.

- **Perfectly black body** is that which absorbs all the radiations incident upon it. Thus absorptive power of a perfectly black body is unity (*i.e.*, 100%). When such a body is heated to high temperature, it would emit radiations of all wavelengths.

## Know the Formulae

- **Rate of conduction of Heat,** $\frac{\Delta Q}{\Delta t} = KA\frac{\Delta T}{\Delta x}$

  where,

  $$\frac{\Delta T}{\Delta x} = \text{temperature gradient}$$

- **Thermal resistance,** $R_{Th} = \frac{T_1 - T_2}{dQ/dt}$
- **Emissive power,** $e' = \int_0^\infty e_\lambda d\lambda$
- **Emissivity,** $\varepsilon = \frac{e}{E}$

## Know the Links

- *www.vedantu.com*
- *www.khanacademy.org*
- *www.learncbse.in*

## MCQ/Fillups/True or False **(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. Heat is associated with**

**(a) kinetic energy of random motion of molecules.**
**(b) kinetic energy of orderly motion of molecules.**
**(c) total kinetic energy of random and orderly motion of molecules.**
**(d) kinetic energy of random motion in some cases and kinetic energy of orderly motion in other.**

**[NCERT Exemp. Q. 11.6, Page 78]**

**Ans. Correct option:** (a)

*Explanation:* When a body is heated its temperature rises and in liquids and gases vibration of molecules about thin mean position increases, hence kinetic energy associated with random motion of molecules increases. So thermal energy or heat associated which the random and translatory motions of molecules.

**Q. 2. A sphere, a cube and a thin circular plate, all of same material and same mass are initially heated to same high temperature.**

**(a) Plate will cool fastest and cube the slowest**
**(b) Sphere will cool fastest and cube the slowest**
**(c) Plate will cool fastest and sphere the slowest**
**(d) Cube will cool fastest and plate the slowest.**

**[NCERT Exemp. Q. 11.8, Page 79]**

**Ans. Correct option:** (c)

*Explanation:* Sphere, cube and plate have a same mass, here clearly the surface area of plate is greater than sphere and cube i.e.,

A sphere < A cube < A plate

Here Emissive energy ∝ Area

So that plate will cool fastest and sphere the slowest.

**Q. 3. Mark the correct options :**

**(a) A system X is in thermal equilibrium with Y but not with Z. System Y and Z may be in thermal equilibrium with each other.**
**(b) A system X is in thermal equilibrium with Y but not with Z. Systems Y and Z are not in thermal equilibrium with each other.**
**(c) A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z must be in thermal equilibrium with each other.**
**(d) A system X is neither in thermal equilibrium with Y nor with Z. The system Y and Z may be in thermal equilibrium with each other.**

**[NCERT Exemp. Q. 11.9, Page 79]**

**Ans. Correct option:** (b) and (d)

*Explanation:* According to Problem :

**(b)** If two system X and Y are in thermal equilibrium *i.e.*, $T_x = T_y$ and X is not in thermal equilibrium with Z, *i.e.*, $T_x \neq T_y$ then clearly $T_y \neq T_z$

**(d)** If X is not in thermal equilibrium with $y$. *i.e.*, $T_x \neq T_y$ and also X is not in thermal equilibrium with Y, *i.e.*, $T_y \neq T_z$

Then we cannot say about equilibrium of Y and Z, they may or may not be in equilibrium.

**Q. 4. 'Gulab Jamuns' (assumed to be spherical) are to be heated in an oven. They are available in two sizes, one twice bigger (in radius) than the other. Pizzas (assumed to be discs) are also to be heated in oven. They are also in two sizes, one twice big (in radius) than the other. All four are put together to be heated to oven temperature. Choose the correct option from the following :**

**(a) Both size gulab jamuns will get heated in the same time.**

**(b) Smaller gulab jamuns are heated before bigger ones.**

**(c) Smaller pizzas are heated before bigger ones.**

**(d) Bigger pizzas are heated before smaller ones.**

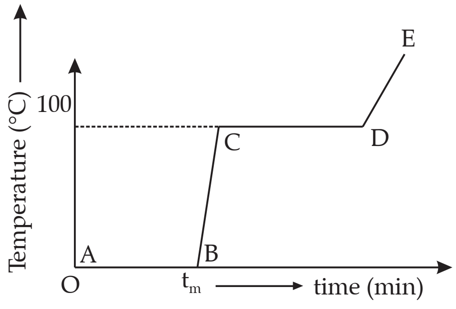
**[NCERT Exemp. Q. 11.10, Page 75]**

**Ans. Correct option:** (b) and (c)

*Explanation:* We know that the least surface area will be heated first because of less heat radiation. So smaller Gulab Jamuns are having least surface area. Hence they will be heated first.

Similarly, smaller Pizzas are heated before bigger ones because they are of small surface areas.

**Q. 5. Refer to the plot of temperature versus time as shown in fig. showing the changes in the state of ice on heating (not to scale). Which of the following is correct?**




**(a) The region AB represents ice and water in thermal equilibrium.**

**(b) At B water starts boiling.**

**(c) At C all the water gets converted into steam.**

**(d) C to D represents water and steam in equilibrium at boiling point.**

**[NCERT Exemp. Q. 11.11, Page 80]**

**Ans. Correct option:** (a) and (d)

*Explanation:*

**(a)** In region AB, a phase change takes place, heat is supplied and ice melts but temperature of the system is 0ºC. It remains constant during process. The heat supplied is used to break bonding between molecules.

**(b)** In region CD, again a phase change takes place from a liquid to a vapour state during which temperature remains constant. If shows water and steam are in equilibrium at boiling point.

**Q. 6. A glass full of hot milk is poured on the table. It begins to cool gradually. Which of the following is correct?**

**(a) The rate of cooling is constant till milk attains the temperature of the surrounding.**

**(b) The temperature of milk falls off exponentially with time.**

**(c) While cooling, there is a flow of heat from milk to the surrounding as well as from surrounding to the milk but the net flow of heat is from milk to the surrounding and that is why it cools.**

**(d) All three phenomenon, conduction, convection and radiation are responsible for the loss of heat from milk to the surrounding.**

**[NCERT Exemp. Q. 11.12, Page 80]**

**Ans. Correct option:** (b) (c) and (d)

*Explanation:* When hot milk spread on table heat is transferred to the surroundings by conduction, convection and radiation. Because the surface of area of poured milk on a table is more than the surface area of milk filled in a glass. Hence, its temperature falls off exponentially according to Newton's law of cooling. heat also will be transferred a from surrounding to the milk but will be lesser than that of transferred from milk to the surroundings.

## Very Short Answer Type Questions (1 mark each)

**Q. 1. Why does a metal bar appear hotter than a wooden bar at the same temperature? Equivalently it also appears cooler than wooden bar if they are both colder than room temperature.**

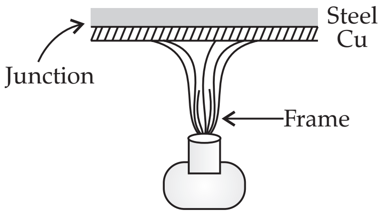
**[NCERT Exemp. Q. 11.16, Page 81]**

**Ans.** Due to difference in conductivity, metals have high conductivity compared to wood. On touch with a finger, heat from the surrounding flows faster to the finger from metals and so one feels the heat. Similarly, when one touches a cold metal the heat from the finger flows away to the surroundings faster.

**Q. 2. These days people use steel utensils with copper bottom. This is supposed to be good for uniform heating of food. Explain this effect using the fact that copper is the better conductor.**

**[NCERT Exemp. Q. 11.17, Page 81]**

**Ans.**



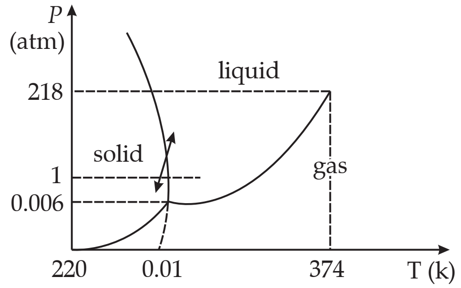
Since copper has a high conductivity in comparison to steel. The junction of copper and steel absorbs heat more quickly than steel and give it to the food in utensils. Therefore, food inside utensils is heated uniformly and quickly.

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. During summers in India, one of the common practice to keep cool is to make ice balls of crushed ice, dip it in flavoured sugar syrup and sip it. For this a stick in inserted into crushed ice and is squeezed in the palm to make it into the ball. Equivalently in winter, in those areas where it snows, people make snow balls and throw around. Explain the formation of ball out of crushed ice or snow in the light of P-T diagram of water. [NCERT Exemp. Q. 11.19, Page 81]**

**Ans.** Refer to the P – T diagram of water and double headed arrow. Increasing pressure at 0°C and 1 atm takes ice into liquid state and decreasing pressure in liquid state at 0°C and 1 atm takes water to ice state.

When crushed ice is squeezed, some of it melts, filling up gap between ice flakes, upon releasing pressure. This water freezes, binding all ice flakes making the ball more stable.




**Q. 2. One day in the morning, Ramesh filled up 1/3 bucket of hot water from geyser, to take bath. Remaining 2/3 was to be filled by cold water (at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh had to attend to something which would take some time, say 5–10 minutes before he could take bath. Now he had two options : (i) fill the remaining bucket completely by cold water and then attend to the work. (ii) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer?**

**[NCERT Exemp. Q. 11.21, Page 85]**

**Ans.** The first option would have kept water warmer because according to Newton's law of cooling, the rate of loss of heat is directly proportional to the difference of temperature of the body and the surrounding and in the first case the temperature difference is less, so rate of loss of heat will be less.

## Long Answer Type Questions (5 marks each)

**Q. 1. A thin rod having length $L_0$ at 0°C and coefficient of linear expansion $\alpha$ has its two ends maintained at temperatures $\theta_1$ and $\theta_2$, respectively. Find its new length.**

**[NCERT Exemp. Q. 11.26, Page 82]**

**Ans.** The rod's temperature varies from $\theta_1$ to $\theta_2$ [from one end to another].

So, mean temperature of rod is

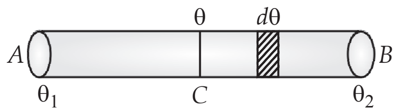
$$\theta = \frac{\theta_1 + \theta_2}{2} \text{ (at C)}$$

So, rate of flow heat A → C and C → B are equal.

$\theta_1 > \theta > \theta_2$

$$\text{therefore, } \frac{d\theta}{dt} = \frac{KA(\theta_1 - \theta)}{L_0/2} = \frac{KA(\theta - \theta_2)}{L_0/2}$$

Here, K = Coefficient of thermal conductivity.




$$\therefore \theta_1 - \theta = \theta - \theta_2$$

$$\theta = \frac{\theta_1 + \theta_2}{2}$$

Using relation. $L = L_0 (1 + \alpha\theta)$

$$\text{Thus, } L = L_0\left[1 + \alpha\left(\frac{\theta_1 + \theta_2}{2}\right)\right]$$

**Q. 2. According to Stefan's law of radiation, a black body radiates energy $\sigma T^4$ from its unit surface area every second where T is the surface temperature of the black body and $\sigma = 5.67 \times 10^{-8}$ W/ m$^2$K$^4$ is known as Stefan's constant. A nuclear weapon may be thought of as a ball of radius 0.5 m. When detonated, it reaches temperature of $10^6$K and can be treated as a black body.**

**(a) Estimate the power it radiates.**

**(b) If surrounding has water at 30°C, how much water can 10% of the energy produced evaporate in 1 s?**

**[$s_w$ = 4186.0 J/kg K and $L_v$ = 22.6 × $10^5$ J/kg]**

**(c) If all this energy U is in the form of radiation, corresponding momentum is p = U/c. How much momentum per unit time does it impart on unit area at a distance of 1 km?**

**[NCERT Exemp. Q. 11.27, Page 82]**

**Ans. (a)** Power radiated,

$$P = \sigma A T^4 = \sigma(4\pi R^2)T^4$$

$$= 5.67 \times 10^{-8} \times 4 \times 3.14 \times (0.5)^2 \times (10^6)^4$$

$$= 1.78 \times 10^{17} \text{ J/s}$$

$$\approx 1.8 \times 10^{17} \text{ J/s}$$

(b) Energy available per second,

$$E = 1.8 \times 10^{17} \text{ J/s}$$

$$= 18 \times 10^{16} \text{ J/s}$$

Energy required to evaporate water =

10% of E = 18 × $10^{16}$ J/s

Energy used (per second) to raise temperature 30° C→100°C and then into vapour (at 100°C)-

$$= mS_w\Delta\theta + mL_v$$

$$= m \times 4186 \times (100 - 30) + m \times 22.6 \times 10^5$$

$$= 2.93 \times 10^5 m + 22.6 \times 10^5 \text{m}$$

$$= m(25.53 \times 10^5 \text{ J/s})$$

From question,

$$m(25.53 \times 10^5) = 1.8 \times 10^{16}$$

or $$m = \frac{1.8 \times 10^{16}}{25.53 \times 10^5}$$

$$= 7.0 \times 10^9 \text{ kg.}$$

**(c)** Momentum per unit time,

$$P = \frac{E}{c}$$

$$= \frac{1.8 \times 10^{17}}{3 \times 10^8}$$

$$= 6 \times 10^8 \text{ kg.m/s}^2$$

Momentum per unit time per unit area,

$$P_1 = \frac{P}{4\pi R^2} = \frac{6 \times 10^8}{4 \times 3.14 \times (10^3)^2}$$

$$P_1 = 47.7 \text{ N/m}^2$$

**Q. 3. A 'thermocol' icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45 °C, and co-efficient of thermal conductivity of thermocol is 0.01 J $s^{-1}m^{-1}K^{-1}$. [Heat of fusion of water = 335 × $10^3$J $kg^{-1}$]. [NCERT Ex. Q. 11.17, Page 301]**

**Ans.** Side of the given cubical ice box, $s$ = 30 cm = 0.3 m

Thickness of the ice box, $l$ = 5.0 cm = 0.05 m

Mass of ice kept in the ice box, m = 4 kg

Time gap, $t$=6h = 6× 60 × 60s

Outside temperature, $T$ = 45°C

Coefficient of thermal conductivity of thermocol, $K$ = 0.01$Js^{-1}m^{-1}K^{-1}$

Heat of fusion of water, $L$= 3 35 × $10^3$ J $kg^{-1}$

Let m′ be the total amount of ice that melts in 6 h.

The amount of heat lost by the food:

$$Q = \frac{KA(T_1 - T_2)t}{l}$$

where,

$A$ = Surface area of the box = $6s^2$

= 6 × $(0.3)^2$

= 0.54 $m^3$

$$Q = \frac{0.01 \times 0.54 \times (45) \times 6 \times 60 \times 60}{0.05} = 104976 \text{ J}$$

But $Q = m'L$

$$\therefore m' = \frac{Q}{L}$$

$$= \frac{104976}{335 \times 10^3} = 0.313 \text{ kg}$$

Mass of ice left = 4 – 0.313 = 3.687 kg

Hence, the amount of ice remaining after 6 h is 3.687 kg

**Q. 4. A brass boiler has a base area 0.15 $m^2$ and thickness 1.0 cm. It boils water at the rate of 6.0 kg $min^{-1}$ when placed on a gas stove. Estimate temperature of the part of the same in contact with the boiler. (Thermal conductivity of brass = 109 $Js^{-1}$ $m^{-1}$ $°C^{-1}$, heat of vaporisation of water = 2256 × $10^3$ J $kg^{-1}$.)**

**[NCERT Ex. Q. 11.17, Page 301]**

**Ans.** Using

$$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{d},$$

we get $$\frac{Q}{t} = \frac{109 \times 0.15}{10^{-2}}(T_1 - 100)$$

$$\frac{Q}{t} = 1635(T_1 - 100) \quad \text{...(i)}$$

Also $$\frac{Q}{t} = \frac{ML}{t}$$

$$= \frac{6}{60} \times 2256 \times 10^3$$

$$\frac{Q}{t} = 225.6 \times 10^3 \quad \text{...(ii)}$$

*i.e.,* $$1635(T_1 - 100) = 225.6 \times 10^3$$

$$T_1 - 100 = \frac{225.6 \times 10^3}{1635}$$

$$T_1 = 137.98 + 100$$

$$T_1 = 237.98°\text{C.}$$

**Q. 5. Explain why :**

**(a) a body with large reflectivity is a poor emitter**

**(b) a brass tumbler feels much colder than a wooden tray on a chilly day**

**(c) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace**

**(d) the earth without its atmosphere would be inhospitality cold**

**(e) heating systems based on circulation of steam are more efficient in warning a building than those based on circulation of hot water**

**[NCERT Ex. Q. 11.19, Page 301]**

**Ans. (a)** This is because a body with large reflectivity is poor absorber of heat, and poor absorbers of heat are poor emitters.

**(b)** When we touch a brass tumbler on a chilly day, heat flows from our body to the tumbler quickly (as thermal conductivity of brass is very high) and as result, it appears colder. On the other hand, as the wood is a bad conductor, heat does not flow to the wooden tray from our body, on touching it.

**(c)** When the red hot iron piece is in the oven, its temperature TK is given by the relation.

$$X = 1\left[\frac{mg}{YA}\right]^{1/3} = 0.5\left[\frac{0.1\times 10}{20\times 10^{11}\times 0.5\times 10^{-6}}\right]^{1/3}$$

But, if the red hot iron piece is in the open having the surrounding temperature $= 1.074 \times 10^{-2}$ m = 1 = 1.074 cm, its energy is radiated according to relation. $6.9 \times 10^{7}$ Pa. As the working principle of optical pyrometer is based on the fact that the brightness of a glowing surface of a body depends upon its temperature. Therefore, pyrometer gives too low a value for the temperature of red iron in the open.

**(d)** The lower layers of earth's atmosphere reflect infrared radiations from earth back to the surface of earth. Thus the heat radiation received by the earth from the sun during the day are kept trapped by the atmosphere. If atmosphere of earth were not there, its surface would become too cold to live.

**(e)** Steam at 100°C possesses more heat than the same mass of water at 100°C. One gram of steam at 100°C possesses 540 calories of heat more than that possessed by 1 gm of water at 100°C. That is why heating systems based on circulation of steam are more efficient than those based on circulation of hot water.

**Q. 6. A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C. The temperature of the surroundings is 20°C.** **[NCERT Ex. Q. 11.20, Page 302]**

**Ans.** We know that, as per Newton's law of cooling,

$$\frac{dT}{dt} = -K(T - T_s)$$

$$\frac{dT}{T - T_s} = -K\,dt$$

$$\int_{T_1}^{T_2} \frac{dT}{T - T_s} = -\int_{t_1}^{t_2} K\,dt$$

$$\log_e [T - T_s]_{T_1}^{T_2} = -K(t_2 - t_1)$$

As it is given in the first part of this question

$T_s = 20°C$,

$T_1 = 80°C$,

$T_2 = 50°C$,

$t_2 - t_1 = 5$ minutes $= 300$ s.

$$\therefore \log_e\left(\frac{50-20}{80-20}\right) = -K(300)$$

$$\log_e\left(\frac{30}{60}\right) = -300\,K$$

$$-0.6931 = -300\,K$$

or $K = 0.002310°C\ s^{-1}$ ...(i)

Using this value of K for the second part of the question,

$$\log_e\left(\frac{30-20}{60-20}\right) = -(0.002310)\times t$$

$$-0.002310t = -1.39629$$

or $t = 600$ s

$t = 10$ minutes.

**Q. 7. Answer the following questions based on the *P*-*T* phase diagram of carbon dioxide:**

**(a) At what temperature and pressure can the solid, liquid and vapour phases of $CO_2$ co-exist in equilibrium?**

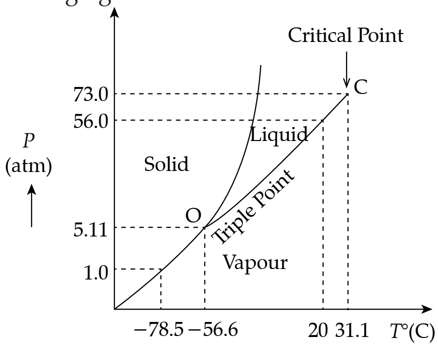
**(b) What is the effect of decrease of pressure on the fusion and boiling point of $CO_2$ ?**

**(c) What are the critical temperature and pressure for $CO_2$ ? What is their significance?**

**(d) Is $CO_2$ solid, liquid or gas at (a) –70 °C under 1 atm, (b) –60 °C under 10 at m, (c) 15 °C under 56 atm?**

**[NCERT Ad. Ex. Q. 11.21, Page 302]**

**Ans. (a)** The *P*-*T* phase diagram for $CO_2$ is shown in the following figure.



O is the triple point of the $CO_2$ phase diagram. This means that at the temperature and pressure corresponding to this point (i.e., at –56.6°C and 5.11 atm), the solid, liquid, and vapour phases of $CO_2$ co-exist in equilibrium.

**(b)** The fusion and boiling points of $CO_2$ decrease with a decrease in pressure.

**(c)** The critical temperature and critical pressure of $CO_2$ are 31.1°C and 73 atm respectively

Even if it is compressed to a pressure greater than 73 atm, $CO_2$ will not liquefy above the critical temperature.

**(d)** It can be concluded from the *P-T* phase diagram of $CO_2$ that:

**(a)** $CO_2$ is gaseous at –70°C, under 1 atm pressure

**(b)** $CO_2$ is solid at –60°C, under 10 atm pressure

**(c)** $CO_2$ is liquid at 15°C , under 56 atm pressure

**Q. 8. Answer the following questions based on the P–T phase diagram of $CO_2$ :**

**(a) $CO_2$ at 1 atm pressure and temperature –60 °C is compressed isothermally. Does it go through a liquid phase?**

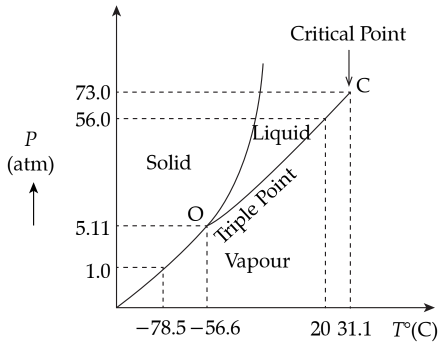
**(b) What happens when $CO_2$ at 4 atm pressure is cooled from room temperature at constant pressure?**

**(c) Describe qualitatively the changes in a given mass of solid $CO_2$ at 10 atm pressure and temperature –65 °C as it is heated up to room temperature at constant pressure.**

**(d) $CO_2$ is heated to a temperature 70 °C and compressed isothermally. What changes in its properties do you expect to observe?**

**[NCERT Ad. Ex. Q. 11.22, Page 302]**

**Ans. (a)** The *P-T* phase diagram for $CO_2$ is shown in the following figure.




At 1 atm pressure and at –60°C, $CO_2$ lies to the left of –56.6°C (triple point C). Hence, it lies in the region of vapour and solid phases.

Thus, $CO_2$ condenses into the solid state directly, without going through the liquid state.

(b) At 4 atm pressure, $CO_2$ lies below 5.11 atm (triple point C). Hence, it lies in the region of vapour and solid phases. Thus, it condenses into the solid state directly, without passing through the liquid state.

(c) When the temperature of a mass of solid $CO_2$ (at 10 atm pressure and at –65 °C ) is increased, it changes to the liquid phase and then to the vapour phase. It forms a line parallel to the temperature axis at 10 atm. The fusion and boiling points are given by the intersection point where this parallel line cuts the fusion and vaporisation curves.

(d) If $CO_2$ is heated to 70°C and compressed isothermally, then it will not exhibit any transition to the liquid state. This is because 70°C is higher than the critical temperature of $CO_2$. It will remain in the vapour state, but will depart from its ideal behaviour as pressure increases.

**TIPS... & TRICKS...**

- *Understand all type of expansions and coefficient of expansions.*
- *Study relations between all type of temperature scales.*
- *Study and understand Latent heat and heat capacities.*
- *Understand about kinds of heat transfer.*
- *Study and understands Kirchhoff's law, wien's law, Newtons law of cooling and Stefan's law.*
- *Study coefficient of Thermal conductivity and thermal resistance.*

# Some Commonly Made Errors

➢ Generally, during calculations students makes mistake by using wrong values of specific heat and coefficient of linear expansion.

## EXPERT ADVICE

☞ *Practice and focus on numericals based on triple point concept.*

☞ *Numerical questions on calorimetry involves lengthy calculations, so avoid calculations mistakes and provide the solutions in steps.*

## OSWAAL LEARNING TOOLS

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| | |
|---|---|
| **Visit :** *https://youtu.be/WzX5tpC8Bs8* **Or Scan the Code** | **Visit :** *https://youtu.be/bwR83HA4Cvk* **Or Scan the Code** |
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CHAPTER

# 12

# THERMODYNAMICS

## Chapter Objective

**This chapter will help you understand :**

- *Thermal equilibrium and definition of temperature (Zeroth law of thermodynamics), heat, work and internal energy, First law of thermodynamics, Isothermal and adiabatic processes.*
- *Second law of thermodynamics : reversible and irreversible processes, Heat engine and refrigerator.*

## TOPIC-1
## Zeroth Law, Heat and First Law



### Quick Review

- **Facts About Specific Heat :** The amounts of heat required to raise the temperature of one unit of unit mass of a substance.

  **(a) Specific heat at constant volume** ($C_V$) = amount of heat required to raise the temperature of one gram of gas through 1°C at constant volume.

  **(b) Specific heat at constant pressure** ($C_P$) = amount of heat required to raise the temperature of one gram of gas through 1°C at constant pressure.

  **(c) Molar specific heat at constant volume/pressure** = amount of heat required to raise the temperature of one gram mole of the gas through 1°C at constant volume/pressure. It is represented by $C_V$ and $C_P$ respectively. Thus

  $$C_V = M \times C_v$$

  and $$C_P = M \times c_P$$

  where M is molecular weight of the gas. $C_v$ and $C_P$ are measured in cal/gram mole/°C.

  **(d)** Out of the two principal specific heats of a gas,

  $$C_P > C_v$$

  **(e)** The ratio of two principle specific heats of a gas is always greater than 1, *i.e.*,

  $$\gamma = \frac{C_p}{C_v} > 1$$

  **(f)** The value of γ depends upon nature of the gas,

  $$\gamma = 1 + \frac{2}{n}$$

  where $n$ is the number of degrees of freedom of the molecules of the gas,

  $$n = 3A - R$$

  where A is no. of atoms in each molecule and R is no. of independent relations among the atoms in a molecule.

- **Internal energy :** The energy possessed by the molecules of a gas by virtue of their particular configuration and molecular motion is called internal energy of the gas. It is of two types :

  **(a) Internal potential energy ($U_P$) :** It is due to molecular configuration, *i.e.,* due to mutual interaction of atoms/ molecules.

  **(b) Internal kinetic energy ($U_k$) :** It is due to motion of the molecules of the gas. Hence,

  $$U = U_P + U_K$$

Internal energy of a **real gas** depends on volume of the gas and also on temperature of the gas.

- **Four thermodynamical operations :** There are following four operations :

**(a) Isothermal changes,** where the temperature remains constant. The pressure and volume of a given mass of gas changes.

Two essential conditions are :

**(i)** Walls of container must be perfectly conducting.

**(ii)** Changes must be slow.

**(b) Adiabatic changes,** where the heat content of a gaseous system remains constant. The pressure and volume of given mass of gas change with consequent change in temperature.

Two essential conditions are :

**(i)** Walls of container must be perfectly insulating.

**(ii)** Changes must be sudden.

**(c) Isobaric changes,** where pressure is kept constant.

**(d) Isochoric changes,** where volume is kept constant.

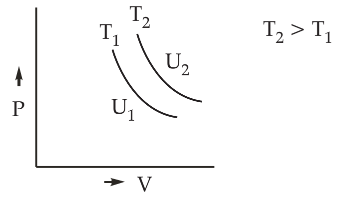
- **Characteristics of isothermal process :**

**(a)** T = constant or $\Delta T = 0$

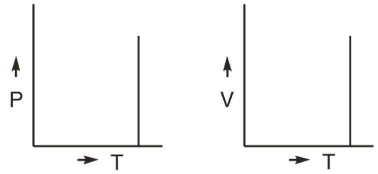
**(b)** U = constant or $\Delta U = 0$

**(c)** Equation of isothermal changes is PV = constant

**(d)** Variation of P with V at constant temperature is represented by Isothermal curves, which are rectangular hyperbola as shown in following figures.




**(e)** The P-T and V-T graphs are straight lines perpendicular to temperature axis. They are shown in figures.




**(f)** Bulk modulus of elasticity under isothermal conditions is given by

$$K_i = \frac{\Delta P}{\Delta V/V} = -P$$

- **Characteristics of adiabatic process :**

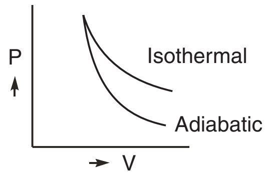
**(a)** Q = constant or $\Delta Q = 0$

**(b)** If S represents entropy, then from $\Delta S = \frac{\Delta Q}{\Delta T} = 0$

*i.e.*, there is no change of entropy in an adiabatic change. That is why an adiabatic process is called isentropic process.

**(c)** Equation of adiabatic changes is $PV^{\gamma}$ = constant, where $\gamma = \frac{C_P}{C_V}$

**(d)** The variation of P with V at constant heat content is represented by an Adiabatic curve, shown in figure, which is also a rectangular hyperbola.




**(e)** Since, it is clear from figure an adiabatic curve is steeper than an isothermal curve. In fact,

slope of adiabatic curve = $\gamma$-times the slope of isothermal curve.

**(f)** Bulk modulus of elasticity, under adiabatic conditions is given as

$$K_a = \frac{\Delta P}{\Delta V/V} = -\gamma P = \gamma K_i$$

- **Zeroth law of thermodynamics** corresponds to the concept of temperature, thermal equilibrium of a system. According to this law, if two systems A and B are separately in thermal equilibrium with a third system C, then A and B will also be in thermal equilibrium with each other.
- **First law of thermodynamics** is basically the law of conservation of energy. According to this law, when a certain amount of heat energy ($dQ$) disappears, an equivalent amount of energy appears in some other form. When ($dU$) is small increase in internal energy and ($dW$) is small amount of external work done by the system in expansion, then

$$dQ = dU + dW$$

## Know the Terms

- **Thermodynamics** is a branch of physics which deals with conversion of heat energy into mechanical work and vice-versa.
- **Thermodynamical system & thermodynamical parameters.**
  A gaseous system is called a thermodynamical system. The state of the system is represented in terms of pressure (P), volume (V), temperature (T) and heat content (Q) of the gas. These four quantities are called thermodynamical parameters of the system.
- **Thermodynamic equilibrium :** A system is said to be in thermodynamic equilibrium when macroscopic variables like pressure, volume, temperature, mass, composition etc. that characterise the system do not change with time.
- **Heat** is the transfer of kinetic energy from one medium or object to another, or from an energy source to a medium or object.
- **Temperature** is the degree of hotness or coldness of a body.
- **Open system :** Exchanges both energy & matter with surroundings.
- **Closed system :** Exchanges only energy with surroundings.
- **Isolated system :** Exchanges neither energy nor matter with surroundings.
- **Equation of state** is the equation connecting pressure, volume and temperature of the gas.

## Know the Formulae

- **Mayer's relation :** $C_p - C_v = \frac{R}{J}$ where R is gas constant for one gram mole of the gas.
- **Equation of state for :**
  - **(a) an ideal gas :** $PV = RT$
  - **(b) a real gas :** $(P + a/V^2)(V - b) = RT$
  - **(c) an isothermal process :** $PV$ = Constant.
  - **(d) an isobaric process :** $\frac{V}{T}$ = Constant.
  - **(e) an isochoric process :** $\frac{P}{T}$ = Constant.
  - **(f) an adiabatic process :** $PV^{\gamma}$ = Constant; $TV^{\gamma-1}$ = Constant and $\frac{P^{\gamma-1}}{T^{\gamma}}$ = constant.
- **Work done during expansion of gas :** $dW = PdV$

or $W = \int_{V_1}^{V_2} PdV$

**(a) In an isothermal process :**

$$W = 2.3026\ RT \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$W = 2.3026\ RT \log_{10}\left(\frac{P_1}{P_2}\right)$$

**(b) In an adiabatic process :**

$$W = \frac{R}{1-\gamma}(T_2 - T_1)$$

$$= \frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$$

Other expressions for work done by a gas in adiabatic expansion may be expressed as

$$W = \frac{R}{1-\gamma}(T_1 - T_2)$$

$$= \frac{RT_1}{\gamma - 1}\left(1 - \frac{T_2}{T_1}\right)$$

$$W = \frac{(C_P - C_V)}{\left(\frac{C_P}{C_V} - 1\right)}(T_1 - T_2) \qquad \left[\because R = C_P - C_V \text{ and } \gamma = \frac{C_P}{C_V}\right]$$

$$= C_V(T_1 - T_2)$$

$$= \frac{C_P}{\gamma}(T_1 - T_2)$$

**(c) In an isobaric process :** $W = P(V_2 - V_1)$

**(d) In a isochoric process :**

As V = constant, $dV = 0, \therefore W = 0$

**(e) In a non-cyclic process :**

W = area of P-V curve = area enclosed between the PV curve and volume axis.

The value of W would depend upon the **path followed** by the system in going from initial state to final state.

**(f) In a cyclic process :**

W = area enclosed by the closed loop representing the cyclic process.

W is positive when loop is traced in clockwise direction and W is negative, when loop is traced in anticlockwise direction.

When a gas **expands,** work is done by the gas. It is taken as **Positive.**

When a gas is **compressed,** work is done on the gas. It is taken as **Negative.**

- If heat is converted into work or work is converted into heat, then W = JQ, where J = Joule's mechanical equivalent of heat = 4.2 joule/calorie.

  The value of W could be $mgh$ or $\frac{1}{2}mv^2$.

## Know the Links

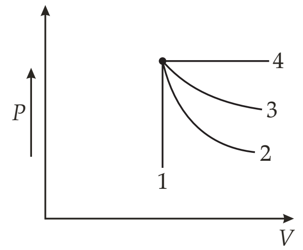
- *www.vedantu.com*
- *www.learncbse.in*
- *www.meritnation.com*

## MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. An ideal gas undergoes four different processes from same initial state as shown in fig. Four processes are adiabatic, isothermal, isobaric and isochoric. Out of 1,2,3 and 4 which one is adiabatic.**



(a) 4 (b) 3
(c) 2 (d) 1

[NCERT Exemp. Q. 12.1, Page 83]

**Ans. Correct option:** (c)

*Explanation:* Graph (1) represent Isochoric process and (2) represent adiabatic, (3) and (4) represents Isothermic and Isobaric process.

**Q. 2. If an average person jogs, he produces 14.5 × 103 cal/min. This is removed by the evaporation of sweat. The amount of sweat evaporated per minute (assuming 1 kg requires $580 \times 10^3\, cal$ for evaporation) is**

**(a) 0.25 kg (b) 2.25 kg**
**(c) 0.05 kg (d) 0.20 kg**

[NCERT Exemp. Q. 12.2, Page 83]

**Ans. Correct option:** (a)

*Explanation:* Amount of sweat evaporated per min

$$= \frac{\text{calories produced per minute}}{\text{no. of calories required for evaporation per kg}}$$

$$= \frac{14.5 \times 10^4}{580 \times 10^3} = 0.25\,\text{kg}$$

**Q. 3. Consider P-V diagram for an ideal gas shown in figure**

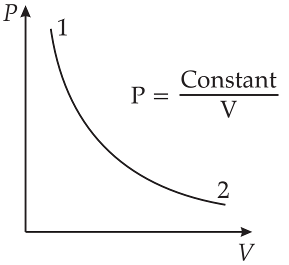
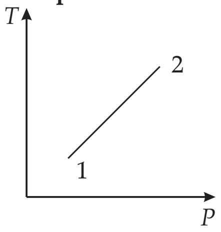
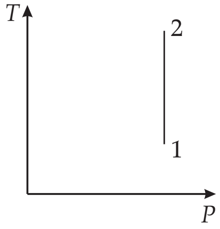



**Fig 12.2**

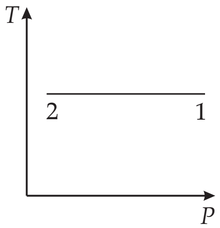
**Out of the following diagrams in figure, which represents the T-P diagram?**



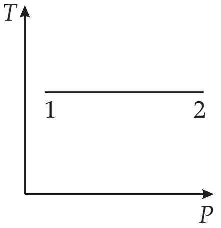

**(i)**




**(ii)**




**(iii)**




**(iv)**

**(a) (iv) (b) (ii)**
**(c) (iii) (d) (i)**

[NCERT Exemp. Q. 12.3, Page 84]

**Ans. Correct option:** (c)

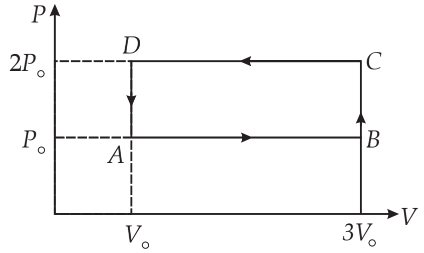
*Explanation:* Given that :

$$P = \frac{\text{Constant}}{V}$$

or PV = Constant

Its Isothermic process or T = Constant and Pressure $P_2 < P_1$

**Q. 4. An ideal gas undergoes cyclic process ABCDA as shown in given P-V diagram.**
**The amount of the work done by the gas is**




**(a)** $6P_0V_0$

**(b)** $-2P_0V_0$

**(c)** $+2P_0V_0$

**(d)** $+4P_0V_0$

[NCERT Exemp. Q. 12.4, Page 84]

**Ans. Correct option:** (b)

*Explanation:*

Work done= $\Delta P \times \Delta V$

$= (2P_0 - P_0)(3V_0 - V_0) = 2P_0V_0$

$\therefore$ Cyclic process is anticlockwise, work done by the gas is negative.

**Q. 5. Consider two containers A and B containing identical gases at the same pressure, volume and temperature. The gas in container A is compressed to half of its original volume isothermally while the gas in container B is compressed to half of its original value adiabatically. The ratio of final pressure of gas in B to that of gas in A is**

**(a)** $2^{\gamma-1}$

**(b)** $\left(\frac{1}{2}\right)^{\gamma-1}$

**(c)** $\left(\frac{1}{1-\gamma}\right)^2$

**(d)** $\left(\frac{1}{\gamma-1}\right)^2$

[NCERT Exemp. Q. 12.5, Page 85]

**Ans. Correct option:** (a)

*Explanation:*

$P_1V_1 = P_2V_2$

$P_2$ or $\frac{P_1V_1}{V_2} = P_1\frac{V_1}{V_2} = 2P_1$

Gas in container B is compresses adiabatically

$P_1V_1^\gamma = P_2V_2^\gamma$

$P_o(2V_o)^\gamma = P_2(V_o)^\gamma$

$$P_2 = P_o\left(2\frac{V_o}{V_o}\right)^{\gamma} = P_o(2)^{\gamma}$$

Hence, ratio of final pressure

$$= \frac{(P_2)_A}{(P_2)_B} = \frac{P_o(2)^{\gamma}}{P_o} = 2^{\gamma-1}$$

**Q. 6. Three copper blocks of masses $M_1$, $M_2$ and $M_3$ kg respectively are brought into thermal contact till they reach equilibrium. Before contact, they were at $T_1$, $T_2$, $T_3$ ($T_1 > T_2 > T_3$). Assuming there is no heat loss to the surroundings, the equilibrium temperature T is (s is specific heat of copper)**

(a) $T = \dfrac{T_1 + T_2 + T_3}{3}$

(b) $T = \dfrac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$

(c) $T = \dfrac{M_1T_1 + M_2T_2 + M_3T_3}{3(M_1 + M_2 + M_3)}$

(d) $T = \dfrac{M_1T_1s + M_2T_2s + M_3T_3s}{M_1 + M_2 + M_3}$

**[NCERT Exemp. Q. 12.6, Page 85]**

**Ans. Correct option:** (b)

*Explanation:* Let us assume that $T_1 > T_2 . T_3$ and $T_1 > T > T_2, T_3$

Now heat lost by $M_1$ = that gained by $M_2$ and $M_3$

$M_1S(T_1 - T) = M_2S(T - T_1) + M_3S(T - T_3)$

$\Rightarrow M_1T_1 + M_2T_2 + M_3T_3 = (M_1 + M_2 + M_3)T$

$$\Rightarrow T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$$

**Q. 7. An ideal gas undergoes isothermal process from some initial state I to final state f. Choose the correct alternatives.**

**(a) dU = 0**

**(b) dQ = 0**

**(c) dQ = dU**

**(d) dQ = dW**

**[NCERT Exemp. Q. 12.8, Page 86]**

**Ans. Correct option:** (a) and (d)

*Explanation:* **(a)** Under isothermal process, T= constant $\therefore \Delta T = 0$

Hence, $\Delta U = nC_v\Delta T = 0$

**(d)** From first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

$\therefore \Delta Q = \Delta W$

**Q. 8. Figure shows the P-V diagram of an ideal gas undergoing a change of state from A to B. Four different parts I, II, III, IV as shown in the figure may lead to the same change of state.**

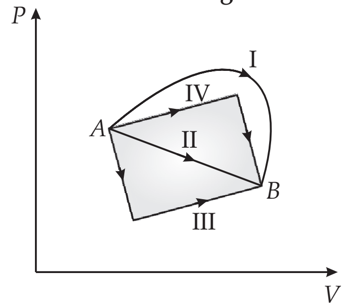



**Fig 12.5**

**(a) Change in internal energy is same in IV and III cases, but not in I and II.**

**(b) Change in internal energy is same in all the four cases.**

**(c) Work done is maximum in case I**

**(d) Work done is minimum in case II.**

**[NCERT Exemp. Q. 12.9, Page 86]**

**Ans. Correct option:** (b) and (c)

*Explanation:*

**(b)** All Process state A to state B, here change in internal energy $\Delta U = U_B - U_A$ = constant to all process.

**(c)** Area between V-axis and graph line is called work done. So area between I and V-axis is maximum.

## Very Short Answer Type Questions **(1 mark each)**

**Q. 1. Can a system be heated and its temperature remains constant?**

**[NCERT Exemp. Q. 12.12, Page 87]**

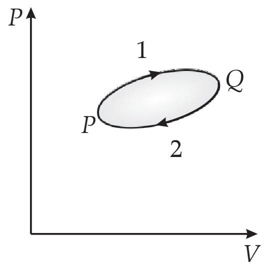
**Ans.** $\Delta Q = AU + W$ (from I law of thermodynamics)

or $\Delta Q = n\,C_v\Delta T + \Delta W$ ($\because \Delta U = nC_v\Delta T$)

If system has constant temperature inspite of heat supplied then

$\Delta T = 0$ and $\Delta Q = \Delta W$. It means heat supplied ($\Delta Q$) to the system is used in doing work ($\Delta W$) against the surroundings.

**Q. 2. A system goes from P to Q by two different paths in the P-V diagram as shown in figure. Heat given to the system in path 1 is 1000 J. The work done by the system along path l is more than path 2 by 100 J. What is the heat exchanged by the system in path 2?**




**[NCERT Exemp. Q. 12.13, Page 87]**

**Ans.** For path 2,

$\Delta Q = \Delta U + \Delta W$ (i)

For path 1,

$1000 = \Delta U + (\Delta W + 100)$

or $\Delta U + \Delta W = 900$

or $\Delta Q = 900$ J

**Q. 3. Is it possible to increase the temperature of a gas without adding heat to it? Explain.**

**[NCERT Exemp. Q. 12.15, Page 87]**

**Ans.** Yes. When the gas undergoes adiabatic compression, its temperature increases, while no heat is given to it.

From First Law of Thermodynamics.

$dQ = dU + dW$

As $dQ = 0$ [for adiabatic process]

so, $dU = -dW$

In compression, work is done on the system So, work done is negative,

*i.e.* $dW = -ve$

or $dU = +ve$

So internal energy of the gas increases due to which its temperature increases.

**Q. 4. Air pressure in a car tyre increases during driving. Explain.**

**[NCERT Exemp. Q. 12.16, Page 87]**

**Ans.** During driving, temperature of the gas increases while its volume remains constant. So, according to Charles's law, at constant V, Pressure, P $\propto$ Temperature (T) Therefore, pressure of gas increases.

## Short Answer Type Questions

**(2 or 3 marks each)**

**Q. 1. Consider a cycle tyre being filled with air by a pump. Let V be the volume of the tyre (fixed) and at each stroke of the pump $\Delta V$ (<<V) of air is transferred to the tube adiabatically. What is the work done when the pressure in the tube is increased from $P_1$ to $P_2$ ?.**

**[NCERT Exemp. Q. 12.19, Page 87]**

**Ans.** Let initial volume of air in tyre be V and after pumping one stroke it becomes (V + *d*V) and pressure increased from P to (P + dP).

Then $$P_1V_1^\gamma = P_2V_2^\gamma$$

$$P(V + dV)\gamma = (P + dP)V\gamma$$

$$PV^\gamma\left(1+\frac{dV}{V}\right)^\gamma = P\left[1+\frac{dP}{P}\right]V^\gamma$$

Volume of tyre remains constant

$$PV^\gamma\left[1+^\gamma\frac{dV}{V}\right] = PV^\gamma\left[1+\frac{dP}{P}\right]$$

or $$\gamma\frac{dV}{V} = \frac{dP}{P}$$

or $$dV = \frac{VdP}{\gamma p} \text{ or } PdV = \frac{VdP}{\gamma}$$

Integrating both sides

$$\int PdV = \int_{P_1}^{P_2}\frac{VdP}{\gamma} \text{ or } \int dW = \int_{P_1}^{P_2}\frac{VdP}{\gamma} \quad [V = \text{constant}]$$

$$W = \frac{(P_2 - P_1)V}{\gamma}$$

**Q. 2. The initial state of a certain gas is ($P_i$, $V_i$, $T_i$). It undergoes expansion till its volume becomes $V_f$ Consider the following two cases:**

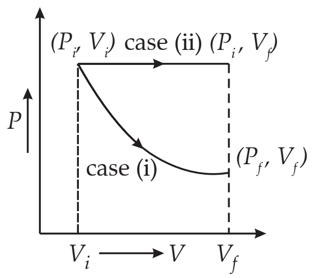
**(a) the expansion takes place at constant temperature.**

**(b) the expansion takes place at constant pressure.**

**Plot the P-V diagram for each case. In which of the two cases, is the work done by the gas more ?**

**[NCERT Exemp. Q. 12.22, Page 88]**

**Ans.**




The P-V diagram for each case is shown in the figure.

**(a)** In case (i) $P_iV_i = P_fV_f$; therefore process is isothermal. Work done = area under the PV curve.

**(b)** The expression is at constant pressure $P_i$, so (ii) is isobaric process.

Area enclosed by graph (i) is less than (ii), so work done is more when the gas expands at constant pressure (case ii).

**Q. 3. A person of mass 60 kg wants to lose 5 kg by going up and down a 10 m high stairs. Assume he burns twice as much fat while going up than coming down. If 1 kg of fat is burnt on expending 7000 kcal, how many times must he go up and down to reduce his weight by 5 kg ?**

**[NCERT Exemp. Q. 12.18, Page 87]**

**Ans.** Height ($h$) = 10 m

Energy produced by burning 1 kg of fat

= 7000 kcal

$\therefore$ Energy produced by burning 5 kg of fat

$= 5 \times 7000 = 35000$ kcal

$= 35 \times 10^6$ cal

Energy utilized in going up and down one time

$$= mgh + \frac{1}{2}mgh = \frac{3}{2}mgh$$

$$= \frac{3}{2} \times 60 \times 10 \times 10 \text{ J}$$

$= 9000$ J

$$= \frac{90000}{4.2} \text{ cal}$$

$$= \frac{3000}{1.4} \text{ cal}$$

∴ Number of times, the person has to go up and down the stairs :

$$= \frac{35\times10^6}{(3000/1.4)}$$

$$= \frac{35\times1.4\times10^6}{3000}$$

$= 16.3 \times 10^3$ times.

**Q. 4. A geyser heats water flowing at the rate of 3 litres per minute from 27°C to 77°C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heating of combustion is $4.0 \times 10^4$ J/g ?**

**[NCERT Ex. Q. 12.1, Page 321]**

**Ans.** The mass of water flowing per unit time

= 3 l/min

= 3000 g per min.

$= \frac{3000}{60} = 50$ g/s

$= \frac{50}{1000}$ kg/s

Heat required to heat this water from 27°C to 77°C.

$= \frac{50}{1000} \times 4.2 \times 10^3 \times 50$ J/s

$= 1.05 \times 10^4$ J/s

Heat generated by the combustion of 1 g of fuel

$= 4.0 \times 10^4$ J

∴ Fuel required per second

$$= \frac{1.05\times10^4}{4.0\times10^4}$$

= 0.2625 g

Fuel required to be burnt per min.

$= 0.2625 \times 60$

= 15.75 g

**Q. 5. What amount of heat must be supplied to $2.0 \times 10^{-2}$ kg of nitrogen (at room temperature) to raise its temperature by 45°C at constant pressure ? (Molecular mass of $N_2$ = 28, R = 8.3 J mol$^{-1}$ K$^{-1}$**

**[NCERT Ex. Q. 12.2, Page 321]**

**Ans.** Mass of nitrogen, $m = 20\times 10^{-2}$ kg = 20 g

Rise in temperature. $\Delta T$ = 45°C

Molecular mass of $N_2$, $M = 28$

Universal gas constant, $R = 8.3$ J mol$^{-1}$ K$^{-1}$

Number of moles, n = m/M

$= (2 \times 10^{-2} \times 10^3) / 28$

= 0 .714

Molar specific heat at constant pressure for nitrogen, $C_p = (7/2)R$

$= (7/2) \times 8.3$

$= 29.05$ J mol$^{-1}$ K$^{-1}$

The total amount of heat to be supplied is given by the relation:

$\Delta Q = nCp\,\Delta T$

$= 0.714 \times 29.05 \times 45$

= 933. 38 J

Therefore, the amount of heat to be supplied is 933. 38 J

**Q. 6. A cylinder with a movable piston contains 3 moles of hydrogen at constant temperature and pressure. The walls of a cylinder are made up of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of a gas increases if the gas is compressed to half its original volume ?**

**[NCERT Ex. Q. 12.4, Page 321]**

**Ans.** Since the process is adiabatic

∴ $P_1V_1^\gamma = P_2V_2^\gamma$

But $\frac{V_1}{V_2} = \frac{2}{1}$

∴ $V_2 = \frac{V_1}{2}$

$\gamma = 7/5$ for hydrogen

∴ Factor by which the pressure of the gas increases

or, $\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$

∴ $\frac{P_2}{P_1} = \frac{(V_1)^\gamma}{\left(V_1/2\right)^\gamma}$

$\frac{P_2}{P_1} = \left(\frac{2}{1}\right)^\gamma$

so, $P_2 = 2^{7/5}$

**Q. 7. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the later case ? (Take 1 cal = 4.19 J)**

**[NCERT Ex. Q. 12.5, Page 321]**

**Ans.** Since, the system is going from A to B and then back to A, it is undergoing a cyclic change. Now in cyclic change there is no change in internal energy ($\Delta$U = 0). From first law of thermodynamic.

$\Delta Q = \Delta U + \Delta W$

$[\therefore \Delta Q = 0]$

In the second case, $\Delta Q$ = 9.35 cal

$= 9.35 \times 4.2 = 39$ J

$\Delta W$ = ?

As, $\Delta U + \Delta W = \Delta Q$

$\Delta W = \Delta Q + \Delta U$

$= 39.3 - 22.3$

= 17.0 J

**Q. 8. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following :**

**(a) What is the final pressure of the gas in A and B ?**

**(b) What is change in the internal energy of the gas ?**

**(c) What is the change in the temperature of the gas ?**

**(d) Do the intermediate states of the system (before setting to the final equilibrium state) lie on its P–V–T surface ?**

**[NCERT Ex. Q. 12.6, Page 321]**

**Ans. (a)** The gas would rush from A to B. The change in pressure or volume will take place under adiabatic conditions. The final pressure in the two cylinders would be 0.5 atm.

**(b)** The change in internal energy of the gas will be zero.

**(c)** The change in temperature will be zero.

**(d)** Since the process is rapid, the intermediate states are not equilibrium states and hence do not satisfy the gas equation. So, the intermediate states of the system do not lie on the P–V–T surface.

**Q. 9. An electric heater supplies heat to a system at a rate of 100 W. If the system performs work at a rate of 75 joules per second. At what rate is the internal energy increasing ?**

**[NCERT Ex. Q. 12.8, Page 322]**

**Ans. Given :** Heat supplied per second,

dQ = 100 J

Work done by the system per second,

dW = 75 J

Increase in internal energy/second dU = ?

From first law of thermodynamics,

dQ = dU + dW

or dU = dQ – dW

or dU = 100 J – 75J

or dU = 25 J $s^{-1}$

or dU = 25 W (1 watt = 1 $Js^{-1}$)

## Long Answer Type Questions

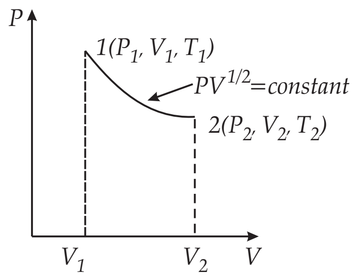
**(5 marks each)**

**Q. 1. Consider a P-V diagram in which the path followed by one mole of perfect gas in a cylindrical container is shown in figure.**

**(a) Find the work done when the gas is taken from state 1 to state 2.**

**(b) What is the ratio of temperature $T_1/T_2$, if $V_2=2V_1$ ?**

**(c) Given the internal energy for one mole of gas at temperature T is (3/2) RT, find the heat supplied to the gas when it is taken from state 1 to 2, with $V_2=2V_1$.**




**[NCERT Exemp. Q. 12.23, Page 88]**

**Ans.** Let $PV^{\frac{1}{2}} = constant = A$

**(a)** Work done by the gas for process 1 to 2

$$\Delta W = \int_{V1}^{V2} PdV = A\int_{V1}^{V2} \frac{dV}{\sqrt{V}} = A\left[\frac{\sqrt{V}}{\frac{1}{2}}\right]_{V_1}^{V_2} = 2A\left(\sqrt{V_2} - \sqrt{V_1}\right)$$

$$= 2P_1V_1^{\frac{1}{2}}\left[\sqrt{V_2} - \sqrt{V_1}\right]$$

**(b)** From ideal gas equation-

$$PV = nRT$$

$$\text{or } T = \frac{PV}{nR} = \frac{P\sqrt{V}\sqrt{V}}{nR}$$

$$T = \frac{A\sqrt{V}}{nR} \qquad \left[P\sqrt{V} = A\right]$$

$$T_1 = \frac{A\sqrt{V_1}}{nR}, T_2 = \frac{A\sqrt{V_2}}{nR}$$

$$\frac{T_1}{T_2} = \frac{\frac{A\sqrt{V_1}}{nR}}{\frac{A\sqrt{V_2}}{nR}} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V_1}{2V_1}} = \frac{1}{\sqrt{2}} \qquad \left[\because V_2 = 2V_1\right]$$

(c) Then, the change in internal energy $= U = \frac{3}{2}RT$

$$\Delta U = U_2 - U_1 = \frac{3}{2}R\left(T_2 - T_1\right) = \frac{3}{2}RT_1\left(\sqrt{2} - 1\right)$$

$$\Delta W = 2P_1V_1^{1/2}\left(\sqrt{V_2} - \sqrt{V_1}\right)$$

$$= 2P_1V_1^{1/2}\left(\sqrt{2} \times \sqrt{V_1} - \sqrt{V_1}\right)$$

$$= 2P_1\sqrt{V_1} \times \sqrt{V_1}\left(\sqrt{2} - 1\right)$$

$$= 2P_1V_1\left(\sqrt{2} - 1\right)$$

$$= 2RT_1\left(\sqrt{2} - 1\right)$$

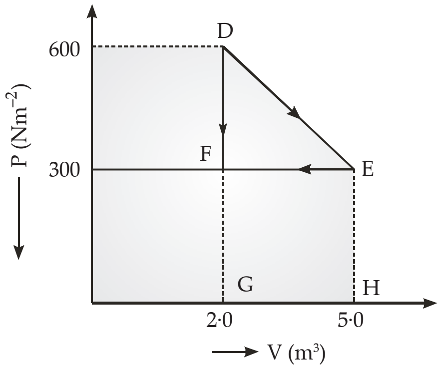
Such that,

$$\Delta Q = \Delta U + \Delta W = \frac{3}{2}RT_1\left(\sqrt{2} - 1\right) + 2RT_1\left(\sqrt{2} - 1\right)$$

$$= \left(\sqrt{2} - 1\right)RT_1\left(\frac{3}{2} + 2\right)$$

Amount of heat supplied, $\Delta Q = \frac{7}{2}RT_1\left(\sqrt{2} - 1\right)$

**Q. 2. A thermodynamic system is taken from an original state to an intermediate state by linear process shown in figure here. Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F.**




**[NCERT Ex. Q. 12.9, Page 322]**

**Ans.** Assume W = total work done by the gas from D to E to F.

When $W_1$ and $W_2$ be the work done by the gas from D to E and E to F respectively, then

$W = W_1 + W_2$ ...(1)

Now $W_1$ = work done during the process from D to E (expansion),

= area DEHGD

= area of ΔDEF + area of rectangle EHGF

$= \frac{1}{2}$ EF × DF + GH × FG ...(2)

Now EF = 5 – 2 = 3 litre

= 3 m$^3$

DF = 600 – 300

= 300 Nm$^{-2}$

FG = 300 – 0

= 300 Nm$^{-2}$

GH = 5 – 2

= 3 m$^3$ ...(3)

∴ From equations (2) and (3),

$$\therefore \quad W_1 = \left[\frac{1}{2}\times 3\times 300 + 3\times 300\right] \text{J} \quad ...(4)$$

$W_2$ = Work done during the process from E to F (compression) = Area EHGF

= – FG × GH

= – (300 – 0) × (5 – 2)

= – 300 × 3 J ...(5)

∴ From equation (1), (4) and (5), we get

$$W = \frac{1}{2} \times 3 \times 300 + 3 \times 300 - 300 \times 3 \times 10^{-3}$$

or W = 3 × 150 J

= 450 J

or W = 450 J

**Q. 3. Consider that an ideal gas (n moles) is expanding in a process given by P = f (V), which passes through a point ($V_0$, $P_0$) Show that the gas is absorbing heat at ($P_0$, $V_0$) if the slope of the curve P = f (V) is larger than the slope of the adiabat passing through ($P_0$, $V_0$).**

**[NCERT Exemp. Q. 12.26, Page 89]**

**Ans.** From question-

Slope of P = f (V), curve at ($V_0$.$P_0$)

$= f(V_0)$

Slope of adiabatic process at ($V_0$.$P_0$)

$$= k(-\gamma)V_0^{-1-\gamma} = -\gamma P_0 / V_0$$

Now heat absorbed in the process P = f (V)

dQ = dV + dW

$$dQ = nC_V dT + PdV$$

$$PV = nRT \quad ...(i)$$

$$\text{or } T = P\frac{V}{nR}$$

Since $P=f(V)$

$$T = (1/nR)\ PV = (1/nR)V\,f(V)$$

$$dT = (1/nR)[f(V) + V\,f'(V)]dV \quad ...(ii)$$

From (I)

$$\frac{dQ}{dV} = nC_V\frac{dT}{dV} + P\frac{dV}{dV}$$

$$= \frac{nC_V}{nR}\times[f(V) + Vf'(V)] + f(V) \text{ [using II]}$$

$$\left[\frac{dQ}{dV}\right]_{V=V_0} = \frac{C_V}{R}[f(V_0) + V_0 f'(V_0)] + f(V_0)$$

$$= f(V_0)\left[\frac{C_V}{R} + 1\right] + V_0 f'(V_0)\frac{C_V}{R}$$

$$\because \quad C_V = \frac{R}{\gamma - 1}$$

$$\text{or} \quad \frac{C_V}{R} = \frac{1}{\gamma - 1}$$

$$\left[\frac{dQ}{dV}\right]_{V=V_0} = \left[\frac{1}{\gamma-1} + 1\right]f(V_0) + \frac{V_0 f'(V_0)}{\gamma - 1}$$

$$= \frac{\gamma}{\gamma-1}P_0 + \frac{V_0}{\gamma-1}f'(V_0)$$

Heat is absorbed when $dQ/dV > 0$ when gas expands, that is when

$$\gamma P_0 + V_0 f'(V_0) > 0$$

$$\text{or } f'(V_0) > \frac{-\gamma P_0}{V_0}$$

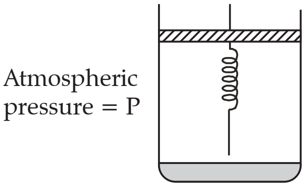
**Q. 4. Consider one mole of perfect gas in a cylinder of unit cross section with a piston attached as shown in fig. A spring (spring constant k) is attached (unstretched length L) to the piston and to the bottom of the cylinder. Initially the spring is unstretched and the gas is in equilibrium. A certain amount of heat Q is supplied to the gas**

**causing an increase of volume from $V_0$ to $V_1$.**

**(a) What is the initial pressure of the system?**

**(b) What is the final pressure of the system?**

**(c) Using the first law of thermodynamics, write down a relation between $Q, P_a, V, V_1$ and $k$.**




**[NCERT Exemp. Q. 12.27, Page 89]**

**Ans. (a)** The piston is in equilibrium at initial position,

$\therefore \quad P_i = P_a$

**(b)** As heat is supplied, the gas expands from V to $V'$

$\therefore$ Volume Increase $= V' - V$

When piston is of unit cross- sectional area, hence, extension in spring

$$x = \frac{V' - V}{Area} = V' - V \quad [\because area = 1]$$

$\therefore$ Force exerted by spring on piston,

$$F = kx$$
$$= k(V' - V)$$

$\therefore$ final pressure,

$$P_f = P_a + kx$$
$$= P_a + k(V' - V)$$

**(c)** From I law of thermodynamics-

$$dQ = dU + dW$$

Final temperature of gas is T, then increase in Internal energy-

$$dU = C_V(T - T_0)$$

Work done $= PdV +$ Increase in P.E. (spring)

$$= P_a(V' - V) + \frac{1}{2}Kx^2$$

Now,

$$dQ = dU + dW$$
$$= C_v(T - T_0) + P_a(V' - V) + \frac{1}{2}Kx^2$$
$$dQ = C_v(T - T_0) + P_a(V' - V) + \frac{1}{2}(V' - V)^2$$

This is the required relation.

# TOPIC-2
## Second law of Thermodynamics

## Quick Review

- **Cyclic & Non-Cyclic processes :**
  - **(i) In a cyclic process,** the system returns to its initial state after passing through various stages of pressure, volume and temperature. P-V curve representing a cyclic process is a closed curve.
  - **(ii) In non-cyclic process,** the series of changes involved do not return the system back to its initial state.
- **Heat Engine :** It is a device which converts heat energy into mechanical energy. It consists of 3 parts :
  - **(a)** Source of heat at higher temperature.
  - **(b)** Working substance.
  - **(c)** Sink of heat at lower temperature.

**Types of Heat Engine :**

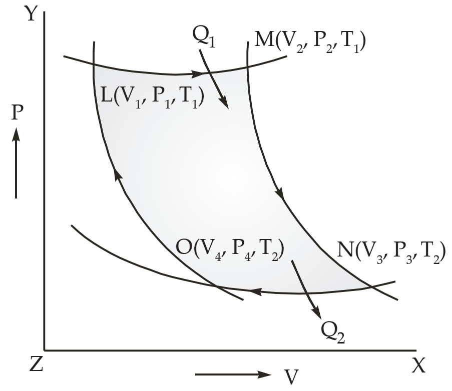
1. **External combustion engine :** Heat is produced by burning fuel outside the main body.
2. **Internal combustion engine :** Heat produced by burning fuel inside in the main body.

- **Second Law of Thermodynamics** represents the direction of flow of heat. According to Clausius statement, it is impossible for a self-acting machine unaided by any external agency, to transfer heat from a body at lower temperature to a body at higher temperature.

  So, heat can be transferred from lower temperature to higher temperature only when some external work is done or energy is supplied to the system. For example, a refrigerator cools things only when electric energy is supplied to it. Further, we deduce from second law, that efficiency of any heat engine can never be 100%.
- **A Carnot engine** is an ideal heat engine, which consists of a **source** of infinite thermal capacity which is maintained at a constant high temperature $T_1$ and a **sink** of heat of infinite thermal capacity maintained at a constant low temperature $T_2$ and an ideal gas acts as the working substance. It consists of a cylinder fitted with an insulating frictionless piston. The cylinder has a conducting base and insulating walls. It can be placed on an insulating pad.

  **Carnot engine :** Works on the principle of Carnot cycle made up of four stages : Isothermal expansion LM; Adiabatic expansion MN; Isothermal compression NO; Adiabatic compression OL as represented in given figure.

If $Q_1$ is the amount of heat absorbed per cycle from the source at temperature $T_1$ ; $Q_2$ is the amount of heat rejected per cycle to the sink at temperature $T_2$;




➢ **Carnot's Theorem :**

**(a)** Working between two given temperatures. $T_1$ of hot reservoir and $T_2$ of cold reservoir, no engine can have efficiency more than that of carnot engine.

**(b)** The efficiency of carnot engine is independent of the nature of working substance.

## Know the Terms

➢ **Reversible process** is a process which can be reversed back to initial state.

➢ **Irreversible process** is a process which cannot be traced back in opposite direction.

➢ **Entropy** of a system is a measure of its molecular disorder. The greater the disorder, greater is the entropy.

➢ **Perpetual motion machine of first kind** disobeys first law of thermodynamics and **perpetual motion machine of second kind** disobeys second law of thermodynamics.

## Know the Formulae

➢ **Work done :**

**in cyclic process** $dQ = dW$

**in non- cyclic process** $dQ \neq dW$

2. **Change in Entropy** $\Delta S = \frac{\Delta Q}{T} = \frac{\text{Heat absorbed}}{\text{Absolute temperature}}$

3. **Efficiency of Heat Engine**

*i.e.,* $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

where, W = useful work done/cycle by the engine

$Q_1$ =amount of heat energy absorbed/cycle from the source

$Q_2$ = amount of heat rejected/cycle to the sink

and $W = Q_1 - Q_2$

Where $T_1$ is temperature of source; $T_2$ is temperature of sink, then

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$\therefore$ $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

4. **Efficiency of Carnot's Engine** $W = Q_1 - Q_2$

Efficiency, $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

## Know the Links

- *www.vedantu.com*
- *www.learncbse.com*
- *www.meritnation.com*

# MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. Which of the processes described below are irreversible?**

**(a) The increase in temperature of an iron rod by hammering it.**

**(b) A gas in a small container at a temperature $T_1$ is brought in contact with a big reservoir at a higher temperature $T_2$ which increases the temperature of the gas.**

**(c) A quasi-static isothermal expansion of an ideal gas in cylinder fitted with a frictionless piston.**

**(d) An ideal gas is enclosed in a piston cylinder arrangement with adiabatic walls. A weight W is added to the piston, resulting in compression of gas. [NCERT Exemp. Q. 12.7, Page 85]**

**Ans. Correct option:** (a), (b) and (d)

*Explanation:*

**(a)** In this case internal energy of the rod is increased from external work done by human which in turn increases its temperature. So the process cannot be treated itself.

**(b)** In this process energy in the form of heat is transferred to the gas in the small container by big reservoir at temperature $T_2$.

**(d)** As the weight is added to the cylinder arrangement in the form of external pressure. Hence, it cannot be reserved back itself.

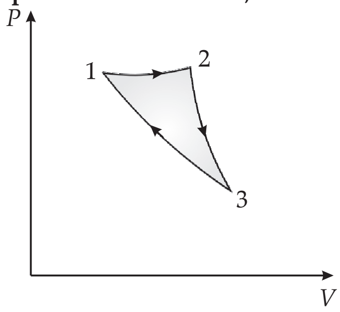
**Q. 2. Consider a cycle followed by an engine as shown in fig.**

**1 to 2 is isothermal**

**2 to 3 is adiabatic**

**3 to 1 is adiabatic**

**Such a process does not exist, because**




**(a) heat is completely converted to mechanical energy in such a process , which is not possible.**

**(b) mechanical energy is completely converted to heat in this process, which is not possible.**

**(c) curves representing two adiabatic processes don't intersect.**

**(d) curves representing an adiabatic process and an isothermal process don't intersect.**

**[NCERT Exemp. Q. 12.10, Page 86]**

**Ans. Correct option:** (a) and (c)

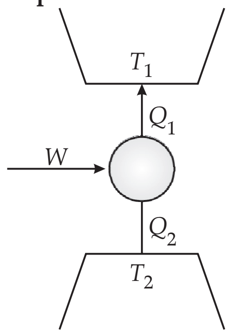
*Explanation:*

**(a)** The given process is a cyclic process i.e., it returns to the original state 1 and change in internal energy in cyclic process is always zero as per cyclic process $v_f = v_i$. So, $\Delta v = v_f - v_i = 0$. Hence, total heat is completely converted to mechanical energy. Such a process is not possible by second law of thermodynamics.

**(c)** Here two curve are intersecting, when the gas expands adiabatically from 2 to 3. It is possible to return to the same state without being heat supplied, hence the process 3 to 1 cannot be adiabatic, so, well concluded that such a process does not exist because curves representing two adiabatic processes do not intersect.

**Q. 3. Consider a heat engine as shown in figure $Q_1$ and $Q_2$ are heat added to heat bath $T_1$ and heat taken from $T_2$ in one cycle of engine. W is the mechanical work done on the engine.**

**If W > 0, then possibilities are:**




**(a) $Q_1 > Q_2 > 0$**

**(b) $Q_2 > Q_2 > 0$**

**(c) $Q_2 < Q_2 > 0$**

**(d) $Q_1 < Q_2 > 0$**

**[NCERT Exemp. Q. 12.11, Page 86]**

**Ans. Correct option:** (a) and (c)

*Explanation:*

$Q_2 + W = Q_1$

or $\quad W = Q_1 - Q_2 \qquad \because W > 0$

or $(Q_1 - Q_2) > 0 \qquad \therefore Q_1 > Q_2$

(a) If $Q_2 > 0$, then $Q_1 > Q_2$
(b) If $Q_2 < 0$, then $Q_2 < Q_1$
(c) ($\because W > 0$) Hence, $Q_2 < Q_1 < 0$

## Very Short Answer Type Questions (1 mark each)

**Q. 1. If a refrigerator's door is kept open, will the room become cool or hot? Explain.**

**[NCERT Exemp. Q. 12.14, Page 87]**

**Ans.** The room will become hot. Amount of heat removed is less than the heat supplied and hence the room, including the refrigerator (which is not insulated from the room) becomes hotter.

**Q. 2. Explain why**

**(a) Two bodies at different temperatures $T_1$ and $T_2$, if brought in thermal contact do not necessarily settle to the mean temperature $(T_1 + T_2) / 2$?**

**(b) The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat. Comment**

**(c) Air pressure in a car tyre increases during driving. Why?**

**(d) The climate of a harbour town is more temperate than that of a town in a desert at the same latitude. Why?**

**[NCERT Ex. Q. 12.3, Page 321]**

**Ans. (a)** When two bodies at different temperatures $T_1$ and $T_2$ are brought in thermal contact, heat flows from the body at higher temperature to the body at lower temperature till equilibrium is achieved, *i.e.*, the temperatures of both the bodies becomes equal. The equilibrium temperature equal to the mean temperature $(T_1 + T_2) / 2$ only when thermal capacities of the two bodies are equal.

**(b)** The coolant in a chemical or nuclear plant should have a high specific heat. This is because higher the specific heat of the coolant, higher is its heat-absorbing capacity and vice-versa. Hence, a liquid having a high specific heat is the best coolant to be used in a nuclear or chemical plant. This would prevent different parts of the plant from getting too hot.

**(c)** When the car is in motion, the temperature of air inside the tyre increases due to motion of the air molecules. According to Charles's law, pressure is directly proportional to the temperature, $P \propto T$. Hence, if the temperature inside a tyre increases, then the air pressure inside the tyre will also increase.

**(d)** A harbour town has a more temperate climate than a town located in a dessert at the same latitude. This is because in a harbour town, the relative humidity is more than in a desert town.

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. A steam engine delivers $5.4 \times 10^8$ J of work per minute and services $3.6 \times 10^9$ J of heat per minute from the boiler. What is the efficiency of the engine ? How much heat is wasted per minute ?**

**[NCERT Ex. Q. 12.7, Page 322]**

**Ans.** Given :

$Q_1$ = Heat absorbed from the boiler per minute $= 3.6 \times 10^9$ J

W = Work done per minute by the steam engine $= 5.4 \times 10^8$ J

$Q_2$ = Heat wasted/rejected per minute = ?

$\eta\%$ = Percentage efficiency of the engine = ?

From formula $\eta\% = \dfrac{W}{Q_1} \times 100$

$$\eta\% = \frac{5.4 \times 10^8 \text{ J}}{3.6 \times 10^9 \text{ J}} \times 100$$

$$= \frac{3}{10} \times 100 = 15\%$$

From the relation :

$$Q_1 = W + Q_2,$$

or $$Q_2 = Q_1 - W$$
$$= 36 \times 10^8 - 5{\cdot}4 \times 10^8$$
$$= 30.6 \times 10^8 \text{ J}$$
$$= 3.06 \times 10^9 \text{ J}$$
$$\approx 3.1 \times 10^9 \text{ J}$$

**Q. 2. A refrigerator is to maintain eatables kept inside at 9°C. Calculate the coefficient of performance if the room temperature is 36°C.**

**[NCERT Ex. Q. 12.10, Page 322]**

**Ans. Given :** $T_1 = 273 + 36 = 309$ K

$T_2 = 9°C = 282$ K

$\beta$ = ?

From the relation,

$$\beta = \frac{T_2}{T_1 - T_2},$$

or $$\beta = \frac{282}{309 - 282} = \frac{282}{27}$$
$$= 10.4$$

**Q. 3. Consider a Carnot's cycle operating between $T_1$= 500 K and $T_2$= 300 K producing 1 kJ of mechanical work per cycle. Find the heat transferred to the engine by the reservoirs.**

**[NCERT Exemp. Q. 12.17, Page 87]**

**Ans.** Given: Work done per cycle, W=1 KJ =1000J

Temperature (source), $T_1$= 500K

Temperature (source), $T_2$= 300K

Efficiency of Carnot's engine,

$$\eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{300}{500} = \frac{2}{5}$$

But $$\eta = \frac{W}{Q_1}$$

or $$Q_1 = \frac{W}{\eta} = \frac{1000}{\frac{2}{5}}$$

$$= \frac{1000 \times 5}{2} = \frac{5000}{2}$$

$$Q_1 = 2500\,J$$

As $Q_1 - Q_2 = W$

$Q_2 = Q_1 - W$

$= 2500 - 1000$

$= 1500$ J

**Q. 4. In a refrigerator one removes heat from a lower temperature and deposits to the surroundings at a higher temperature. In this process, mechanical work has to be done, which is provided by an electric motor. If the motor is of 1 kW power, and heat is transferred from −3°C to 27°C, find the heat taken out of the refrigerator per second assuming its efficiency is 50% of a perfect engine.**

**[NCERT Exemp. Q. 12.20, Page 87]**

**Ans.** Given: $T_1$ = 27°C = 27 + 273 = 300K

$T_2$= −3°C = −3 + 273 = 270K

Efficiency,

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{270}{300}$$

$$= 1 - 0.9 = 0.1 = \frac{1}{10}$$

Efficiency of refrigerator is 50% of percentage engine.

$$\eta' = 50\% \, of \, \eta = 0.5 \times \frac{1}{10} = \frac{1}{20}$$

∴ Coefficient of performance,

$$\beta = \frac{Q_2}{W} = \frac{1 - \eta'}{\eta'}$$

$$\beta = \frac{1 - \frac{1}{20}}{\frac{1}{20}} = \frac{1 - 0.05}{0.05}$$

$$= \frac{0.95}{0.05} = 19$$

$Q_2$ = 19% of work done by motor on refrigerator

= 19 × 1 kW

Q = 19 K J/s

**Q. 5. If the co-efficient performance of a refrigerator is 5 and operates at the room temperature (27°C), find the temperature inside the refrigerator.**

**[NCERT Exemp. Q. 12.21, Page 88]**

**Ans.** Given:

$$\beta = 5, \; T_1 = 27^\circ C = 27^\circ + 273 = 300\,K$$

$$\beta = \frac{T_2}{T_1 - T_2}$$

$$5 = \frac{T_2}{300 - T_2} \Rightarrow T_2 = 1500 - 5T_2$$

$$T_2 + 5T_2 = 1500$$

$$6T_2 = 1500$$

$$T_2 = \frac{1500}{6} = 250\,K$$

$$T_2 = 250 - 273$$

$$T_2 = -23^\circ C$$

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. A cycle followed by an engine (made of one mole of perfect gas in a cylinder with a piston) is shown in figure.**

**A to B : volume constant**

**B to C : adiabatic**

**C to D : volume constant**

**D to A : adiabatic**

$V_C = V_D = 2V_A = 2V_B$

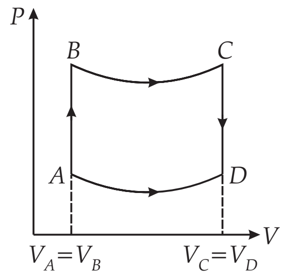
**(a) In which part of the cycle heat is supplied to the engine from outside?**

**(b) In which part of the cycle heat is being given to the surrounding by the engine?**

**(c) What is the work done by the engine in one cycle? Write your answer in term of $P_A, P_B, V_A$.**

**(d) What is the efficiency of the engine?**

**[γ = 5⁄3 for the gas], ($C_v = \frac{3}{2}R$ for one mole)**




**[NCERT Exemp. Q. 12.24, Page 88]**

**Ans. (a)** Heat supplied to engine in part AB-

dV=0 or dW=PdV=0,

By first law of thermodynamics- dQ=dU+dW dQ=dU

Hence, heat energy supplied to system does not work, but increase the internal energy of system.

$$\therefore\ P = \frac{nRT}{V},\ \because V = constant \text{ or } P \propto T$$

So pressure increases with increase in temperature *i.e.*, internal energy increases. Therefore, heat is supplied to the engine is process AB.

**(b)** Heat is being given to the surrounding by the engine in part CD. In this process CD, dV=0, pressure decreases so temperature also decreases(P∝T),

**(c)** Work done in each part-

$$W_{AB} = \int_{V_A}^{V_B} PdV = 0,\ W_{CD} = \int_{V_C}^{V_D} PdV = 0,$$

$$W_{BC} = \int_{V_B}^{V_C} PdV = K\int_{V_B}^{V_C} \frac{dV}{V^{\gamma}} = \frac{K}{1-\gamma}\left[V^{1-\gamma}\right]_{V_B}^{V_C}$$

$$= \frac{1}{1-\gamma}[PV]_{V_B}^{V_C}$$

$$= \frac{(P_C V_C - P_B V_B)}{1-\gamma}$$

Similarly,

$$W_{DA} = \frac{P_A V_A - P_D V_D}{1-\gamma}$$

$$\because\ P_B V_B^{\gamma} = P_C V_C^{\gamma}$$

$$\text{or } P_C = P_B\left(\frac{V_B}{V_C}\right)^{\gamma}$$

$$= P_B\left(\frac{1}{2}\right)^{\gamma}$$

$$P_C = 2^{-\gamma} P_B$$

Similarly, $P_D = 2^{-\gamma} P_A$

Total work done in one cycle (ABCDA)-

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= 0 + W_{BC} + 0 + W_{DA}$$

$$= \left(\frac{P_C V_C - P_B V_B}{1-\gamma}\right) + \left(\frac{P_A V_A - P_D V_D}{1-\gamma}\right)$$

$$= \frac{1}{1-r}\ [P_B V_B (2^{-\gamma+1} - 1) - P_A V_A (2^{-\gamma+1} - 1)]$$

$$= \frac{1}{1-r}\ (2^{1-\gamma})\ [P_B - P_A] V_A$$

$$= \frac{3}{2}\left(1 - \left(\frac{1}{2}\right)^{\frac{2}{3}}\right)\ (P_B - P_A) V_A$$

**(d)** Heat supplied during process A. B

$$dQ_{AB} = dU_{AB}$$

$$Q_{AB} = \frac{3}{2} nR(T_B - T_A) = \frac{3}{2}(P_B - P_A) V_A$$

$$\text{Efficiency} = \frac{\text{Net Workdone}}{\text{Heat Supplied}} = \left[1 - \left(\frac{1}{2}\right)^{\frac{2}{3}}\right]$$

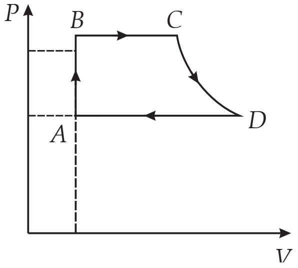
**Q. 2. A cycle followed by an engine (made of one mole of an ideal gas in a cylinder with a piston) is shown in figure. Find heat exchanged by the engine, with the surroundings for each section of the cycle. ($C_v$ = (3/2) R)**

**(a) AB : constant volume**

**(b) BC : constant pressure**

**(c) CD :adiabatic**

**(d) DA :constant pressure**




**[NCERT Exemp. Q. 12.25, Page 89]**

**Ans.** **(a)** For process AB- volume= constant

∴ Work done, dW=0

From 1st law of thermodynamics-

$$dQ = dU + dW = dU + 0$$

$$dQ = dU$$

$$= nC_V dT = nC_V(T_B - T_A) \qquad [n=1]$$

$$= \frac{3}{2}R(T_B - T_A)$$

$$= \frac{3}{2}(RT_B - RT_A)$$

$$= \frac{3}{2}(P_B V_B - P_A V_A)$$

Heat exchanged in the system

$$= \frac{3}{2}(P_B V_B - P_A V_A) = \frac{3}{2}V_A(P_B - P_A)$$

$$\because (V_B = V_A)$$

**(b)** For process BC−Pressure = constant

$$dQ = dU + dW$$

$$= \frac{3}{2}R(T_C - T_B) + P_B(V_C - V_B)$$

$$= \frac{3}{2}(P_C V_C - P_B V_B) + P_B(V_C - V_B)$$

$$= \frac{5}{2}P_B(V_C - V_B)$$

$$\because (P_B = P_C \text{ and } V_B - V_A)$$

Heat exchanged in the system $= \frac{5}{2}P_B(V_C - V_B)$

**(c)** For process CD,

Heat exchanged is zero because CD is adiabatic.

**(d)** For process DA,

Heat exchanged can be calculated as for BC

similarly, $dQ = \frac{5}{2}P_A(V_A - V_D)$

## TIPS... & TRICKS...

- *Study and understand Zeroth and First low of Thermodynamics.*
- *Learn about Internal energy, Isothermal process. adiabatic process, Isobaric and Isochoric pracesses.*
- *Use formula for work done in Isothermal and adiabatic process.*
- *Study and understand about heat engine and carnot engine.*
- *Study second law of thermodynamics and concept of Ideal engine.*
- *Understand carnot's theorem and efficiency of cannot's engine.*

## Some Commonly Made Errors

- Sometimes, students forget to write down the formula when working on numericals.
- Students get confused in using the right sign convention during calculation of efficiencies.

### EXPERT ADVICE

- *Always remember that work done by the system is negative and work done on the system is positive.*
- *Equilibrium of a system in mechanics means the net external force and torque on the system are zero.*
- *In isothermal quasi–static processes, heat is absorbed or given out by the system even though at every stage the gas has the same temperature as that of the surrounding reservoir.*
- *Interpretation of carnot cycle is required.*
- *Intensive study of all laws is important.*

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| | |
|---|---|
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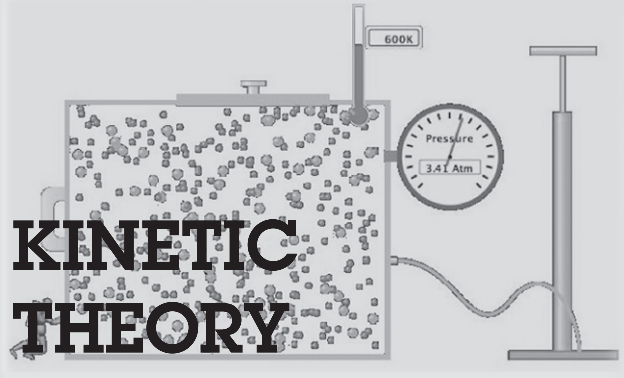
| **Visit :** *https://youtu.be/0Oq7bCSDPxE* **Or Scan the Code** | **Visit :** *https://goo.gl/bFmGsQ* **Or Scan the Code** |

CHAPTER

# 13 KINETIC THEORY




## Chapter Objective

**This chapter will help you understand :**

➢ *Equation of state of a perfect gas, Work done in compressing a gas. Kinetic theory of gases–assumptions, Concept of pressure, Kinetic interpretation of temperature, r.m.s. speed of gas molecules.*

➢ *Degrees of freedom, Law of equi-partition of energy (statement only) and application of specific heat capacities of gases, Concept of mean free path.*

## TOPIC-1
## Equation of State and Kinetic Theory of Gases



### Quick Review

➢ **Ideal Gases :**

**(a)** Ideal gas or perfect gas is that gas which strictly obeys gas laws, like Boyle's law, Charles' law etc.

**(b)** For ideal gas, the size of the gas molecules is almost zero and the volume of the gas molecule is also almost zero.

**(c)** There is no force of attraction or repulsion amongst the molecules of an ideal gas.

**(d)** There is no intermolecular potential energy for the molecules of an ideal gas.

**(e)** The molecules of an ideal gas consists only kinetic energy.

**(f)** The ideal gas cannot be liquefied or solidify, which supports the absence of intermolecular forces of ideal gas at very low pressure and high temperature.

**(g)** The internal energy of an ideal gas does not depend on volume.

**(h)** The internal energy of an ideal gas depends upon the temperature alone.

**(i)** The specific heat of an ideal gas is independent of temperature.

**(j)** No gas available in the universe is strictly an ideal gas.

**(k)** The gases such as $H_2$, $O_2$, $N_2$ etc. and monoatomic inert gases behave very similar to ideal gases at very low pressure and high temperature

**(l)** The real gases at low pressure and high temperature behave as ideal gases due to negligible intermolecular force of attraction and volume of gas molecules.

➢ **Real Gases :**

**(a)** Real gases which are actually found in nature are known as real gases.

**(b)** The size of the molecules of a real gas is finite and hence the volume of the molecules of a real gas is finite.

**(c)** There is a force of attraction or repulsion between the molecules of a real gas. The intermolecular force between molecules is attractive for large intermolecular separation and repulsive for small intermolecular separation.

**(d)** The molecules of a real gas have potential energy as well as kinetic energy.

**(e)** The internal energy of a real gas depends on pressure, volume and temperature of the gas.

**(f)** The real gas can be liquefied and solidified.

**(g)** The real gas do not obey gas equation but obey Van der Waal's gas equation :

$$\left(P + \frac{n^2a}{V^2}\right)(V - nb) = nRT$$

where *a* and *b* are Van der Waal's gas constants of a real gas.

**(h)** The real gases like $CO_2$, $NH_3$, $SO_2$ etc. obey Van der Waal's equation at high pressure and low temperature.

**(i)** In Van der Waal's equation, the value of *a* depends upon the intermolecular force and the nature of the gas.

**(j)** The value of *b* depends upon the size of the gas molecules and represents the volume occupied by the molecules of a gas.

**(k)** In Van der Waal's equation $(V - b)$ shows volume available to the molecules of the real gas, which is the effective volume of the gas.

**(l)** Real gases do not obey the gas laws at all temperatures.

- **Boyle's law :** It states that the volume V of the given mass of a gas is inversely proportional to its pressure P, when temperature is kept constant, *i.e.*,

$$V \propto 1/P \text{ or } V = K/P \text{ ...(Here, T is constant)}$$

or
$$PV = K = \text{Constant.}$$

- **Charles's law :** It states that the pressure remaining constant, the volume of the given mass of a gas is directly proportional to its kelvin temperature, *i.e.*, $V \propto T$, if P is constant.

or,
$$V = KT \text{ ...(Here, P is constant)}$$

or,
$$\frac{V}{T} = K = \text{Constant.}$$

- **Assumptions of Kinetic Theory of Gases :**

**(a)** A gas consists of a very large number of molecules which are perfectly elastic spheres and are identical in all respects for a given gas and are different for different gases.

**(b)** The molecules of a gas are in a state of continuous, rapid and random motion.

**(c)** The volume occupied by the molecules is negligible in comparison to the volume of the gas.

**(d)** The molecules do not exert any force of attraction or repulsion on each other, except during collision.

**(e)** The collisions of the molecules with themselves and with the walls of the vessel are perfectly elastic.

**(f)** Molecular density is uniform throughout the gas.

**(g)** A molecule moves along a straight line between two successive collisions.

**(h)** The collisions are almost instantaneous.

## Know the Terms

- **Gram mole and kilogram mole :**

  **(i)** The molecular weight expressed in gram is known as gram mole (g mol). The molecular weight expressed in kilograms is known as kilogram mole (kg mol).

  **(ii)** The mass of 1 mole of a gas equal to its molecular weight in gram. And 1 kg mol = 1,000 g mole.

- **Most probable speed** of the molecules of a gas as that speed which is possessed by maximum fraction of total number of molecules of the gas.
- **Mean speed or average speed** is the average speed with which molecules of a gas move.
- **Root mean square speed** is defined as the square root of the mean of the squares of random velocities of individual molecules of a gas.

## Know the Formulae

- **Boyle's Law :** $PV = \text{constant}$
- **Charles's Law :** $\frac{V}{T} = \text{constant}$
- **Standard gas equation :** $PV = nRT$

  where *n* is the number of moles contained in the given ideal gas of volume V, pressure P and temperature T.
- **Gas constant :**

  (i) R is a universal gas constant and *r* is a gas constant for 1 gram of a gas.

  (ii) The universal gas constant is defined as the work done by (or on) a gas per mole per Kelvin *i.e.*

$$R = \frac{PV}{nT}$$

$$= \frac{\text{Pressure} \times \text{Volume}}{\text{No. of moles} \times \text{Temperature}}$$

$$= \frac{\text{Work done}}{\text{No. of moles} \times \text{Temperature}}$$

(iii) The value of R for every gas at S.T.P. = 8·31 J $mole^{-1}$ $K^{-1}$ = 1.98 cal. $mol^{-1}$ $K^{-1}$.
(iv) Dimensional formula for R = $[ML^2T^{-2}K^{-1}]\ mol^{-1}$.

➢ **Most probable speed :** $c_{mp} = \sqrt{\frac{2k_BT}{m}}$

➢ **Average speed** $c_{av} = \sqrt{\frac{8k_BT}{\pi m}}$

➢ **Root mean square speed** $c_{rms} = \sqrt{\frac{3k_BT}{m}}$

$k_B$ = Boltzman constant, T = Temperature, $m$ = mass

➢ **Ratio among speeds,** $c_{mp} : c_{av} : c_{rms} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}.$

➢ **Van der Waal's equation for one mole of a gas,** $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

## Know the Links

☞ *www.vedantu.com*
☞ *www.learncbse.in*
☞ *www.examplar.com*
☞ *https://school.aglasem.com*

## MCQ/Fillups/True or False **(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. A cubic vessel (with faces horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 $ms^{-1}$ in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground**

**(a) remains the same because 500 $ms^{-1}$ is very much smaller than $v_{rms}$ of the gas.**
**(b) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls.**
**(c) will increase by a factor equal to $(v^2_{rms} + (500)^2)/v^2_{rms}$ where $v_{rms}$ was the original mean square velocity of the gas.**
**(d) will be different on the top wall and bottom wall of the vessel.**

**[NCERT Exemp. Q. 13.1, Page 90]**

**Ans. Correct option:** (b)

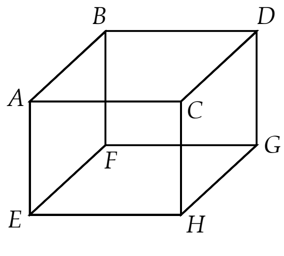
*Explanation:*

$$P = \frac{nRT}{V} = P$$

P remains unaffected as n, R, T and V are constant.

**Q. 2. 1 mole of an ideal gas is contained in a cubical volume V. ABCDEFGH at 300 K as shown in figure. One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time,**

**(a) the pressure on EFGH would be zero.**
**(b) the pressure on all the faces will the equal.**
**(c) the pressure of EFGH would be double the pressure on ABCD.**
**(d) The pressure on EFGH would be half that of ABCD.**




**[NCERT Exemp. Q. 13.2, Page 90]**

**Ans. Correct option:** (d)

*Explanation:* The momentum transferred to the face ABCD = 2 mv and gas molecule is absorbed by the face EFGH. Hence it does not rebound. So, momentum transferred to the EFGH = mv.

**Q. 3. Boyle's law is applicable for an**

**(a) adiabatic process.**
**(b) isothermal process.**
**(c) isobaric process.**
**(d) isochoric process.**

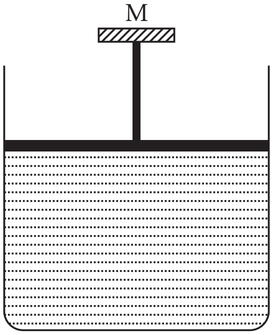
**[NCERT Exemp. Q. 13.3, Page 91]**

**Ans. Correct option:** (b)

*Explanation:*

Since $V \propto \frac{1}{\rho}$ at constant temperature.

**Q. 4. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction as shown in fig. If the temperature is increased.**




**(a) both P and V of the gas will change.**

**(b) only P will increase according to Charles's law.**

**(c) V will change but not P.**

**(d) P will change but not V.**

**[NCERT Exemp. Q. 13.4, Page 91]**

**Ans. Correct option:** (c)

*Explanation:*

$$p = \frac{F}{A} = \frac{mg}{A} = \text{constant}$$

$\therefore$ $V \propto T$ (at constant) pressure

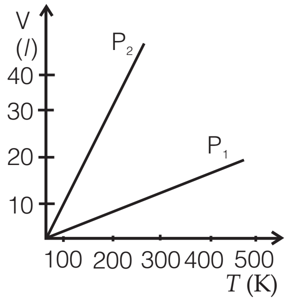
**Q. 5. Volume versus temperature graphs for a given mass of an ideal gas are shown in figure at two different values of constant pressure. What can be inferred about relation between $P_1$ & $P_2$ ?**

**(a) $P_1 > P_2$**

**(b) $P_1 = P_2$**

**(c) $P_1 < P_2$**

**(d) data is insufficient**




**[NCERT Exemp. Q. 13.5, Page 91]**

**Ans. Correct option:** (a)

*Explanation:*

$V \propto T$

$$\frac{V}{T} = \text{constant} = \frac{1}{P}$$

In graph, slope at $P_2$ is more than slope at $P_1$,

$\therefore$ $P_1 > P_2$

**Q. 6. 1 mole of $H_2$ gas is contained in a box of volume V = 1.00 $m^3$ at T = 300 K. The gas is heated to a temperature of T = 3000 K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal)**

**(a) Same as the pressure initially.**

**(b) 2 times the pressure initially.**

**(c) 10 times the pressure initially.**

**(d) 20 times the pressure initially.**

**[NCERT Exemp. Q. 13.6, Page 92]**

**Ans. Correct option:** (d)

*Explanation:* At constant volume $P \propto nT$

$$\therefore \quad \frac{P_2}{P_1} = \frac{n_2T_2}{n_1T_1} = \frac{2n}{n} \times \frac{3000}{300} = 20$$

$\Rightarrow P_2 = 20\,P_1$

**Q. 7. A vessel of volume V contains a mixture of 1 mole of Hydrogen and 1 mole of Oxygen (both considered as ideal). Let $f_1(v)dv$, denote the fraction of molecules with speed between $v$ and $(v + dv)$ with $f_2(v)dv$, similarly for oxygen. Then :**

**(a) $f_1(v) + f_2(v) = f(v)$ obeys the Maxwell's distribution law**

**(b) $f_1(v)$, $f_2(v)$ will obey the Maxwell's distribution law separately**

**(c) Neither $f_1(v)$, nor $f_2(v)$ will obey the Maxwell's distribution law**

**(d) $f_2$ (v) and $f_1$(v) will be the same.**

**[NCERT Exemp. Q. 13.7, Page 92]**

**Ans. Correct option:** (b)

*Explanation:* The masses of hydrogen and oxygen molecules $dx = f(v)$, which are having speeds between $v$ and $v + dv$. The Maxwell Boltzmann speed distribution function NV = $\frac{dn}{dv}$ depends an the mass of the gas molecules.

For each function $f_1(v)$ and $f_2(v)$ $n$ will be different, hence each function $f_1(v)$ and $f_2(v)$ will obey the Maxwell's distribution law separately.

**Q. 8. An inflated rubber balloon contains one mole of an ideal gas, has a pressure p, volume V and temperature T. If the temperature rises to 1.1. T, and the volume is increases to 1.05 V, the final pressure will be**

**(a) 1.1 p** **(b) p**

**(c) less than p** **(d) between p and 1.1 p**

**[NCERT Exemp. Q. 13.8, Page 92]**

**Ans. Correct option:** (d)

*Explanation:* Ideal gas equation PV = nRT. Here n is the numbers of moles,

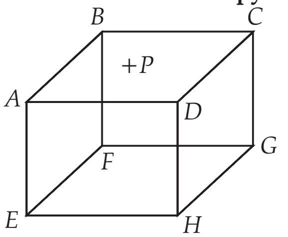
$$\text{So,} \quad n = \frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2} \quad \text{or} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore \quad P_2 = P_1V_1\,\frac{T_2}{V_2T_1}$$

here $P_1 = p$, $V_2 = 1.05$ V, $V_1 = V$, $T_2 = 1.1$ T, $T_1 = T$

$$\therefore \quad P_2 = \frac{P \times V \times 1.1T}{1.05V \times T} = \frac{1.1}{1.05}P = 1.05\,p$$

**Q. 9. ABCDEFGH is a hollow cube made of an insulator as shown in fig). Face ABCD has positive charge on it. Inside the cube, we have ionized hydrogen.**
**The usual kinetic theory expression for pressure**
**(a) will be valid**
**(b) will not be valid since the ions would experience forces other than due to collisions with the walls**
**(c) will not be valid since collisions with walls would not be elastic**
**(d) will not be valid because isotropy is lost**



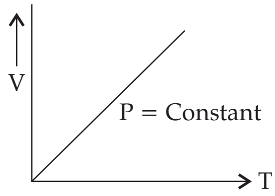

**[NCERT Exemp. Q. 13.9, Page 92]**

**Ans. Correct option:** (b) and (d)

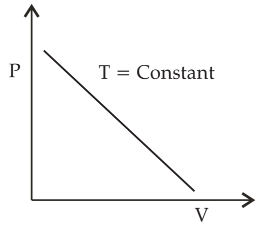
*Explanation:* According to the problem ionized hydrogen is present inside the cube, they are having charge, Now, due to the preference of Positive charge on the surface ABCD hydrogen ions would experience forces other then the forces due to collision with the walls of container. So, these forces must be of electrostatic nature Hence, Isotropy of system is last at only are face ABCD because of the pressure of external positive charge. The usual expression for pressure are the basis of Kinetic they will be valid.

**Q. 10. Which of the following diagrams (figure) depicts ideal as behaviour ?**

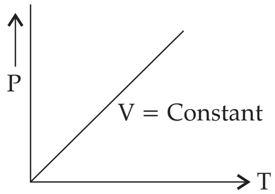
**(a)**



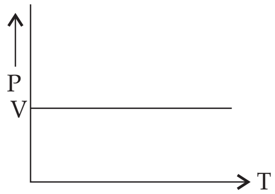

**(b)**




**(c)**




**(d)**




**[NCERT Exemp. Q. 13.12, Page 93]**

**Ans. Correct option:** (a) and (c)

*Explanation:*

**(a)** At constant P, V∝T it is charles' law.

**(c)** At constant V, P∝T it is pressure law

## Very Short Answer Type Questions **(1 mark each)**

**Q. 1. The volume of a given mass of a gas at 27°C, 1 atm is 100 cc. What will be its volume at 327°C ?**

**[NCERT Exemp. Q. 13.15, Page 94]**

**Ans.** Given that

$$T_1 = 27 + 273 = 300 \text{ K}$$
$$T_2 = 327 + 273 = 600 \text{ K}$$
$$V_1 = 100 \text{ cc}, V_2 = ?$$

If P is constant then V∝T or $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\Rightarrow \quad V_2 = \frac{V_1 T_2}{T_1} = \frac{100 \times 600}{300}$$

$$= 200 \text{ c.c.}$$

**Q. 2. The molecules of a given mass of a gas have root mean square speeds of 100 ms$^{-1}$ at 27°C and 1.00 atmospheric pressure. What will be the root mean square speeds of the molecules of the gas at 127°C and 2.0 atmospheric pressure?**

**[NCERT Exemp. Q. 13.16, Page 94]**

**Ans.**

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{P_2T_1}{P_1T_2} = \frac{2 \times 300}{400} = \frac{3}{2}$$

$$P_1 = \frac{1}{3}\frac{M}{V_1}c_1^{-2};\ P_2 = \frac{1}{3}\frac{M}{V_2}c_2^{-2}$$

$$\therefore c_2^2 = c_1^2 \times \frac{V_2}{V_1} \times \frac{P_2}{P_1}$$

$$= (100)^2 \times \frac{2}{3} \times 2$$

$$c_2 = \frac{200}{\sqrt{3}} m s^{-1}$$

**Q. 3. Two molecules of a gas have speed of 9 × 10$^{16}$ ms$^{-1}$ and 1 × 10$^{16}$ ms$^{-1}$ respectively. What is the root mean source speed of these molecules ?**

**[NCERT Exemp. Q. 13.17, Page 94]**

**Ans.** Root mean square speed

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + ....V_n^2}{n}}$$

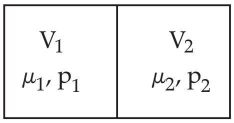
For two molecules

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2}{2}} = \sqrt{\frac{(9\times10^6)^2 + (1\times10^6)^2}{2}}$$

$$= \sqrt{\frac{(81\times1)\times10^{12}}{2}} = \sqrt{41}\times10^6\,\text{ms}^{-1}$$

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. The container shown in Fig. 13.6 has two chambers, separated by a partition, of volumes $V_1$ = 2.0 litre and $V_2$ = 3.0 litre. The chambers contain $\mu_1$ = 4.0 and $\mu_2$ = 5.0 moles of a gas at pressures $P_1$ = 1.00 atm and $P_2$ = 2.00 atm. Calculate the pressure after the partition is removed and the mixture attains equilibrium.**




**[NCERT Exemp. Q. 13.20, Page 94]**

**Ans.** $V_1$ = 2.0 litre, $V_2$ = 3.0 litre

$\mu_1$ = 4.0 moles, $\mu_2$ = 5.0 moles

$P_1$ = 1.00 atm, $P_2$ = 2.00 atm

$P_1V_1 = \mu_1 RT_1$, $P_2V_2 = \mu_2 RT_2$

$\mu = \mu_1 + \mu_2 \qquad V = V_1 + V_2$

For 1 mole, $PV = \frac{2}{3}E$

For $\mu_1$ moles, $P_1V_1 = \frac{2}{3}\mu_1 E_1$

For $\mu_2$ moles, $P_2V_2 = \frac{2}{3}\mu_2 E_2$

Total energy is $(\mu_1E_1 + \mu_2E_2) = \frac{3}{2}(P_1V_1 + P_2V_2)$

$$PV = \frac{2}{3}E_{total} = \frac{2}{3}\mu E_{per\ mole}$$

$$P(V_1 + V_2) = \frac{2}{3}\times\frac{3}{2}(P_1V_1 + P_2V_2)$$

$$P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}$$

$$= \left(\frac{1.00\times2.0 + 2.00\times3.0}{2.0 + 3.0}\right)\text{atm}$$

$$= \frac{8.0}{5.0} = 1.60\ atm.$$

**Comment:** This form of Ideal gas law represented by Equation marked* becomes very useful for adiabatic changes.

**Q. 2. When air is pumped into a cycle tyre the volume and pressure of the air in the tyre both are increased. What about Boyle's law in this case?**

**[NCERT Exemp. Q. 13.23, Page 95]**

**Ans.** When air is pumped, more molecules are pumped in. Boyle's law is stated for situation where number of molecules remain constant.

**Q. 3. A gas mixture consists of molecules of types A, B and C with masses $m_A > m_B > m_C$. Rank the three of molecules is decreasing under of :**

**(a) Average KE (b) rms speeds.**

**[NCERT Exemp. Q. 13.21, Page 94]**

**Ans. (a)** K.E. $= \frac{3}{2}$ kT

or K.E. $\propto \sqrt{T}$

Which remains same for all three types of molecules

**(b)** $v_{rms} = \sqrt{\frac{3RT}{m}} = \sqrt{\frac{3kT}{m}}$

or $v_{rms} \propto \frac{1}{\sqrt{m}}$

here $m_A > m_B > m_C$

so $(v_{rms})_C > (v_{rms})_B > (v_{rms})_A$

**Q. 4. Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP : 1 atm pressure, 0°C). Show that it is 22.4 litres. [NCERT Ex. Q. 13.2, Page 338]**

**Ans.** Here $n$ =1 mol, T = 273 K, R = 8.31 J/mole$^{-1}$ K$^{-1}$

P = 1 atm = 0.76 mm of Hg

$= 0.76 \times 9.8 \times 13.6 \times 10^3$ N/m$^2$

Using the relation

PV = $n$RT,

or, $V = \frac{nRT}{P}$

$$= \frac{1\times8.31\times273}{0.76\times9.8\times(13.6\times10^3)}$$

$= 22.4 \times 10^{-3}$ m$^3$

= 22.4 lit.

**Q. 5. Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m$^3$ at a temperature of 27°C and 1 atm pressure.**

**[NCERT Ex. Q. 13.6, Page 339]**

**Ans.** Here P = 1 atm

$= 1.01 \times 10^5$ Pa

V = 25 m$^3$

$k_B = (1.38 \times 10^{-23})$ J/K

T = 273 + 27

= 300 K

Using, PV = $nk_BT$, we get

$$n = \frac{PV}{k_BT}$$

$$= \frac{(1.01\times10^5)(25)}{(1.38\times10^{-23})(300)}$$

$= 6.1 \times 10^{26}$ molecules.

**Q. 6. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contains equal number of respective molecules ? Is the root mean square speed of molecules the same in the three cases ? If not, in which case is $v_{rms}$ the largest ? [NCERT Ex. Q. 13.8, Page 339]**

**Ans. (a)** Yes, because according to Avogadro's hypothesis, equal volume of all the gases have same number of molecules under the condition of same temperature and pressure.

**(b)** Using $v_{rms} = \sqrt{\frac{3kT}{m}}$, we get $v_{rms} \propto \frac{1}{\sqrt{m}}$, *i.e.,* $v_{rms}$ will not be same in the three cases because it depends upon mass of the gas. It will be large for neon.

**Q. 7. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the r.m.s. speed of a helium gas atom at –20°C ? (Atomic mass of Ar = 39.9 u and that of He = 4.0 u.) [NCERT Ex. Q. 13.9, Page 339]**

**Ans.** R.M.S. speed of argon at temperature T

$$v = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}}$$

R.M.S. for helium at temperature – 20°C or 253K is

$$v' = \sqrt{\frac{3R\times 253}{4.0}}$$

But $v = v'$ (given)

$$\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R\times 253}{4.0}}$$

or

$$\frac{T}{39.9} = \frac{253}{4.0}$$

$$T = 2523.7K.$$

**Q. 8. From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 $cm^3/s$. The diffusion of another gas under the same condition is measured to have an average rate of 7.2 $cm^3/s$. Identify the gas.**

**[Hint : Use Graham's law of diffusion : $R_1/R_2 = (M_2/M_1)V_2$, where $R_1$, $R_2$ are diffusion rates of gases 1 and 2, and $M_1$ and $M_2$ their respective molecular masses. This law is a simple consequence of kinetic theory.]**

**[NCERT Ad. Ex. Q. 13.12, Page 340]**

**Ans.** Using Graham's law of diffusion

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

*i.e.,*

$$\left(\frac{R_1}{R_2}\right)^2 = \frac{M_2}{M_1}$$

$$M_2 = M_1\left(\frac{R_1}{R_2}\right)^2$$

$$= 2\left(\frac{28.7}{7.2}\right)^2$$

$$= 2 \times 15.89 \approx 32.$$

The gas is identified as oxygen.

**Q. 9. Given below are densities of some solids and liquids. Give rough estimates of size of their atoms:**

| Substance | Atomic mass (u) | Density ($10^3$. $kgm^{-3}$) |
|---|---|---|
| Carbon (diamond) | 12.01 | 2.22 |
| Gold | 197.00 | 19.32 |
| Nitrogen (liquid) | 14.01 | 1.00 |
| Lithium | 6.94 | 0.53 |
| Fluorine(liquid) | 19.00 | 1.14 |

**(Hint: assume the atoms to be tightly packed in a solid or liquid phase, and use the known value of Avogadro's number. You should, however, not take the actual numbers you obtain for various atomic sizes too literally. Because of the crudeness of the tight packing approximation, the results only indicate that atomic sizes are in the range of a few Å]. [NCERT Ex. Q. 13.14, Page 340]**

**Ans.** Volume of each atom $= \frac{4}{3}\pi r^3$

Volume of N number of molecules $= \frac{4}{3}\pi r^3 N$ ...(i)

Volume of one mole of a substance $= \frac{M}{P}$ ...(ii)

from (i) and (ii)

$$\frac{4}{3}\pi r^3 N = \frac{M}{P}$$

$$r = \sqrt[3]{\frac{3M}{4\pi PN}}$$

**For carbon :** $r = \left(\frac{3\times 12.01\times 10^{-3}}{4\pi\times 2.22\times 10^3\times 6.023\times 10^{23}}\right)^{\frac{1}{3}}$

$= 1.29$ Å

**For gold :** $r = \left(\frac{3\times 197\times 10^{-3}}{4\pi\times 19.32\times 10^3\times 6.023\times 10^{23}}\right)^{\frac{1}{3}}$

$= 1.59$ Å

**For liquid nitrogen :**

$$r = \left(\frac{3\times 14.01\times 10^{-3}}{4\pi\times 1.00\times 10^3\times 6.023\times 10^{23}}\right)^{\frac{1}{3}}$$

$= 1.77$ Å

**For lithium :** $r = \left(\frac{3\times 6.94\times 10^{-3}}{4\pi\times 0.53\times 10^3\times 6.023\times 10^{23}}\right)^{\frac{1}{3}}$

$= 1.73$ Å

**For liquid Fluorine :**

$$r = \left(\frac{3\times 19\times 10^{-3}}{4\pi\times 1.14\times 10^3\times 6.023\times 10^{23}}\right)^{\frac{1}{3}}$$

$= 1.88$ Å

# Long Answer Type Questions (5 marks each)

**Q. 1. Explain why**
**(a) there is no atmosphere on moon.**
**(b) there is fall in temperature with altitude.**
**[NCERT Exemp. Q. 13.27, Page 95]**

**Ans.** **(a)** The moon has small gravitational force and hence the escape velocity is small. As the moon is in the proximity of the Earth as seen from the Sun, the moon has the same amount of heat per unit area as that of the Earth. The air molecules have large range of speeds. Even though the rms speed of the air molecules is smaller than the escape velocity on the moon, a significant number of molecules have speed greater than escape velocity and they escape. Now rest of the molecules arrange the speed distribution for the equilibrium temperature. Again a significant number of molecules escape as their speeds exceed escape speed. Hence, over a long time the moon has lost most of its atmosphere.

At 300 K,

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{7.3 \times 10^{-26}}} = 1.7\, km/s$$

$V_{esc}$ for moon = 4.6 km/s

**(b)** As the molecules move higher their potential energy increases and hence kinetic energy decreases and hence temperature reduces.

At greater height more volume is available and gas expands and hence some cooling takes place.

**Q. 2. A box of $1.00 m^3$ is filled with nitrogen at 1.50 atm at 300K. The box has a hole of an area $0.010\ mm^2$. How much time is required for the pressure to reduce by 0.10 atm, if the pressure outside is 1 atm.**
**[NCERT Exemp. Q. 13.30, Page 96]**

**Ans.** $V_{ix}$ = speed of molecule inside the box along $x$ direction

$n_1$ = number of molecules per unit volume

In time $\Delta t$, particles moving along the wall will collide if they are within $(V_{ix}\Delta t)$ distance. Let $a$ = area of the wall. No. of particles colliding in time $\Delta t = \frac{1}{2} n_i (V_{ix} \Delta t) a$ (factor of 1/2 due to motion towards wall).

In general, gas Is In equilibrium as the wall is very large as compared to hole.

$$\therefore V_{1x}^2 + V_{1y}^2 + V_{1z}^2 = V^2_{rms}$$

$$\therefore V_{1x}^2 = \frac{V^2_{rms}}{3}$$

$$\frac{1}{2} m V^2_{rms} = \frac{3}{2} kT \Rightarrow V^2_{rms} = \frac{3kT}{m}$$

$$\therefore V^2_{1x} = \frac{kT}{m}$$

$\therefore$ No. of particles colliding in time $\Delta t = \frac{1}{2} n_1 \sqrt{\frac{kT}{m}} \Delta t\ a.$

If particles collide along hole, they move out. Similarly outer particles colliding along hole will move in.

$\therefore$ Net particle flow in time $\Delta t = \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}} \Delta t\ a$ as temperature is same in and out.

$$pV = \mu RT \Rightarrow \mu = \frac{PV}{RT}$$

$$n = \frac{\mu N_A}{V} = \frac{PN_A}{RT}$$

After some time $\tau$ pressure changes to $p_1'$ inside

$$\therefore n_1' = \frac{P_1' N_A}{RT}$$

$n_1 V - n_1' V$ = no.of particle gone out

$$= \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}} \tau a$$

$$\therefore \frac{P_1 N_A}{RT} V - \frac{P_1' N_A}{RT} V = \frac{1}{2}(P_1 - P_2)\frac{N_A}{RT}\sqrt{\frac{kT}{m}} a$$

$$\therefore \tau = 2\left(\frac{P_1 - P_1'}{P_1 - P_2}\right)\frac{V}{a}\sqrt{\frac{m}{kT}}$$

$$= 2\left(\frac{1.5 - 1.4}{1.5 - 1.0}\right)\frac{5 \times 1.00}{0.01 \times 10^{-6}}\sqrt{\frac{46.7 \times 10^{-27}}{1.38 \times 10^{-23} \times 300}}$$

$$= 1.38 \times 10^5 s$$

**Q. 3. Consider a rectangular block of wood moving with a velocity $v_0$ in a gas at temperature $T$ and mass density $\rho$. Assume the velocity is along $x$-axis and the area of cross-section of the block perpendicular to $v_0$ is A. Show that the drag force on the block is $4\rho A v_0 \sqrt{\frac{kT}{m}}$, where $m$ is the mass of the gas molecule.**
**[NCERT Exemp. Q. 13.31, Page 96]**

**Ans.** $n$ = no. of molecules per unit volume

$v_{rms}$ = rms speed of gas molecules

When block is moving with speed $v_o$, relative speed of molecules w.r.t. front face = $v + v_o$

Coming head on, momentum transferred to block per collision

= $2m(v + v_o)$, where $m$ = mass of molecule.

No. of collision in time $\Delta t = \Delta t \frac{1}{2}(v + v_o) n \Delta t A$,

where A = area of cross section of block and factor of 1/2 appears due to particles moving towards block.

$\therefore$ Momentum transferred in time $\Delta t = m(v + v_o)^2 nA\Delta t$ from front surface.

Similarly momentum transferred in time $\Delta t = m(v - v_o)^2 nA\Delta t$ from back surface.

$\therefore$ Net force (drag force) $= mnA\left[(v + v_o) - (v - v_o)^2\right]$

from front $= mnA(vv_o) = (4mnAv)v_o$

$= (4\rho A v)v_o$

We also have $\frac{1}{2}mv^2 = \frac{1}{2}kT$ ($\because$ $v$ is the velocity along $x$-axis)

Therefore, $v = \sqrt{\frac{kT}{m}}$.

Thus, drag $= 4\rho A\sqrt{\frac{kT}{m}}v_o$.

**Q. 4. Consider an ideal gas with following distribution of speeds.**

| Speed (m/s) | % of molecules |
|---|---|
| 200 | 10 |
| 400 | 20 |
| 600 | 40 |
| 800 | 20 |
| 1000 | 10 |

**(i) Calculate $V_{rms}$ and hence h. (m = 3.0 × $10^{-26}$ kg)**

**(ii) If all the molecules with speed 1000 m/s escape from the system, calculate new $V_{rms}$ and hence T.**

**[NCERT Exemp. Q. 13.28, Page 95]**

**Ans.**

**(i)** $$v_{rms}^2 = \frac{\sum_i n_i v_i^2}{\sum_i n_i}$$

$$= \frac{10\times(200)^2 + 20\times(400)^2 + 40\times(600)^2 + 20\times(800)^2 + 10\times(1000)^2}{10+20+40+20+10}$$

= 10 × 400 + 20 × 1600 + 40 × 3600 + 20 × 6400 + 10 × 10000

= 4 × 1000 + 32 × 1000 + 144 × 1000 + 128 × 1000 + 100 × 1000

= 408 × 1000

$\therefore$ $V_{rms} = \sqrt{408\times1000}$ = 639 m/sec

now, $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

$$\therefore \text{T} = \frac{mv_{rms}^2}{3K} = \frac{3\times10^{-26}\times4.08\times10^5}{3\times1.38\times10^{-23}}$$

= 2.96 × $10^2$ = 296 K.

**(ii)** remaining particles root mean speed

$$v_{rms}^2 = \frac{10\times(200)^2 + 20\times(400)^2 + 40\times(600)^2 + 20\times(800)^2}{90}$$

= 342000

$\therefore$ $v_{rms} = \sqrt{342000}$ = 584 m/sec.

$$\therefore \text{T} = \frac{mv_{rms}^2}{3k} = \frac{3\times10^{-26}\times342000}{3\times1.38\times10^{-23}}$$

= 248 K.

**Q. 5. Figure shows plot of PV/T versus P for 1.00 × $10^{-3}$ kg of oxygen gas at two different temperatures.**

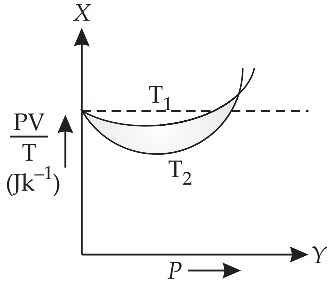
**(a) What does the dotted line signify ?**

**(b) Which is true : $T_1 > T_2$ or $T_1 < T_2$ ?**

**(c) What is the value of $\frac{PV}{T}$ where the curves meet on the y-axis ?**

**(d) If we obtained similar plots for 1·00 × $10^{-3}$ kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the Y-axis ? If not, what mass of hydrogen yields the same value of PV/T (for low pressure-high temperature region of the plot) ?**

**[Molecular mass of hydrogen ($H_2$) = 2.02 u, of $O_2$ = 32.0 u, R = 8.31 $mol^{-1}K^{-1}$.]**




**[NCERT Ex. Q. 13.3, Page 338]**

**Ans. (a)** The dotted plot signifies the ideal gas behaviour as PV/T = constant.

**(b)** $T_1$ is greater than $T_2$ because behaviour of a real gas approaches the ideal gas behaviour when temperature is increased.

**(c)** Here, $\frac{PV}{T} = nR = \frac{8.31}{(32\times10^{-3})} \times 1 \times 10^{-3}$

= 0.26 $JK^{-1}$

($\because$ R = 8.31 $Jmol^{-1}K^{-1}$, $O_2$ = 32 × $10^{-3}$ kg and mass of $O_2$ is given as 1 × $10^{-3}$ kg)

**(d)** No, this is because PV/T depends upon the volume of gas and volume of equal masses of different gases are different. For hydrogen

$$\frac{PV}{T} = \frac{8.31\times10^{-3}}{2.02\times10^{-3}} = 4{\cdot}11 \text{ JK}^{-1}$$

The value is not same.

Mass of hydrogen for same value of $\frac{PV}{T}$, *i.e.,* 0·26 $JK^{-1}$ is given by,

$$= \frac{2.02\times10^{-3}}{8.31} \times 0.26$$

= 6.32 × $10^{-5}$ kg

**Q. 6. An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder. (R = 8.31 $Jmol^{-1}K^{-1}$, molecular mass of $O_2$ = 32$\mu$)**

**[NCERT Ex. Q. 13.4, Page 338]**

**Ans.** Initially in the oxygen cylinder

$V_1$ = 30 litres

= 30 × $10^{-3}$ $m^3$

$P_1$ = 15 atm

= 15 × 1·01 × $10^5$ Pa.

$$T_1 = 27 + 273$$
$$= 300\text{ K}$$

If the cylinder contains $n_1$ mole of oxygen gas then

$$P_1V_1 = n_1RT_1$$

or $$n_1 = \frac{P_1V_1}{RT_1}$$

$$= \frac{15\times1.01\times10^5\times30\times10^{-3}}{8.3\times300}$$

$$= 18.253$$

For oxygen, molecular weight M = 32 g

Initial mass of cylinder

$$m_1 = n_1M$$
$$= 18{\cdot}253 \times 32$$
$$= 584{\cdot}1\text{ g}$$

Finally in the oxygen cylinder, let $n_2$ moles of oxygen be left

$$V_2 = 30 \times 10^{-3}\text{ m}^3$$
$$P_2 = 11 \times 1.01 \times 10^5\text{ Pa.}$$
$$T_2 = 17 + 273 = 290\text{ K.}$$

Now, $$n_2 = \frac{P_1V_1}{RT_1}$$

$$= \frac{15\times1.01\times10^5\times30\times10^{-3}}{8.3\times300}$$

$$= 13.847$$

Final mass of oxygen gas in the cylinder

$$m_2 = 13.847 \times 32 = 443.1\text{ g}$$

Mass of the oxygen gas taken out

$$= m_1 - m_2 = 584.1 - 443.1$$
$$= 141.0\text{ g}$$

**Q. 7. An air bubble of volume 1.0 cm$^3$ rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35°C ? (Given, 1 atm = 1·01 × 10$^5$ Pa)**

**[NCERT Ex. Q. 13.5, Page 338]**

**Ans.** When the air bubble is at 40 m depth, then

$$V_1 = 1.0\text{ cm}^3$$
$$= 1.0 \times 10^{-6}\text{ m}^3$$
$$T_1 = 12°\text{C}$$
$$= 12 + 273 = 285\text{ K}$$
$$P_1 = 1\text{ atm} + h\rho g$$
$$= 1.01 \times 10^5 + 40 \times 10^3 \times 9{\cdot}8$$
$$= 4{,}93{,}000\text{ Pa}$$

When the air bubble reaches at the surface of lake, then

$$V_2 = ?$$
$$T_2 = 35°\text{C} = 35 + 273\text{K}$$
$$= 308\text{ K}$$
$$P_2 = 1\text{ atm} = 1{\cdot}01 \times 10^5\text{ Pa}$$

Now $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

or $$V_2 = \frac{P_1V_1T_2}{T_1P_2}$$

$$V_2 = \frac{493000\times1.0\times10^{-6}\times308}{285\times1.01\times10^5}$$

$$= 5.27 \times 10^{-6}\text{ m}^3.$$

**Q. 8. A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom ?**

**[NCERT Ad. Ex. Q. 13.11, Page 340]**

**Ans.** Suppose $h$ cm of Hg flows out to equalise the pressure length of Hg thread = $(76 - h)$ and air $= (24 + h)$

[∴ Length of air column = 100 – 76 = 24 cm]

Pressure of the trapped air

$$= 76 - (76 - h)$$
$$= h\text{ cm of Hg}$$

According to Boyle's law

$$P_1V_1 = P_2V_2$$

*i.e.*, $$76 \times 15 = h(24 + h)$$

*i.e.*, $$h^2 + 24h - 1140 = 0$$

$$h = -\frac{24 \pm \sqrt{576 + 4560}}{2}$$

$$= 23.8\text{ cm or } -47{\cdot}8\text{ cm}$$

Taking the positive value $h = 23.8$ cm.

**Q. 9. A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by so called law of atmosphere's**

$$n_2 = n_1 \exp\left[\frac{-mg(h_2 - h_1)}{(k_BT)}\right]$$

**where $n_2$, $n_1$ refer to number density at heights $h_2$ and $h_1$ respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column :**

$$n_2 = n_1 \exp\left\{-mgN_A(\rho - \rho')(h_2 - h_1)/\rho RT\right\}$$

**where ρ is the density of suspended particle and ρ′, that of the surrounding medium [$N_A$ is Avogadro's number, and R the universal gas constant.]** **[NCERT Ad. Ex. Q. 13.13, Page 340]**

**Ans.** According to the law of atmosphere

$$n_2 = n_2 \exp\left[\frac{-mg(h_2 - h_1)}{k_BT}\right]$$

When the suspension is in sedimentation equilibrium in a liquid column, the weight mg of the suspended particle needs to be replaced by its apparent weight.

Now the apparent weight of the suspended particle.

$$mg' = \text{Actual weight} - \text{Upthrust}$$
$$= mg - V\rho' g$$

$$= mg - \frac{m}{p}\,\rho' g = mg\left(\frac{\rho - \rho'}{\rho}\right)$$

Also, $k_B = \frac{R}{N_A}$

Replacing mg by mg′ and putting the value of $k_B$ in equation (i), we get

$$n_2 = n_1 \exp\left[\frac{-mg'(h_2 - h_1)}{k_B T}\right]$$

or $$n_2 = n_1 \exp\left[\frac{-mgN_A(\rho - \rho')(h_2 - h_1)}{\rho RT}\right]$$

# TOPIC-2
## Law of Equi-partition energy and Brownian Motion

### Quick Review

- **Gay Lussac's Law or Regnault's Law :** When volume of a certain mass of a gas is kept constant, the pressure P exerted by gas is directly proportional to temperature T of gas *i.e.* P & T.
- **Avogadro's Law :** It states that equal volumes of all gases. Under identical conditions of temperature and pressure contain the same no. of molecules. $n_1 = n_2$
- **Graham's Law of Diffusion—**It states that rates of diffusion of two gases are inversely proportional to the square roots of their densities $r \propto \frac{1}{\sqrt{\rho}}$ or $\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$
- **Dalton's Law of Partial Pressure :** It states that total pressure exerted by a mixture of non-reactive ideal gases is equal to sum of partial pressures which each would exert, if it alone occupied the same volume at the given temperature.

$$P_1 + P_2 + P_3 \ldots\ldots\ldots\ldots = P$$

- **Law of Gaseous Volumes :** It states that when gases react together, they do so in volumes which will be a simple ratio to one another and also to the volumes of product.
- **Law of Equipartition of energy :** It states that the energy for each degree of freedom in thermal equilibrium is $\frac{1}{2}k_B T$.
- **Brownian Motion :** It is defined as continuous zig-zag motion of particles of macroscopic size ($\approx 10^{-5}$ m) suspended in water or air or some other fluid. Brownian motion increases :

  **(a)** When size of suspended object is smaller.

  **(b)** When density of fluid is smaller.

  **(c)** When temperature of medium is higher.

  **(d)** When viscosity of medium is smaller.

### Know the Terms

- **Pressure** exerted by gas is due to continuous bombardment of gas molecules against the walls of container.
- **Degrees of freedom** of a dynamic system is defined as the total no. of co-ordinates or independent quantities required to describe completely the position & configuration of the system.
- **Mean free path** is the average distance covered between two successive collisions by the gas molecule moving along the straight line.
- **Absolute zero of temperature** may be defined as that temperature at which the root mean square velocity of gas molecules reduces to zero.

### Know the Formulae

- **Pressure exerted by Ideal Gas.** $P = \frac{1}{3}\,mnc^2$

or $$P = \frac{1}{3}\frac{M}{V}c^2 = \frac{1}{3}\rho c^2$$

- **Relation between Pressure & K.E. of gas**

$$P = \frac{2}{3}E$$

- **Average K.E. of translation of 1 mole**

$$= \frac{1}{2}Mc^2 = \frac{3}{2}RT.$$

- **Average K.E. of translation per molecule of gas**

$$= \frac{1}{2}mc^2 = \frac{3}{2}k_BT.$$

- **Boyle's Law** PV = Constant at constant T.
- **Charles's Law** $V \propto c^2 \therefore V \propto T$ at constant pressure
- **Avogadro's Law** $n_1 = n_2$ at equal temperature , pressure and volume.
- **Graham's Law of diffusion** $r \propto \frac{1}{\sqrt{\rho}}$ or $\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$
- **Dalton's Law of partial pressure**

$$P_1 + P_2 + P_3 + .... = P$$

- **Degrees of freedom**

For
**(a)** Mono-atomic gas = 3
**(b)** Di-atomic gas = 5
**(c)** Tri-atomic gas = 7
**(d)** Non-linear triatomic gas = 6

- **Law of equipartition of energy** $E_t = \frac{1}{2}k_BT$

- **Specific Heat Capacity of :**

**(a)** Monoatomic gas $\gamma = \frac{C_P}{C_V}$

$$\gamma = \frac{5}{3} = 1.67$$

**(b)** Diatomic gas $\gamma = \frac{C_P}{C_V} = \frac{7}{5} = 1.4$

**(c)** Triatomic gas

Linear gas molecules $\gamma = \frac{9}{7} = 1.28$

Non-linear gas molecules $\gamma = \frac{4}{3} = 1.33.$

**(d)** Polyatomic gas $\gamma = \left(1 + \frac{2}{n}\right)$, where $n$ is the degree of freedom

**(e)** Solids $c = 3R = 24.93 J\ Mole^{-1} K^{-1}$
**(f)** Water $c = 9R = 74.7\ J\ mole^{-1} K^{-1}$

- **Mean free path**

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + ............ + \lambda_n}{n} = \frac{ct}{n}$$

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2} = \frac{R_B T}{\sqrt{2}n\pi d^2 P}$$

Here, $d$ is diameter, P is the pressure, T is temperature & $k_B$ is Boltzmann constant.

## Know the Links

- *www.learncbse.in*
- *www.vedantu.com*
- *www.examfear.com*

# MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. Diatomic molecules like hydrogen have energies due to both translational as well as rotational motion. From the equation in kinetic theory $pV = \frac{2}{3}E$, E is**

**(a) the total energy pen unit volume**

**(b) Only the translational part o energy because rotational energy is very small compared to the translational energy**

**(c) Only translational part of the energy because during collisions with the well pressure relates to change in linear momentum**

**(d) The translational part of the energy because rotational energies of molecules can be with sign and its average our all the molecules is zero.**

**[NCERT Exemp. Q. 13.10, Page 92]**

**Ans. Correct option:** (c)

*Explanation:* According to kinetic theory equation $PV = \frac{2}{3}E$. Here E is representing only translational part of energy. Internal energy contains all types of energies like translational, rotational vibrational etc. But the molecules of an θ ideal gas is treated as point masses in kinetic theory, so its kinetic energy is only due to translational or vibrational motion. Here we assumed that the walls only exert perpendicular forces a molecules. They do not exert any parallel force hence there there will not be any type of rotation present. The wall produces only change in translational motion.

**Q. 2. In a diatomic molecule, the rotational energy at a given temperature.**

**(a) obeys Maxwell's distribution**

**(b) have the same value for all molecules**

**(c) equals the translational kinetic energy for each molecule**

**(d) is $\left(\frac{2}{3}\right)^{rd}$ the translational kinetic energy for each molecule.** **[NCERT Exemp. Q. 13.11, Page 93]**

**Ans. Correct option:** (a) and (d)

*Explanation:*

**(a)** Translational kinetic energy and rotational kinetic energy both obey Maxwell's distribution independent of each other.

**(d)** Here 2 rotational and 3 translational energies are associated with each molecule. Translational kinetic energy of each molecule

$$K_T = \frac{3}{2}kT$$

and rotational kinetic energy $K_R = 2\left(\frac{1}{2}kT\right)$

now $$\frac{K_R}{K_T} = \frac{kT}{\frac{3}{2}kT} = \frac{2}{3}$$

$$\therefore K_R = \frac{2}{3}kT$$

**Q. 3. When an ideal gas is compressed adiabatically. Its temperature rises the molecules on the average have more kinetic energy than before. The kinetic energy increases.**

**(a) because of collisions with mouing parts of the wall only**

**(b) because of collisions with the entire wall**

**(c) because the molecules gets accelerated in their motion inside the volume**

**(d) because of redistribution of energy amongst the molecules**

**[NCERT Exemp. Q. 13.13, Page 93]**

**Ans. Correct option:** (a)

*Explanation:* Since the gas is ideal and the collisions of the molecules are elastic. When the molecules collides with the moving parts of the wall. its kinetic energy increases. But the total kinetic energy of the system will remain conserved. When the gas is compressed adiabatically, the total work done on the gas increases, its internal energy which in turn increases the KE of gas molecules and hence, the collision between molecules also increases.

## Very Short Answer Type Questions (1 mark each)

**Q. 1. Calculate the number of atoms in 39.4 g gold. Molar mass of gold is 197g mole$^{-1}$.**

**[NCERT Exemp. Q. 13.14, Page 94]**

**Ans.** $\therefore$ Molar mass of gold is 197 g mole$^{-1}$, the number of atoms = 6.0 x $10^{23}$

$\therefore$ No. of atoms in 39.4g $= \frac{6.0 \times 10^{23} \times 39.4}{197} = 1.2 \times 10^{23}$

**Q. 2. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature *T*. Neglecting all vibrational modes, calculate the total internal energy of the system. (Oxygen has two rotational modes.)**

**[NCERT Exemp. Q. 13.18, Page 94]**

**Ans.** $O_2$ has 5 degrees of freedom. Therefore, energy per mole = $\frac{5}{2}RT$

$\therefore$ For 2 moles of $O_2$, energy = $5RT$

Neon has 3 degrees of freedom

$\therefore$ Energy per mole = $\frac{3}{2}RT$

$\therefore$ For 4 mole of neon, energy = $4 \times \frac{3}{2}RT = 6RT$

$\therefore$ Total energy = $11RT$.

**Q. 3. Calculate the ratio of the mean free paths of the molecules of two gases having molecular diameters 1 Å and 2 Å. The gases may be considered under identical conditions of temperature, pressure and volume.**

**[NCERT Exemp. Q. 13.19, Page 94]**

**Ans.** $l \alpha \frac{1}{d^2}$

$d_1 = 1\mathring{A}$, $\alpha_2 = 2\mathring{A}$

$l_1 : l_2 = 4 : 1$

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. A balloon has 5.0 g mole of helium at 7°C. Calculate (a) the number of atoms of helium in the balloon, (b) the total internal energy of the system.**

**[NCERT Exemp. Q. 13.24, Page 95]**

**Ans. (a)** $\mu = 5.0T$

$T = 280K$

No. of atoms = $\mu N_A = 5.0 \times 6.02 \times 10^{23}$

$= 30 \times 10^{23}$

**(b)** Average kinetic energy per molecule = $\frac{3}{2}kT$

$\therefore$ Total internal energy = $\frac{3}{2}kT \times N$

$= \frac{3}{2} \times 30 \times 10^{23} \times 1.38 \times 10^{-23} \times 280$

$= 1.74 \times 10^4$ J

**Q. 2. Calculate the number of degrees of freedom of molecules of hydrogen in 1 cc of hydrogen gas at NTP. [NCERT Exemp. Q. 13.25, Page 95]**

**Ans.** Volume occupied by 1 gram mole of gas at NTP = 22400cc

$\therefore$ Number of molecules in 1cc of hydrogen

$= \frac{6.023 \times 10^{23}}{22400} = 2.688 \times 10^{19}$

As each diatomic molecule has 5 degrees of freedom, hydrogen being diatomic also has 5 degrees of freedom

$\therefore$ Total no. of degrees of freedom = $5 \times 2.688 \times 10^{19}$

$= 1.344 \times 10^{20}$

**Q. 3. An insulated container containing monoatomic gas of molar mass m is moving with a velocity vo. If the container is suddenly stopped, find the change in temperature.**

**[NCERT Exemp. Q. 13.25, Page 95]**

**Ans.** Loss in kinetic energy

$\Delta K = \frac{1}{2}(mn)v_0^2$

It temperature changes ΔT.

then $n \times \frac{3}{2} R \times \Delta T = \frac{1}{2}mnv_0^2$

$\therefore \quad \Delta T = \frac{mv_0^2}{3R}$

**Q. 4. We have 0.5 g of hydrogen gas in a cubic chamber of size 3 cm kept at NTP. The gas in the chamber is compressed keeping the temperature constant till a final pressure of 100 atm. Is one justified in assuming the ideal gas law in the final state ? (Hydrogen molecules can be consider as sphere of radius)Å. [NCERT Exemp. Q. 13.22, Page 95]**

**Ans.** Volume of 1 molecule = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times (10^{-10}) = 4.2 \times 10^{-30}$ m$^3$

Number of mole in 0.5 g $H_2$ gas

$= \frac{0.5}{2} = 0.25$ moles

$\therefore$ Volume of $H_2$ molecules in 0.25 moles

$= 0.25 \times 6.023 \times 10^{23} \times 4.2 \times 10^{-30}$

$V_1 = 1.05 \times 6.023 \times 10^{-7} = 6.3 \times 10^{-7}$ m$^3$

Now, $P_1V_1 = P_2V_2$

$V_2 = \frac{P_1V_1}{P_2} = \frac{1}{100} (3 \times 10^{-2})^3$

$= \frac{27 \times 10^{-6}}{100} = 2.7 \times 10^{-7}$ m$^3$

# Long Answer Type Questions

**(5 marks each)**

**Q. 1. Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3Å.**

**[NCERT Ex. Q. 13.1, Page 338]**

**Ans.** Given: diameter,

$$d = 3\text{Å},\ r = \frac{d}{2} = 1.5\text{Å}$$

$$= 1.5 \times 10^{-8} \text{ cm}$$

$$\text{Molecular volume V} = \frac{4}{3}\pi r^3 \text{N}$$

(Here, N = Avogadro's number)

$$= \frac{4}{3} \times \frac{22}{7} \times (1.5 \times 10^{-8})^3 \times (6.023 \times 10^{23})$$

$$= 8.52 \text{ cc}$$

Let V′ be actual volume occupied by 1 mole of $O_2$ at STP = 22400cc.

$$\therefore \frac{V}{V'} = \frac{8.52}{22400} = 3.8 \times 10^{-4}$$

**Q. 2. Estimate the average thermal energy of a helium atom at (i) room temperature (27°C), (ii) the temperature on the surface of the sun (6000 K), (iii) temperature of 10 million kelvin (the typical core temperature in the case of star).**

**[NCERT Ex. Q. 13.7, Page 339]**

**Ans. (i)**

$$\text{K.E.}_{avg} = \frac{3}{2}\text{kT}$$

$$= \frac{3}{2}(1.38 \times 10^{-23})(27 + 273)$$

$$= 6.21 \times 10^{-21} \text{ J}$$

**(ii)**

$$\text{K.E.}'_{avg} = \frac{3}{2}\text{kT}'$$

$$= \frac{3}{2}(1.38 \times 10^{-23}) \times 6000$$

$$= 1.242 \times 10^{-19} \text{ J}$$

**(iii)**

$$\text{K.E.}''_{avg} = \frac{3}{2}\text{kT}''$$

$$= \frac{3}{2}(1.38 \times 10^{-23}) \times 10^7$$

$$= 2.07 \times 10^{-16} \text{ J}$$

**Q. 3. Estimate the mean free path and collision frequency of nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17°C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collisions. (Molecular mass of $N_2$ = 28.0 u).**

**[NCERT Ex. Q. 13.10, Page 339]**

**Ans.** From Maxwell's correction, the mean free path (λ) of a gas molecule is given by

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$$

where $d$ = diameter of a molecule.

and

$$n = \frac{N}{V} = \frac{\text{Number of molecules}}{\text{Volume of gas}}$$

At 2 atm pressure, the volume of the gas of mass M is given by $V = \frac{RT}{P}$, T = 273 + 17 = 290 K.

$$\therefore \quad n = \frac{N}{V} = \frac{NP}{RT} \quad \text{(i)}$$

Now given : N = $6.023 \times 10^{23}$ mole$^{-1}$

P = 2 atm

= $2 \times 1.013 \times 10^5$ Nm$^{-2}$.

R = 8.3 JK$^{-1}$ mol$^{-1}$.

From equ (i)

$$n = \frac{6.023 \times 10^{-23} \times 2 \times 1.013 \times 10^{25}}{8.3 \times 290}$$

$$= 5.07 \times 10^{25}$$

Here, $r = 1\text{Å} = 1 \times 10^{-10}$ m

So, $d = 2r = 2 \times 10^{-10}$ m

Mean free path,

$$\lambda = \frac{1}{\sqrt{2} \times 3.142 \times (2 \times 10^{-10})^2 \times 5.07 \times 10^{25}}$$

$$= 1.0 \times 10^{-7} \text{ m}$$

The *r.m.s.* velocity is given by

$$c = \sqrt{\frac{3RT}{M}}$$

Putting R = 8.31 Jmol$^{-1}$ K$^{-1}$, T = 290 K,

M = $28 \times 10^{-3}$ kg

In equation (i),

$$c = \sqrt{\frac{3 \times 8.31 \times 290}{28 \times 10^{-3}}}$$

$$= 5.08 \times 10^2 \text{ ms}^{-1}$$

$$= 5.10 \times 10^2 \text{ ms}^{-1}.$$

∴ Collision frequency (ν) is given by

$$\nu = \frac{c}{\lambda} = \frac{5.1\times10^2}{1.0\times10^{-7}}$$

$= 5.1 \times 10^9\ s^{-1}$.

Suppose τ be the time between two successive collisions,

$$\therefore \qquad \tau = \frac{\lambda}{c} = \frac{1.0\times10^{-7}\,m}{5.1\times10^2\,ms^{-1}}$$

$= 2{\cdot}0 \times 10^{-10}$ s. (ii)

Also suppose $t$ = time taken for the collision.

$$\therefore \qquad t = \frac{d}{c} = \frac{2\times10^{-10}}{5.10\times10^2}$$

$= 4 \times 10^{-13}$ s (iii)

(ii)/(iii) gives $\frac{\tau}{t} = \frac{2.0\times10^{-10}\,s}{4\times10^{-13}\,s}$

$= 500$

or τ = 500 t, *i.e.*, the time taken between successive collisions is 500 times the time taken for a collision to take place, *i.e.*, the two molecules are in contact. This shows that the molecule in a gas moves nearly free for most of the time.

**Q. 4. Ten small planes are flying at a speed of 150 km/h in total darkness in an air space that is 20 × 20 × 1.5 km$^3$ in volume. You are in one of the planes, flying at random within this space with no way of knowing where the other planes are. On the average about how long a time will elapse between near collision with your plane. Assume for this rough computation that a safety region around the plane can be approximated by a sphere of radius 10 m. [NCERT Exemp. Q. 13.29, Page 96]**

**Ans.** Planes can be considered as the motion of molecules in confined space. Time of relaxation for mean free path λ is distance between two planes travelled between the collision or just to avoid accident.

$$\text{time} = \frac{\text{distance }(\lambda)}{\text{speed }(v)} = \frac{1}{\sqrt{2}n.\pi d^2 \times v}$$

now $n$ (number of particles per unit volume)

$$= \frac{N}{V} = \frac{10}{20\times20\times1.5\ km^3} = 0.0167\ km^{-3}$$

$d = 2 \times 10 = 20$ m $= 20 \times 10^{-3}$ km, v = 150 km/hr

$$\therefore \text{ time} = \frac{1}{\sqrt{2}n\pi d^2 v}$$

$$= \frac{1}{1.414\times0.0167\times3.14\times20\times20\times10^{-6}\times150}$$

= 225 hrs.

**TIPS... & TRICKS...**

- *Understand the concept of Ideal gas and real gas.*
- *Study about Boyle's law, charle's law and Pressure law.*
- *Study and understand about most probable speed. Average speed and root mean square speed.*
- *Study and understand about Avogodro law and Avogadro number.*
- *Learn about Degree of freedom.*
- *Study Law of Equi-Partition of energy.*

## Some Commonly Made Errors

- Students make mistakes while converting the units of pressure. Always remember / atom = 101325 Pa.

### EXPERT ADVICE

- *Make a short summary for specific heat capacity for monoatomic, diatomic and polyatomic gases.*
- *Remember pressure of fluid is not only exerted on the wall, it is exerted everywhere in a fluid.*
- *Kinetic theory explains the behaviour of gases. Brief understanding of behaviour of gases in required. Study them using graph for different processes.*

## OSWAAL LEARNING TOOLS

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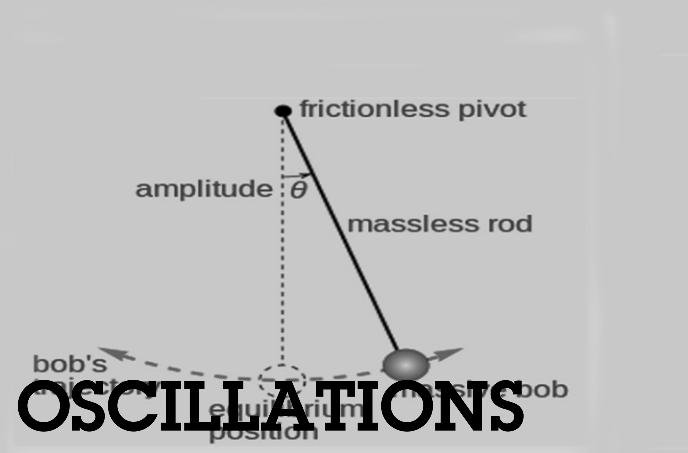
| **Visit :** *https://youtu.be/8jelON52wMw* **Or Scan the Code** | **Visit :** *https://youtu.be/nx9V0bfYijs* **Or Scan the Code** |
| **Visit :** *https://goo.gl/WKmKFj* **Or Scan the Code** | **Visit :** *https://goo.gl/CYRgTq* **Or Scan the Code** |

CHAPTER

# 14 OSCILLATIONS




## Chapter Objective

**This chapter will help you understand :**

- *Periodic motion - time period, frequency, displacement as a function of time, periodic function. Simple harmonic motion (S.H.M.) and its equation, phase, oscillations of a loaded spring - restoring force and force constant.*
- *Energy in S.H.M., Kinetic and potential energies, simple pendulum. Derivation of expression for its time period.*
- *Free, forced and damped oscillations (qualitative ideas only), resonance.*

## TOPIC-1
## Periodic Functions and Simple Harmonic Motion (S.H.M.)

### Quick Review



- **Harmonic Oscillations :** Those oscillations which can be expressed in terms of single harmonic function. *i.e.* (sine function or cosine function).

$$y = a \sin \omega t \text{ or } y = a \cos \omega t.$$

- **Non-Harmonic Oscillations :** Those oscillations which cannot be expressed in terms of single harmonic function *i.e.*,

$$y = a \sin \omega t + b \sin 2\omega t.$$

- **Periodic Functions :** Those functions which are used to represent periodic motion *i.e.*,

$$f(t) = f(t + T) = f(t + 2T).$$

sine & cosine functions are periodic functions.

- **Phase :** Phase of vibrating particle at any instant is a physical quantity which completely expresses the position and direction of motion of particle at that instant with respect to its mean position.
- **Some Facts :**

**(a)** In oscillatory motion, the phase of a vibrating particle is the argument of sine or cosine function involved to represent the generalized equation of motion of the vibrating particle.

**(b)** When the displacement of the particle executing a vibratory motion is represented by $y = a\sin(\omega t + \phi)$, then $(\omega t + \phi)$ is called phase of the vibrating particle.

**(c)** $\phi$ is called the initial phase of the vibrating particle.

- **Simple Harmonic Motion :** It is a special type of periodic motion, in which a particle moves to and fro repeatedly about a mean (*i.e.*, equilibrium) position under a restoring force, which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean (*i.e.*, equilibrium) position at that instant, *i.e.*,

$$F = -ky$$

where $k$ is known as force constant. Here negative sign represent that the restoring force (F) is always directed towards the mean position.

- **Geometrical interpretation of S.H.M. :** S.H.M. is defined as the projection of a uniform circular motion on any diameter of a circle of reference.

**(a)** S.H.M. may be linear and angular S.H.M.

**(b)** The linear S.H.M. is always along a straight line about a fixed point on a line, whereas the angular S.H.M. is always along an arc of a circle about a fixed point on the arc.

**(c)** The linear S.H.M. is controlled by force law, where $F = -ky$, where k is the restoring force constant, *i.e.*, force per unit displacement. Where as the angular S.H.M. is controlled by torque law, where $\tau = -C\theta$, where C is the restoring torque constant, *i.e.*, restoring torque per unit twist.

- **Characteristics of S.H.M. :**

**(a)** **Displacement :** The displacement of a particle executing linear S.H.M. at an instant is defined as the distance of the particle from the mean position at that instant.

**(b)** **Velocity : i**s defined as the time rate of change of the displacement of the particle at the given instant.

**(c)** **Amplitude :** The maximum displacement on either side of mean position.

**(d)** **Acceleration :** It is defined as the time rate of change of the velocity of the particle at the given instant.

**(e)** **Time Period :** It is defined as the time taken by the particle executing S.H.M. to complete one vibration.

- **Restoring Force & Force Constant :** It is the force required to give unit displacement to the body.

$$F = -ky$$ Here, $k$ is force constant.

- **Simple Pendulum :** It is most common example of S.H.M. An ideal simple pendulum consists of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.

$$\text{Time period,} = 2\pi\sqrt{\frac{l}{g}}$$

## Know the Terms

- **Periodic motion :** The motion which is identically repeated after a fixed interval of time. The time interval after which the motion is repeated is known as period of motion. *e.g.* The revolution of earth around the sun, its period is one year.
- **Oscillatory motion or vibratory motion :**

**(a)** The motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position), in a definite interval of time.

**(b)** In such a motion, the body is confined within well defined limits (called extreme positions) on either side of mean position. *e.g.*,
The motion of the pendulum of a wall clock is oscillatory motion.

- **Inertia Factor—**
  **Linear S.H.M :** Inertia factor stands for mass of the body executing S.H.M.
  **Angular S.H.M :** Inertia factor stands for moment of Inertia of the body executing S.H.M.

**Some terms related to periodic motion :**

**Time period :** It is the least interval of the time after which the periodic motion of a body repeats itself and is denoted by T.

**Frequency :** It is defined as the no. of periodic motions executed by the body per second.

**Angular frequency :** It is equal to the product of frequency of the body with factor $2\pi$. *i.e.* $\omega = 2\pi\nu$.

**Displacement :** It is the change in physical quantity under consideration with time in a periodic motion.

- **Spring Factor—**
  **Linear S.H.M :** Spring factor stands for force per unit displacement.
  **Angular SH.M :** Spring factor stands for restoring torque per unit twist.

## Know the Formulae

- **Periodic Motion.**

**(a)** **Frequency,** $$\nu = \frac{1}{T}$$

**(b)** **Angular Frequency**, $$\omega = \nu \times 2\pi$$

or $$\omega = \frac{2\pi}{T}$$

- $$\text{Phase} = (\omega t + \phi) = \left(\frac{2\pi t}{T} + \phi\right) = (2\pi\nu + \phi)$$

or $$\phi = \frac{2\pi t}{T} + \phi$$

➢ **Simple Harmonic Motion :**

**(a) Differential Equation,**

**(i)** Linear S.H.M. $= \frac{d^2y}{dt^2} + \omega^2 y$ where $\omega^2 = k/m$, here, $m$ is the mass of the body

**(ii)** Angular S.H.M. $= \frac{d^2y}{dt^2} + \omega^2\theta = 0$ where $\omega^2 = C/I$, here, I = moment of inertia

**(b) General *eq*uation—**

**(i)** Linear S.H.M. $y = y_0 \sin(\omega t + \phi)$

**(ii)** Angular S.H.M. $\theta = \theta_0 \sin(\omega t + \phi_0)$

**(c) Displacement,** $y = A\sin\omega t$

or $y = A\cos\omega t$

**(d) Velocity,** $v = \omega\sqrt{A^2 - y^2}$

**(e) Acceleration,** $a = \frac{dv}{dt} = -\omega^2 A \sin\omega t$

**(f) Time Period,** $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ or $2\pi\sqrt{\frac{I}{C}}$

➢ **Oscillations :**

**(a) Loaded Spring :**

**(i) Horizontal Direction,**

$$T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\frac{m}{k}}$$

Frequency, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

**(ii) Vertical Direction,** $T = 2\pi\sqrt{\frac{l}{g}}$

Frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

**(b) Loaded Spring Combinations :**

**Case I :** Two springs in parallel, $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$,

If, $k_1 = k_2 = k$, $T = 2\pi\sqrt{\frac{m}{2k}}$

**Case II :** Two springs in series, $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 \times k_2}}$,

If, $k_1 = k_2 = k$, $T = 2\pi\sqrt{\frac{2m}{k}}$

➢ **Spring Constant :** $k = \frac{F}{y}$

**(i)** In parallel, $k = k_1 + k_2$,

**(ii)** In series, $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

## Know the Links

- *www.vedantu.com*
- *www.learncbse.in*
- *www.topper.com*
- *www.britannica.com*

# MCQ/Fillups/True or False

**(1 mark each)**

## (A) Multiple Choice Questions

**Q. 1. The displacement of a particle is represented by the equation $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is**

**(a) simple harmonic with period $2\pi/\omega$.**
**(b) simple harmonic with period $\pi/\omega$.**
**(c) period but not simple harmonic.**
**(d) non – periodic.**

**[NCERT Exemp. Q. 14.1, Page 97]**

**Ans. Correct option:** (b)

*Explanation:*

$$y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right) = 3\cos\left[-\left(2\omega t - \frac{\pi}{4}\right)\right]$$

$$= 3\cos\left(2\omega t - \frac{\pi}{4}\right) \qquad \{\because \cos(-\theta) = \cos\theta$$

This shows simple harmonic motion with time period $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

**Q. 2. The displacement of a particle is represented by the equation y = sin$^3$ ωt. The motion is**

**(a) non – periodic.**
**(b) periodic but not simple harmonic.**
**(c) simple harmonic with period $2\pi/\omega$.**
**(d) simple harmonic with period $\pi/\omega$.**

**[NCERT Exemp. Q. 14.2, Page 97]**

**Ans. Correct option:** (b)

*Explanation:*

as $\frac{d^2y}{dt^2}$ is not proportional to $y$.

∴ It is not SHM but it is a periodic motion with period $2\pi/\omega$.

**Q. 3. The relation between acceleration and displacement of four particles are given below:**

**(a)** $a_x = +2x$.

**(b)** $a_x = +2x^2$.

**(c)** $a_x = -2x^2$.

**(d)** $a_x = -2x$.

**Which one of the particles is executing simple harmonic motion?**

**[NCERT Exemp. Q. 14.3, Page 98]**

**Ans. Correct option:** (d)

*Explanation:* Because for SHM, $a_x \propto (-x)$

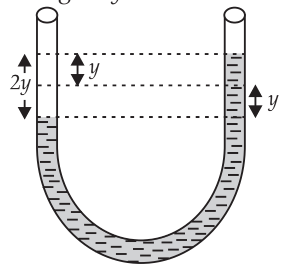
**Q. 4. Motion of an oscillating liquid column in a U – tube is**

**(a) periodic but not simple harmonic.**
**(b) non – periodic.**
**(c) simple harmonic and time period is independent of the density of the liquid.**
**(d) simple harmonic and time-period is directly proportional to the density of the liquid.**

**[NCERT Exemp. Q. 14.4, Page 98]**

**Ans. Correct option:** (c)

*Explanation:* Restoring force = weight of liquid column of height $2y$




$F = -A \times 2y \times \rho \times g$

$= -2A\rho g y$

$\Rightarrow F \propto -y$

Motion is S.H.M. with force constant $k = 2A\rho g$

$\Rightarrow$ Time period

$$T = 2\pi\sqrt{\frac{A \times 2h \times \rho}{2A\rho g}} = 2\pi\sqrt{\frac{h}{g}}$$

which is independent of the density of the liquid.

**Q. 5. A particle is acted simultaneously by mutually perpendicular simple harmonic motions $x = a\cos\omega t$ and $y = a\sin\omega t$. The trajectory of motion of the particle will be**

**(a) an ellipse.**
**(b) a parabola.**
**(c) a circle.**
**(d) a straight line.**

**[NCERT Exemp. Q. 14.5, Page 98]**

**Ans. Correct option:** (c)

*Explanation:* $x = a\cos\omega t$ ...(i)

$y = a\sin\omega t$ ...(ii)

Squaring and adding (i) & (ii),

We get

$$x^2 + y^2 = a^2\cos^2\omega t + a^2\sin^2\omega t$$
$$= a^2$$

The trajectory of motion will be a circle.

**Q. 6. The displacement of a particle varies with according to the relation**

**y = a sin ωt + b cos ωt.**

**(a) The motion is oscillatory but not S.H.M.**
**(b) The motion is S.H.M. with amplitude a + b.**
**(c) The motion is S.H.M. with amplitude $a^2 + b^2$.**
**(d) The motion is S.H.M. with amplitude $\sqrt{a^2 + b^2}$.**

**[NCERT Exemp. Q. 14.6, Page 98]**

**Ans. Correct option:** (d)

*Explanation:* Let

Let

$a = A\cos\phi$ (i)

$b = A\sin\phi$ (ii)

Squaring and adding (i) & (ii)

$a^2 + b^2 = A^2$

$y = a\sin\omega t + b\cos\omega t$ (iii)

'$y$' can be written as

$$y = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$$
$$= A \sin (\omega t + \phi)$$

This is an equation of SHM with amplitude $A = \sqrt{a^2 + b^2}$

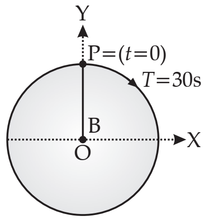
**Q. 7. Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the $x$-projection of the radius vector of the rotating particle P is**

**(a)** $x(t) = B\sin\left(\frac{2\pi t}{30}\right)$.

**(b)** $x(t) = B\sin\left(\frac{\pi t}{15}\right)$.

**(c)** $x(t) = B\sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$.

**(d)** $x(t) = B\sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$.




**[NCERT Exemp. Q. 14.8, Page 99]**

**Ans. Correct option:** (a)

***Explanation:*** Projection of OP on $x$-axis at time $t$ is

$$x(t) = B\cos\left(\frac{\pi}{2} - \frac{2\pi}{T}t\right) = B\sin\left(\frac{2\pi}{T}t\right)$$

$$x(t) = B\sin\left(\frac{2\pi}{30}t\right) \qquad \{\because T = 30s\}$$

**Q. 8. The equation of motion of a particle is $x = a \cos (\alpha t)^2$.**
**The motion is**

**(a) Periodic but not oscillatory.**
**(b) Periodic and oscillatory.**
**(c) Oscillatory but not periodic.**
**(d) Neither periodic nor oscillatory.**

**[NCERT Exemp. Q. 14.9, Page 99]**

**Ans. Correct option:** (c)

***Explanation:*** Given that $x(t) = \text{a} \cos (\alpha t)^2$

$x(t + \text{T}) = \text{a} \cos [\alpha(t + \text{T})]^2$

$= \text{a} \cos [\alpha^2 t^2 + \alpha^2 \text{T}^2 + 2\alpha^2 t\text{T}] \neq x(t)$

Hence, it is not periodic, now cosine function and x varies between –a to + a, the motion is oscillatary.

**Q. 9. A particle executing S.H.M. has a maximum speed of 30 cm/s and a maximum acceleration of 60 cm/s$^2$. The period of oscillation**

**(a)** $\pi s$. **(b)** $\frac{\pi}{2} s$.

**(c)** $2\pi$ **s** **(d)** $\frac{\pi}{t} s$.

**[NCERT Exemp. Q. 14.10, Page 99]**

**Ans. Correct option:** (a)

***Explanation:*** $v_{mass} = \omega A$ ...(i)

$a_{max} = \omega^2 A$ ...(ii)

Divide equ. (ii) by (i)

$$\frac{a_{max}}{v_{max}} = \frac{\omega^2 A}{\omega A} = \omega$$

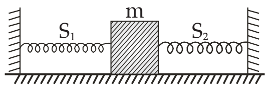
$$\therefore \frac{a_{max}}{v_{max}} = \frac{2\pi}{T}$$

$$T = 2\pi\left(\frac{v_{max}}{a_{max}}\right) = 2\pi\left(\frac{30}{60}\right) = \frac{2\pi}{2} = \pi s$$

**Q. 10. When a mass m is connected individually to two springs, $S_1$ and $S_2$, the oscillation frequencies are $\nu_1$ and $\nu_2$. If the same mass is attached to the two springs as shown in figure, the oscillation frequency would be**

**(a)** $\nu_2 + \nu_2$ **(b)** $\sqrt{\nu_1^2 + \nu_2^2}$.

**(c)** $\left(\frac{1}{\nu_1} + \frac{1}{\nu_2}\right)^{-1}$ **(d)** $\sqrt{\nu_1^2 - \nu_2^2}$.




**[NCERT Exemp. Q. 14.11, Page 99]**

**Ans. Correct option:** (b)

***Explanation:*** If $k_1$ and $k_2$ be the spring constant of spring $S_1$ and $S_2$ resp., Then

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \qquad \text{(i)}$$

$$\nu_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} \qquad \text{(ii)}$$

If $k$ be the effective spring constant.

$k = k_1 + k_2$ ($\because$ springs are parallel)

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$= \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$
$$= \frac{1}{2\pi}\sqrt{\frac{k_1}{m} + \frac{k_2}{m}}$$
$$= \frac{1}{2\pi}\sqrt{4\pi^2\nu_1^2 + 4\pi^2\nu_2^2}$$
$$\{by\,(i)\,\&\,(ii)\}$$
$$= \sqrt{\nu_1^2 + \nu_2^2}$$

**Q. 11. The rotation of earth about its axis is**

**(a) periodic motion.**
**(b) simple harmonic motion.**
**(c) periodic but not simple harmonic motion.**
**(d) non-periodic motion.**

**[NCERT Exemp. Q. 14.12, Page 100]**

**Ans. Correct option:** (a) and (c)

***Explanation:***

**(a)** Rotation of earth about its axis repeats its motion after a fixed interval of time, so its motion is Periodic.

(c) This motion does not follow S.H.M. equation a ∝.t hence, this motion is not a S.H.M.

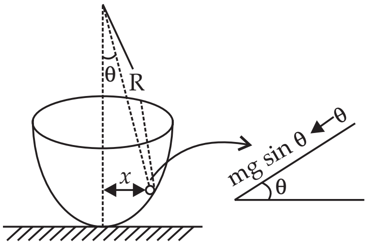
**Q. 12. Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower point is**

**(a) simple harmonic motion.**
**(b) non-periodic motion.**
**(c) periodic motion.**
**(d) periodic but not S.H.M.**

**[NCERT Exemp. Q. 14.13, Page 100]**

**Ans. Correct option:** (a) and (c)

*Explanation:*




Restoring force F = $-mg \sin\theta$

As θ is small, $\sin\theta = \theta$

so $ma = mg\dfrac{x}{R}$

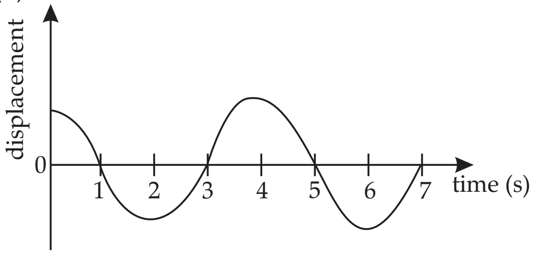
or $a = -\left(\dfrac{g}{R}\right)x$

⇒ $a \propto -x$

So, motion of the ball is S.H.M. and Periodic.

**Q. 13. Displacement vs. Time curve for a particle executing S.H.M. is shown in figure. Choose the correct statements.**

**(a) Phase of the oscillator is same at t = 0 s and t = 2 s.**
**(b) Phase of the oscillator is same at t = 2 s and t = 6 s.**
**(c) Phase of the oscillator is same at t = 1 s and t = 7 s.**
**(d) Phase of the oscillator is same at t = 1 s and t = 5 s.**




**[NCERT Exemp. Q. 14.14, Page 100]**

**Ans. Correct option:** (b) and (d)

*Explanation:*

**(b)** It is clear from the curve that points corresponding to t = 25 s and t = 6 s are separated by a distance belonging to one time period. Hence, these points must be in same phase.

**(d)** Points belong to $t$ = 1$s$ and t = 5s are at separation of one time period, hence must be in phase.

**Q. 14. Which of the following statements is/are true for a simple harmonic oscillator?**

**(a) Force acting is directly proportional to displacement from the mean position and opposite to it.**
**(b) Motion is periodic.**
**(c) Acceleration of the oscillator is constant.**
**(d) The velocity is periodic.**

**[NCERT Exemp. Q. 14.15, Page 100]**

**Ans. Correct option:** (a), (b) and (d)

*Explanation:*

**(a)** In S.H.M. $F \propto -x$

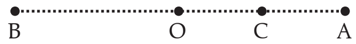
**(b)** Displacement equation of S.H.M.

$x = a \sin \omega t$ shows Periodic motion

**(d)** Velocity $v = \dfrac{dx}{dt} = a\omega \cos \omega t$

shows Periodic motion

**Q. 15. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart as shown in fig. Take the direction from A to B as the + ve direction and choose the correct statements.**

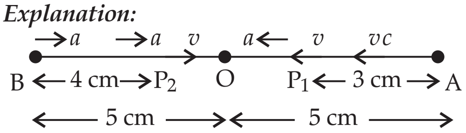



**(a) The sign of velocity, acceleration and force on the particle when it is 3 cm away from A going towards B are positive.**
**(b) The sign of velocity of the particle at C going towards O is negative.**
**(c) The sign of velocity, acceleration and force on the particle when it is 4 cm away from B going towards A are negative.**
**(d) The sign of acceleration and force on the particle when it is at point B is negative.**

**[NCERT Exemp. Q. 14.18, Page 101]**

**Ans. Correct option:** (a), (c) and (d)

*Explanation:*




**(a)** When the particle is 3 cm away from a going towards B. So, velocity is towards AB, i.e., Positive. In S.H.M., acceleration is always towards mean position (O) it means both force and acceleration act towards O, now positive sign.

**(c)** When the particle is 4 cm away from B going towards A velocity is negative and acceleration is towards mean position. (O), hence negative.

**(d)** Acceleration is always towards mean position (O). When the particle is at B, acceleration and force are towards BA that is negative.

**Q. 16. Which of the following relationships between the acceleration $a$ and the displacement $x$ of a particle involve S.H.M.**

**(a)** $a = 0.7x$ **(b)** $a = -200x^2$
**(c)** $a = -10x$ **(d)** $a = 100x^3$.

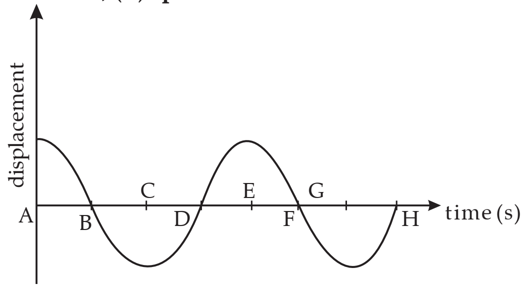
**[NCERT Ex. Q. 14.6, Page 363]**

**Ans. Correct option:** (c)

*Explanation:* In S.H.M. $a \propto -x$.

# Very Short Answer Type Questions (1 mark each)

**Q. 1. Displacement versus time curve for a particle executing S.H.M. is shown in figure. Identify the points marked at which (i) velocity of the oscillator is zero, (ii) speed of the oscillator is maximum.**

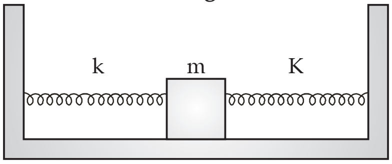



**[NCERT Exemp. Q. 14.19, Page 102]**

**Ans. (i)** Velocity of the oscillator is zero at extreme positions namely A, C, E and G.

**(ii)** Speed of oscillator is maximum is at mean position namely B, D, F and H.

**Q. 2. Two identical springs of spring constant K are attached to a block of mass m and to fixed supports as shown in figure. When the mass is displaced from equilibrium position by a distance x towards right, find the restoring force.**




**[NCERT Exemp. Q. 14.19, Page 102]**

**Ans.** Forces on the springs are

$F_1 = -kx$ (left spring)

$F_1 = -kx$ (right spring)

Restoring Force, $F = F_1 + F_2 = -2kx$

$\therefore$ $F = 2kx$ towards left

**Q. 3. What are the two basic characteristics of a simple harmonic motion?**

**[NCERT Exemp. Q. 14.21, Page 102]**

**Ans. (i)** Acceleration is directly proportional to displacement.

**(ii)** Direction of acceleration is opposite to that of displacement.

**Q. 4. When will the motion of a simple pendulum be simple harmonic?**

**[NCERT Exemp. Q. 14.22, Page 102]**

**Ans.** The motion of a simple pendulum will be simple harmonic when the bob is displaced from mean position in a way such that $\sin\theta = \theta$.

**Q. 5. What is the ratio of maximum acceleration to the maximum velocity of a simple harmonic oscillator?**

**[NCERT Exemp. Q. 14.23, Page 102]**

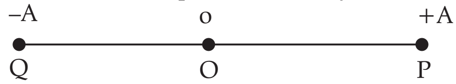
**Ans.** $$\frac{a_{max}}{v_{max}} = \frac{\omega^2 A}{\omega A} = \omega$$

**Q. 6. What is the ratio between the distance travelled by the oscillator in one time period and amplitude?**

**[NCERT Exemp. Q. 14.24, Page 102]**

**Ans.** In the diagram shown a particle is executing SHM between points P and Q.

The particle starts from mean position 'O' moves to amplitude position 'P', then particle turn back and moves from P to Q, finally the particle turn back again and return to mean position 'O'. So one oscillation is complete in this way.




Total distance travelled while it goes from O to P; P to O, O to Q, Q to O.

= OP + PO + OQ + QO

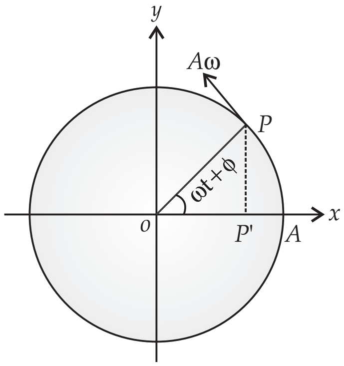
= A + A + A + A

= 4 A

Amplitude = OP = A

$$\therefore \frac{\text{distance}}{\text{amplitude}} = \frac{4A}{A} = 4$$

**Q. 7. In figure below, what will be the sign of the velocity of the point *P*, which is the projection of the velocity of the reference particle P. P is moving in a circle of radius *R* in anticlockwise direction?**




**[NCERT Exemp. Q. 14.25, Page 102]**

**Ans.** Projection of P on x-axis

$$x = A\cos(\omega t + \phi)$$

$$v_x = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

$\therefore$ sign of velocity is negative

**Q. 8. Show that for a particle executing S.H.M. velocity and displacement have a phase difference of $\pi/2$.**

**[NCERT Exemp. Q. 14.26, Page 102]**

**Ans.** Displacement, $y = A\sin\omega t$

{particle is executing SHM}

$$v = \frac{dy}{dt} = A\omega\cos\omega t = A\omega\sin\left(\omega t + \frac{\pi}{2}\right)$$

$\therefore$ phase difference between y and $v$ is

$$\Delta\phi = \left\{\left(\omega t + \frac{\pi}{2}\right) - \omega t\right\} = \frac{\pi}{2}$$

**Q. 9. The length of a seconds pendulum on the surface of Earth is 1m. What will be the length of a second's pendulum on the moon?**

**[NCERT Exemp. Q. 14.28, Page 103]**

**Ans.** For Earth, $T = 2\pi\sqrt{\dfrac{l}{g_{earth}}}$

For Moon, $T' = 2\pi\sqrt{\dfrac{l}{g_{moon}}}$

Since $T = T'$

So, $\dfrac{l_{moon}}{g_{moon}} = \dfrac{l_{earth}}{g_{earth}}$ or $l_{moon} = l_{earth}\left(\dfrac{g_{moon}}{g_{earth}}\right) = 1 \times \dfrac{1}{6}$

as $g_{moon} = \dfrac{1}{6} g_{earth} = \dfrac{1}{6} m$

**Q. 10. Which of the following example represent periodic motion :**

**(i) A swimmer completing one (return) trip from one bank of a river to other and back,**

**(ii) A freely suspended bar magnet displaced from its N-S direction and released,.**

**(iii) A hydrogen molecule rotating about its centre of mass.**

**(iv) An arrow released from a bow.**

**[NCERT Ex. Q. 14.1, Page 362]**

**Ans. (i)** It is not a periodic motion. Though the motion of a swimmer is to and fro but will not have a definite period.

**(ii)** It is a periodic motion because a freely suspended magnet if once displaced from N-S direction and let it go, it oscillates about its position. Hence, it is S.H.M. also.

**(iii)** It is a periodic motion.

**(iv)** It is not a periodic motion.

**Q. 11. Which of the following examples represent (nearly) simple harmonic motion, and which represent periodic but not S.H.M. :**

**(i) The motion of earth about its axis.**

**(ii) Motion of an oscillating mercury column in a U-tube.**

**(iii) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lowermost position.**

**(iv) General vibration of a polyatomic molecule about its equilibrium position.**

**[NCERT Ex. Q. 14.2, Page 362]**

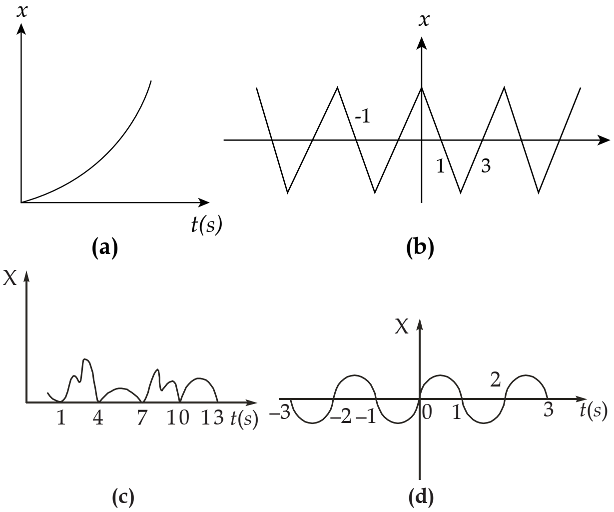
**Ans. (i)** It is periodic but not S.H.M. because it is not to and fro motion about a fixed point.

**(ii)** It is S.H.M.

**(iii)** It is S.H.M.

**(iv)** It is a periodic motion but not S.H.M. A polyatomic gas molecule has a number of natural frequencies and its general motion is the resultant of S.H.M. of number of different frequencies. The resultant motion is periodic but not S.H.M.

**Q. 12. Given below are four $x$–$t$ plots for linear motion of a particle. Which of the plots represent periodic motion ? What is the period of motion (in case of periodic motion) ?**




**[NCERT Ex. Q. 14.3, Page 362]**

**Ans. (a)** This figure does not represent periodic motion as the motion neither repeats nor comes to mean position.

**(b)** This figure represents periodic motion with period equal to 2 s.

**(c)** This figure does not represents periodic motion because it does not repeat identically.

**(d)** This figure represents periodic motion having period equal to 2 s.

**Q. 13. Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion ? (Give the time period for each case of periodic motion, ω is any positive constant.)**

**(i) $\sin \omega t - \cos \omega t$**

**(ii) $\sin^3 \omega t$**

**(iii) $3 \cos\left(\dfrac{\pi}{4} - 2\omega t\right)$**

**(iv) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$**

**(v) $e^{-\omega^2 t^2}$**

**(vi) $1 + \omega t + \omega^2 t^2$[NCERT Exemp. Q. 14.4, Page 363]**

**Ans.** The function will represent a periodic motion, if it is identically repeated after a fixed interval of time and will represent S.H.M. if it can be written uniquely in the form of A cos $\left(\dfrac{2\pi}{T} + \phi\right)$ or A sin $\left(\dfrac{2\pi}{T} + \phi\right)$, where T is the time period.

**(i)** $\sin \omega t - \cos \omega t \;=\; \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right]$

$$= \sqrt{2}\left[\sin\omega t\cos\frac{\pi}{4} - \cos\omega t\sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

It is a S.H.M. and its Time period is $2\pi/\omega$.

**(ii)** $\sin^3 \omega t = \frac{1}{4}[3\sin\omega t - \sin 3\omega t]$

Here each term $\sin \omega t$ and $\sin 3\omega t$ individually represents S.H.M. But (ii) which is the outcome of the superposition of two S.H.Ms. will only be periodic but not S.H.M. Its time period is $2\pi/\omega$.

**(iii)** $3\cos\left(\frac{\pi}{4} - 2\omega t\right) = 3\cos\left(2\omega t - \frac{\pi}{4}\right)$

[$\therefore \cos(-\theta) = \cos\theta$]

Clearly it represents S.H.M. and its time period is $2\pi/2\omega$ *i.e* $\pi/\omega$.

**(iv)** $\cos \omega t + \cos 3\omega t + \cos 5\omega t$. It represents the periodic but not S.H.M. Its time period is $2\pi/\omega$.

**(v)** $e^{-\omega^2 t^2}$ : it is an exponential function which never repeats itself. Therefore, it represents non-periodic motion.

**(vi)** $1 + \omega t + \omega^2 t^2$ also represents non-periodic motion.

**Q. 14. A particle is in linear S.H.M. between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is:**

**(a) at the end A,**

**(b) at the end B,**

**(c) at the mid point of AB going towards A,**

**(d) at 2 cm away from B going towards A,**

**(e) at 3 cm away from A going towards B, and**

**(f) at 4 cm away from B going towards A.**

**[NCERT Ex. Q. 14.5, Page 363]**

**Ans. (a) End A :** Velocity = 0; acceleration +*ve*, force +*ve*.

**(b) End B :** Velocity = 0; acceleration – *ve*; force – *ve*.

**(c)** Velocity – *ve*; acceleration = 0, force = 0 [mean position].

**(d)** Velocity – *ve*; acceleration = – *ve*; force – *ve*.

**(e)** Velocity + *ve*; acceleration + *ve*; force + *ve*

**(f)** Velocity – *ve*; acceleration + *ve*; force + *ve*.

## Short Answer Type Questions

**(2 or 3 marks each)**

**Q. 1. Show that the motion of a particle represented by y = sin ωt – cos ωt is simple harmonic with a period of 2π/ω.[NCERT Exemp. Q. 14.30, Page 103]**

**Ans.** $y = \sin\omega t - \cos\omega t$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\sin\omega t - \sin\frac{\pi}{4}\cos\omega t\right)$$

$$\therefore y = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

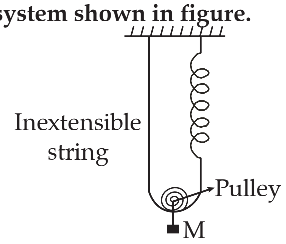
$\therefore$ (sin ωt–cos ωt) represents SHM.

$$y = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right) = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4} + 2\pi\right)$$

$$= \sqrt{2}\sin\left(\omega\left(t + \frac{2\pi}{\omega}\right) - \frac{\pi}{4}\right)$$

$$\therefore \text{Time period} = \frac{2\pi}{\omega}$$

**Q. 2. Find the time period of mass *M* when displaced from its equilibrium position and then released for the system shown in figure.**

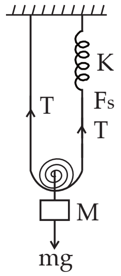



**[NCERT Exemp. Q. 14.29, Page 103]**

**Ans.** Mg = T + T {system is in equilibrium}

$\therefore$ Mg = 2T

because of mass hanging spring elongated by 2*l*. Where *l* is the distance moved by hanging mass




$T = F_s$ (In the spring)

$T = 2kl$

$$\therefore Mg = 2(2kl) = 2k(2l)$$

displacing mass through $y$ distance downwards

Restoring force

$$F = Mg - 2k(2l + 2y)$$

$$= Mg - (2k)(2l) - 4ky$$

or $\quad F = Mg - Mg - 4ky = -4ky$

$$M\frac{d^2y}{dt^2} = -4ky$$

$$\frac{d^2y}{dt^2} = -\frac{4k}{M}y \;\; \left\{\text{comparing with } \frac{d^2y}{dt^2} = -\omega^2 y\right\}$$

$$\therefore \omega = \sqrt{\frac{4k}{M}} \text{ or } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{4k}}$$

**Q. 3. A mass of 2 kg is attached to the spring constant 50 Nm$^{-1}$. The block is pulled to a distance of 5 cm from its equilibrium position at $x = 0$ on a horizontal frictionless surface from rest at $t = 0$. Write the expression for its displacement at anytime $t$. [NCERT Exemp. Q. 14.33, Page 103]**

**Ans.** Given: $m = 2$ kg, $k = 50$ Nm$^{-1}$, A = 5 cm

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{2}} = \sqrt{25} = 5s^{-1}$$

At t = 0, x = 0

$\therefore$ displacement at any time $t$

$x = A\sin\omega t \quad or \quad x = 5\sin 5t$

**Q. 4. Consider a pair of identical pendulums, which oscillate with equal amplitude independently such that when one pendulum is at its extreme position making an angle of 2° to the right with the vertical. The other pendulum makes an angle of 1° to the left of the vertical. What is the phase difference between the pendulums?**

**[NCERT Exemp. Q. 14.34, Page 103]**

**Ans.** Supposing $\theta_0$ is Angular amplitude of each of the pendulums.

$\theta_1 = \theta_0 \sin(\omega t + \delta_1)$

$\theta_2 = \theta_0 \sin(\omega t + \delta_2) \qquad$ {SHM of pendulums}

One pendulum is making 2° angle with vertical, $\theta_0 = 2°$. *For I pendulum* $\theta_1 = 2°$

$\therefore 2° = 2° \sin(\omega t + \delta_1)$

$\sin(\omega t + \delta_1) = 1$

$\omega t + \delta_1 = 90°$

For other pendulum, $\theta_2 = -1°$

$\therefore -1° = 2° \sin(\omega t + \delta_2)$

or $\sin(\omega t + \delta_2) = -\frac{1}{2}$

$\omega t + \delta_2 = -30°$

$\therefore (\omega t + \delta_1) - (\omega t + \delta_2) = 90° - (-30°)$

$\delta_1 - \delta_2 = 120°$

**Q. 5. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s, what is the weight of the body?**

**[NCERT Ex. Q. 14.8, Page 364]**

**Ans.** Given : mass, $m = 50$ kg,

maximum extension, $y = 20 - 0 = 20$ cm

$= 0.2$ m

Time, $t = 0.6$ s.

Force, $F = mg = 50 \times 9.8$ N

$$\therefore k = \frac{F}{y} = \frac{50 \times 9.8}{0.2} = 2450 \text{ N/m.}$$

Now, $T = 2\pi\sqrt{\frac{m}{k}}$ or $m = \frac{T^2 k}{4\pi^2}$

$$m = \frac{(0.6)^2 \times 2450}{4 \times (3.14)^2} = 22.36 \text{ kg}$$

so, weight of the body $= mg$

$= 22.36 \times 9.8$

$W = 219.1$ N.

**Q. 6. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with S.H.M. with an angular frequency of 200 rad/min. What is its maximum speed ? [NCERT Ex. Q. 14.14, Page 365]**

**Ans. Given,** $a = \frac{1}{2}$ $m$, $\omega = 200$ rad/min.

$$v_{max} = r\omega = \frac{1}{2} \times 200$$

$= 100$ m/min.

## Long Answer Type Questions **(5 marks each)**

**Q. 1. A person normally weighing 50 kg stands on a massless platform which oscillates up and down harmonically at a frequency of 2.0 s$^{-1}$ and an amplitude 5.0 cm. A weighting machine on the platform gives the persons weight against time.**

**(a) Will there be any change in weight of the body, during the oscillation?**

**(b) If answer to part (a) is yes, what will be the maximum and minimum reading in the machine and at which position?**

**[NCERT Exemp. Q. 14.35, Page 103]**

**Ans. (a)** Yes, change of weight will be there.

**(b)** Given mass $= m = 50$ kg, $\nu = 2$s$^{-1}$,

A = 5 cm = 0.05 m

Let the platform vibrate between two extreme positions O

P and Q about mean position O.

$a_{max}$ = Maximum acceleration towards mean position O

$a_{max} = \omega^2 A = (2\pi\nu)^2 A$

$a_{max} = 4\pi^2\nu^2 A = 4 \times (3.14)^2 \times 2^2 \times 0.05$

$= 7.9$ ms$^{-1}$

At P, restoring force ($ma_{max}$) and weight (mg) are directed towards mean position. So net weight at P given by

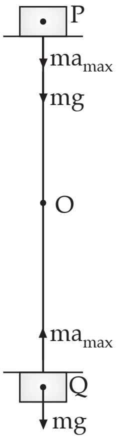
$W_1 = mg + ma_{max} = m(g + a_{max})$

$= 50 (9.8 + 7.9) = 885$ N

Similarly, net weight at Q

$W_2 = mg + ma_{max} = m(g + a_{max})$

$= 50 (9.8 + 7.9) = 95$ N




Weight is maximum at top position and minimum at lowermost position.

**Q. 2. A body of mass *m* is attached to one end of a mass less spring which is suspended vertically from a fixed point. The mass is held in hand so that the spring is neither stretched nor compressed. Suddenly the support of the hand is removed. The lowest position attained by the mass during oscillation is 4 cm below the point, where it was held in hand.**

**(a) What is the amplitude of oscillation?**

**(b) Find the frequency of oscillation?**

**[NCERT Exemp. Q. 14.35, Page 103]**

**Ans. (a)**

$$\text{Amplitude} = \frac{\text{distance between 2 extreme positions}}{2}$$

$$= \frac{4\,\text{cm}}{2} = 2\,\text{cm}$$

**(b)** Freq, $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

At mean position, $mg = k(2)$ or $\frac{k}{m} = \frac{g}{2}$

$$\therefore\ \nu = \frac{1}{2\pi}\sqrt{\frac{g}{2}} = \frac{1}{2\times 3.14}\sqrt{\frac{980}{2}}$$

$$= \frac{1}{6.28}\sqrt{490} = 3.5\,s^{-1}$$

**Q. 3. A cylindrical log of wood of height H and area of cross-section A floats in water. It is pressed and then released. Show that the log would execute S.H.M. with a time period.**

$$T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

**Where *m* is mass of the body and ρ is density of the liquid. [NCERT Exemp. Q. 14.37, Page 104]**

**Ans.** Given, $m$ = mass of cylinder

$h$ = height of cylinder

$h_1$ = length of cylinder dipping in liquid in equilibrium position

ρ = density of liquid

A = area of cross section of cylinder

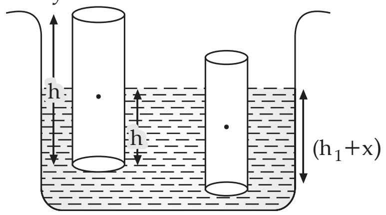
At equilibrium,

$mg$ = buoyant force

= weight of water displaced by body

$= \rho(Ah_1)g$ (i)

log is pressed gently through small distance x vertically and released.




$$F_B = \rho A(h_1 + x)g$$

∴ Net restoring force, $F$ = Buoyant Force – weight

$= \rho A(h_1 + x)g - mg$

$= \rho A(h_1 + x)g - \rho(Ah_1)g$ {from (i)}

$= (A\rho g)x$

∴ F and x are in opposite direction.

$F = -(A\rho g)x$

$$a = \frac{-(A\rho g)}{m}x \quad \text{(ii)}$$

for standard SHM a = $\omega^2 x$ (iii)

∴ by (ii) & (iii)

$$\omega^2 = \frac{A\rho g}{m} \text{ or } \omega = \sqrt{\frac{A\rho g}{m}}$$

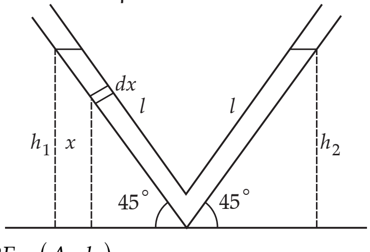
$$\therefore T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

**Q. 4. One end of a V-tube containing mercury is connected to a suction pump and the other end to atmosphere. The two arms of the tube are inclined to horizontal at an angle of 45° each. A small pressure difference is created between two columns when the suction pump is removed. Will the column of mercury in V-tube execute simple harmonic motion? Neglect capillary and viscous forces. Find the time period of oscillation.**

**[NCERT Exemp. Q. 14.38, Page 104]**

**Ans.** Let the liquid in the length $dx$ is at a height x.

Its mass = $A\rho dx$




$$PE = (A\rho dx)gx$$

The PE of the left column $= \int_0^{h_1} A\rho gx dx$

$$= A\rho g\left[\frac{x^2}{2}\right]_0^{h_1} = A\rho g\frac{h_1^2}{2} = A\rho g\frac{l^2\sin^2 45^\circ}{2}$$

$h_1 = h_2 = l\sin 45^\circ$ where $l$=length in one arm of the tube.

$$\text{Total } PE = A\rho g l^2 \sin^2 45^\circ = \frac{A\rho g l^2}{2}$$

If the change in liquid level in the tube in left side is $y$,
then length of liquid on left side = $l - y$
and right side = $l + y$

$$\text{Total } PE = A\rho g(l-y)^2\sin^2 45^\circ + A\rho g(l+y)^2\sin^2 45^\circ$$

$$\text{Change in } PE = (PE)_F - (PE)_i$$

$$= \frac{A\rho g}{2}\left[(l-y)^2 + (l+y)^2 - l^2\right]$$

$$= \frac{A\rho g}{2}\left[l^2 + y^2 - 2ly + l^2 + y^2 + 2ly - l^2\right] = \frac{A\rho g}{2}\left[y^2 + l^2\right]$$

$$\text{Change in } KE = \frac{1}{2}A\rho 2l\dot{y}^2$$

Change in total energy = 0

$$\Delta(PE) + \Delta(KE) = 0$$

$$\frac{A\rho g}{2}\left[2y^2 + l^2\right] + A\rho l\dot{y}^2 = 0$$

Differentiating both wrt time

$$A\rho g\left[0 + 2y\dot{y}\right] + 2A\rho l\dot{y}\ddot{y} = 0$$

$$2A\rho g y + 2A\rho l\ddot{y} = 0$$

$$l\ddot{y} + gy = 0$$

$$\ddot{y} + \frac{g}{l}y = 0$$

$$\text{or} \qquad \ddot{y} = -\frac{g}{l}y$$

$$\therefore \omega^2 = \frac{g}{l} \text{ or } \omega = \sqrt{\frac{g}{l}} \text{ or } T = 2\pi\sqrt{\frac{l}{g}}$$

**Q. 5. A tunnel is dug through the centre of the Earth. Show that a body of mass '*m*' when dropped from one end of the tunnel will execute simple harmonic motion.**

**[NCERT Exemp. Q. 14.39, Page 104]**

**Ans.** Acceleration due to gravity inside Earth

$$g' = \frac{GM}{R^3}x = \frac{g}{R}x$$

Here $x$ = distance of the point from centre of earth ($x < R$)

If, block of mass $m$ is placed along the diameter inside the earth.

So, force on block

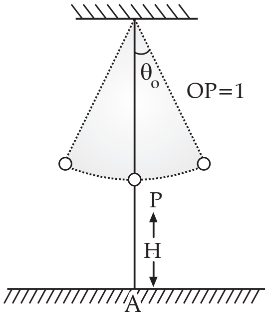
$$F = -\frac{mg}{R}x = -kx$$

$$\therefore k = \frac{mg}{R} \quad \text{(motion will be SHM)}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/R}}$$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

**Q. 6. A simple pendulum of time period 1 s and length *l* is hung from a fixed support at O, such that the bob is at a distance *H* vertically above A on the ground as shown in fig.**
**The amplitude is $\theta_o$. The string snaps at $\theta = \theta_o/2$. Find the time taken by the bob to hit the ground. Also find distance from A where bob hits the ground. Assume $\theta_o$ to be small so that $\sin\theta = \theta$ and $\cos\theta = 1$.**




**[NCERT Exemp. Q. 14.40, Page 104]**

**Ans.** Let $t = 0$ when $\theta = \theta_v$. Then

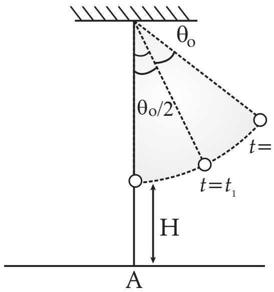
$\theta = \theta_v \cos wt$

Given, $T = 1$ s $\therefore \omega = 2\pi\ rads^{-1}$

At time $t_1$, let $\theta = \frac{\theta_o}{2}$

$$\therefore \cos 2\pi t_1 = \frac{1}{2} \text{ or } t_1 = \frac{1}{6}s$$

$$\therefore \dot{\theta} = -\theta_0 2\pi \sin 2\pi t \quad \left[\text{where } \dot{\theta} = \frac{d\theta}{dt}\right]$$




At $t_1 = 1/6$ s

$$\dot{\theta} = -\theta_0 2\pi \sin\frac{2\pi}{6}$$

$$= -\sqrt{3}\pi\theta_0$$

Thus, the linear velocity, $u = -\sqrt{3}\pi\theta_0 l$ perpendicular to the string

The vertical component is

$$u_y = -\sqrt{3}\pi\theta_0 l\sin\theta_0$$

horizontal component is

$$u_x = -\sqrt{3}\pi\theta_0 l\cos\theta_0$$

The time when it snaps, vertical height,

$$H' = H + l\left(1 - \cos\frac{\theta_0}{2}\right)$$

Let the time required for fall be t, then

$$H' = u_y t + \left(\frac{1}{2}\right)gt^2$$

{g is also in –ve direction}

$$\frac{1}{2}gt^2 + \sqrt{3}\pi\theta_0 l\sin\theta_0 t - H' = 0$$

$$\therefore t = \frac{-\sqrt{3}\pi\theta_0 l\sin\theta_0 \pm \sqrt{3\pi^2\theta_0^2 l^2\sin^2\theta_0 + 2gH'}}{g}$$

$$= \frac{-\sqrt{3}\pi l\theta_0^2 + \sqrt{3\pi^2\theta_0^4 l^2 + 2gH'}}{g}$$

$t \simeq \sqrt{\frac{2H'}{g}}$ {Neglecting terms of order $\theta_0^2$ and higher}

Now, $H' \simeq H + l(1-1) = H \therefore t \simeq \sqrt{\frac{2H'}{g}}$

Distance travelled in $x$-direction is $u_x t$ to the left where snapped.

$$X = \sqrt{3}\pi\theta_0 l\cos\theta_0\sqrt{\frac{2H}{g}}$$

$$X = \sqrt{3}\pi\theta_0 l\sqrt{\frac{2H}{g}} = \sqrt{\frac{6H}{g}}\theta_0 l\pi$$

At the time of snapping, the bob was at

$l\sin\left(\frac{\theta_0}{2}\right) \simeq l\left(\frac{\theta_0}{2}\right)$ distance from A

$\therefore$ the distance from A is

$$\frac{l\theta_0}{2} - \sqrt{\frac{6H}{g}}l\theta_0\pi = l\theta_0\left[1 - \sqrt{\frac{6H}{g}}\right]$$

**Q. 7. The motion of particle executing SHM is described by the displacement function –**

$$x(t) = A\cos(\omega t + \phi)$$

**If the initial (t=0) position of the particle is 1 cm and the initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM: $x = B\sin(wt + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions ?**

**[NCERT Ex. Q. 14.7, Page 364]**

**Ans.** Given:

$t = 0, x = 1$ cm, $v = \omega$ cm/s, $\omega = \pi$/s.

$x(t) = A\cos(\omega t + \phi)$

or $1 = A\cos(\pi \times 0 + \phi)$

$= A\cos\phi = 1$ (i)

Velocity, $v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$

or $\omega = -A\omega\sin(\pi \times 0 + \phi)$

$1 = -A\sin\phi$ or $A\sin\phi = -1$ (ii)

Squaring the eq$^n$.(s) (I) & (II), then adding –

$A^2(\cos^2\phi + \sin^2\phi) = 1 + 1 = 2$

$$A^2 = 2 \text{ or } A = \sqrt{2}\,cm$$

Now, Dividing eq$^n$. (II) by eq$^n$. (I).

$\tan\phi = -1$

$$\phi = -\frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

If we choose sine function instead of cosine function, then,

$x(t) = B\sin(wt + \alpha)$ (iii)

$x = 1cm$

or $1 = B\sin(0 + \alpha)$

$B\sin\alpha = 1$ (iv)

Velocity,

$v(t) = \frac{dx(t)}{dt} = B\omega\cos(\omega t + \alpha)$ [using eqn. (iii)]

At $t = 0$, $v(t) = \omega$ cm/s

$\omega = +B\omega\cos(0 + \alpha)$

$B\cos\alpha = +1$ (v)

Squaring (iv) & (v), then adding –

$B^2\sin^2\alpha + B^2\cos^2\alpha = 1 + 1$ or

$B^2(\sin^2\alpha + \cos^2\alpha) = 2$

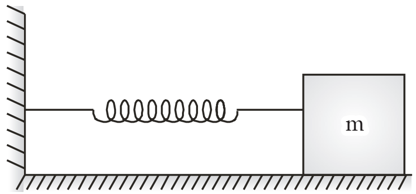
$\therefore B = \pm\sqrt{2}\,cm$

Hence, amplitude of motion $= \sqrt{2}\,cm$.

Now, Dividing eq$^n$. (IV) by (V),

$\frac{B\sin\alpha}{B\cos\alpha} = \frac{1}{1}$ or $\tan\alpha = 1$ or $\alpha = \frac{\pi}{4}$

**Q. 8. A spring having a spring constant 1200 $Nm^{-1}$ is mounted on a horizontal table as shown in figure. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.**




**[NCERT Exemp. Q. 14.9, Page 364]**

**Ans.** **(i)** Using, $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, we get

$$\nu = \frac{1}{2\pi}\sqrt{\frac{1200}{3}}$$
$$= 3.18\ s^{-1}$$

**(ii)** Maximum acceleration $= r\omega^2$
$= r(2\pi\nu)^2$
$= 4\pi^2\nu^2 r$
$= 4\pi^2 (3.18)^2 \times 0.02$
$= 7.98\ ms^{-2}$

**(iii)** Maximum speed $= r\omega$
$= r2\pi\nu$
$= 2\pi \times 3.18 \times 0.02$
$= 0.4\ ms^{-1}$.

**Q. 9. In the above question, let us take the position of mass when the spring is in natural length, *i.e,* unstretched as $x = 0$, and the direction from left to right as the positive direction of $x$-axis, Given $x$ as a function of time t for the oscillating mass if at the moment we start the stop watch ($t = 0$); the mass is :**
**(a) at the mean position,**
**(b) at the maximum stretched position, and**
**(c) at the maximum compressed position.**
**In what way do these functions for S.H.M. differ from each other, in frequency, in amplitude or the initial phase ?**

**[NCERT Ex. Q. 14.10, Page 364]**

**Ans. (a)** When $t = 0$, the mass is at mean position.

Using, $x = r\sin(\omega t + \phi)$,
$\omega = 2\pi f$
$= \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}}$
$= 20\ rad\ s^{-1}$.

We get $x = r\sin(20 \times t + 0)$
$= 2\sin 20\,t$

**(b)** at $t = 0$, mass is at extreme stretched position.

Then, $x = 2\sin\left(20 \times t + \frac{\pi}{2}\right)$
$= 2\cos 20t$

**(c)** at $t = 0$, mass is at extreme compressed position

Then $x = 2\sin\left(20 \times t + \frac{3\pi}{2}\right)$
$= -2\cos 20t$.

These functions differ in initial phase.

**Q. 10. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ($t = 0$) position of the particle, the radius of the circle and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anti-clockwise in every case : ($x$ is in cm and $t$ is in $s$).**

**(a)** $x = -2\sin\left(3t + \frac{\pi}{3}\right)$

**(b)** $x = \cos\left(\frac{\pi}{6} - t\right)$

**(c)** $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

**(d)** $x = 2\cos(\pi t)$

**[NCERT Ex. Q. 14.12, Page 364]**

**Ans. (a)**

$$x = -2\sin\left(3t + \frac{\pi}{3}\right) = 2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right) = 2\cos\left(3t + \frac{5\pi}{6}\right)$$

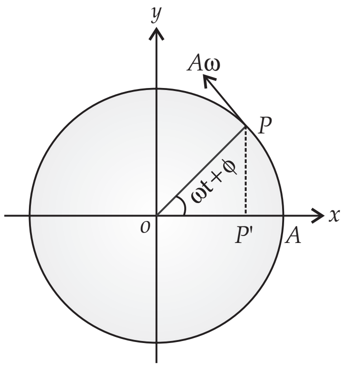
Comparing with standard SHM equation,
$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$$

$\therefore$ Amplitude, $A = 2$ cm

Phase angle, $\phi = \frac{5\pi}{6} = 150^\circ$

Angular velocity, $w = \frac{2\pi}{T} = 3$ rad/s.

Corresponding reference circle is –




**(b)** $x = \cos\left(\frac{\pi}{6} - t\right)$

$x = \cos\left(t - \frac{\pi}{6}\right)$ $[\because \cos(-\theta) = \cos\theta]$

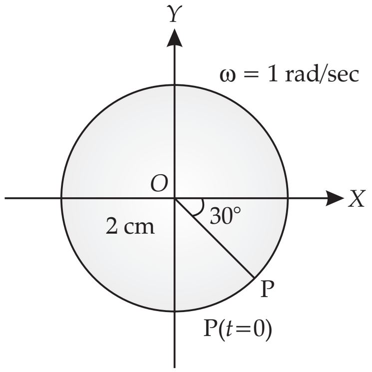
Comparing with standard SHM equation,
$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right),$$

$\therefore$ Amplitude, $A = 1$

Phase angle, $\phi = \frac{-\pi}{6} = -30^\circ$

Angular velocity, $\omega = \frac{2\pi}{T} = 1$ rad/s.

Corresponding reference circle is –



**(c)** $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

$x = -3\cos\left(2\pi t + \frac{\pi}{4} + \frac{\pi}{2}\right)$

or $x = -3\cos\left(2\pi t + \frac{3\pi}{4}\right)$

Comparing with standard SHM equation,

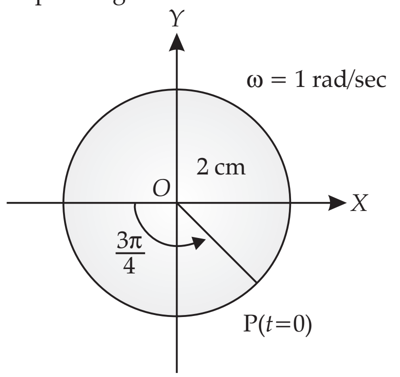
$x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$

Amplitude, A = 3 cm

Phase angle, $\phi = \frac{3\pi}{4} = 135^\circ$

Angular velocity, $\omega = \frac{2\pi}{T} = 2\pi$ rad/s

Corresponding reference circle is




**(d)** $x = 2\cos \pi t$

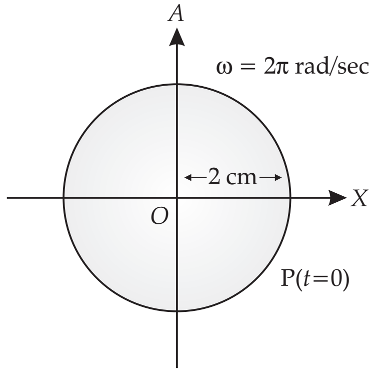
Comparing with standard SHM equation

∴ Amplitude, $A$ = 2 cm

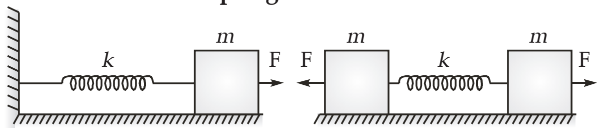
Phase angle, $\phi = 0$

Angular velocity, $\omega = \pi$ rad/s.

Corresponding reference circle is –




**Q. 11. Figure (a) shows a spring of force constant $k$ clamped rigidly at one end and a mass $m$ attached to its free end. A force applied at the free end stretches the spring.**




(a) (b)

**Figure (b) shows the same spring with both ends free and attached to a mass $m$ at either end. Each end of the spring in figure (b) is stretched by the same force F.**

**(i) What is the maximum extension of the spring in the two cases ?**

**(ii) If the mass in figure (a) and the two masses in figure (b) are released free, what is the period of oscillations in each case ?**

**[NCERT Ex. Q. 14.13, Page 365]**

**Ans. (i) (a)** Using $F = ky$,

The maximum extension,

$$y = \frac{F}{k}$$

**(b)** Here the springs have no rigid support, so, each mass behaves as a support for the spring against force applied on the other end.

∴ Maximum extension,

$$y = \frac{F}{k}$$

**(ii) (a)** Here time period,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

**(b)** In figure (b), we have a two body system of spring constant $k$ and reduced mass,

$$v = \frac{m \times m}{m + m} = \frac{m}{2}$$

Inertia factor = m/2

Spring factor = $k$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{2k}}$$

**Q. 12. The acceleration due to gravity on the surface of moon is 1.7 ms$^{-2}$. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s?**

**(g on the surface of earth is 9.8 ms$^{-2}$)**

**[NCERT Ex. Q. 14.15, Page 365]**

**Ans.** Acceleration due to gravity on the surface of moon, g = 1.7 ms$^{-2}$

Acceleration due to gravity on the surface of earth, g = 9.8 ms$^{-2}$

Time period of a simple pendulum on earth T = 3.5 s

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where,

$l$ is the length of the pendulum

$$\therefore l = \frac{T^2}{(2\pi)} \times g$$

$$= \frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8\,m$$

The length of pendulum remains constant

On moon's surface, time period, $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$T' = 2\pi\sqrt{\frac{(3.5)^2 \times 9.8}{4 \times (3.14)^2 \times 1.7}} = 8.4\ s$$

Hence, the period of the simple pendulum on the surface of moon is 8.4s.

**Q. 13. Answer the following questions :**

**(a) Time period of a particle in S.H.M. depends on the force constant k and mass *m* of the particle :**

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

**A simple pendulum executes S.H.M. approximately. Why then is the time period of a pendulum independent of the mass of the pendulum ?**

**(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{\frac{l}{g}}$. Think of a qualitative argument to appreciate this result.**

**(c) A man with a wrist watch on his hand falls from the top of a tower. Does the watch give correct time during the free fall.**

**(d) What is the frequency of oscillation of simple pendulum in a cabin that is freely falling under gravity? [NCERT Ex. Q. 14.16, Page 365]**

**Ans. (a)** In case of a simple pendulum,

$$F = -\frac{mg}{l}y = -ky$$

$$\therefore \quad k = \frac{mg}{l},$$

where $k$ = spring factor or force constant which is proportional to the mass $m$ of the particle. Also from the formula time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

or

$$T = 2\pi\sqrt{\frac{m}{mg/l}} = 2\pi\sqrt{\frac{l}{g}}$$

Hence $m$ cancels out in denominator as well as in numerator. That is why the time period of simple pendulum is independent of the mass of the bob.

**(b)** The effective restoring force that brings the bob back to its mean position in case of simple pendulum is $F = -mg\sin\theta$.

If $\theta$ is small, $\sin\theta \approx \theta$ and the expression for time period of the simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Now for larger values of $\theta$, we will take $F = -mg\sin\theta$ which is definitely $< mg\theta$ as $\sin\theta < \theta$ for larger values of $\theta$. So for larger values of $\theta$, effective value of $g$ decreases.

Therefore T increases beyond $T = 2\pi\sqrt{\frac{l}{g}}$

**(c)** Yes. The time shown by the wrist watch of a man falling from the top of a tower is not affected by the fall since a wrist watch does not work on the principle of a simple pendulum, it is not affected by the acceleration due to gravity during free fall, its working depends on spring action.

**(d)** From the formula the frequency of oscillation of simple pendulum is given by

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}},$$

*i.e.,* $\quad \nu \propto \sqrt{g}$

As $g = 0$, *i.e.*, gravity disappears for a freely falling body, so frequency is zero.

**Q. 14. A simple pendulum of length *l* and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed *v*. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period ?**

**[NCERT Ex. Q. 14.17, Page 365]**

**Ans.** As the pendulum is oscillating in the radial direction, it is acted upon by an additional acceleration equal to the centripetal acceleration of the car on the circular track of radius R, *i.e.*, $v^2/R$ where $v$ = uniform speed of the car. So, there are two accelerations acting on the pendulum in mutually perpendicular directions. *i.e.*,

**(i)** acceleration due to gravity ($g$)

**(ii)** $a_c$ = centripetal acceleration ($v^2/R$)

When $a$ be the resultant acceleration of the pendulum, then

$$a = \sqrt{g^2 + a_c^2}$$

$$a = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$$

$$a = \sqrt{g^2 + \frac{v^4}{R^2}}$$

When T be the time period of oscillation of the pendulum, then

$$T = 2\pi\sqrt{\frac{l}{a}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$$

$$\text{or } T = 2\pi\sqrt{\frac{l}{\left(g^2 + \frac{v^4}{R^2}\right)^{\frac{1}{2}}}}$$

**Q. 15. A cylindrical piece of cork of density of base area A and height floats in a liquid of density $\rho_l$ .The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$**

**Where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).**

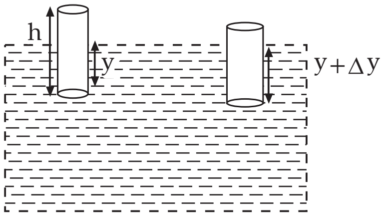
**[NCERT Ex. Q. 14.10, Page 364]**

**Ans.** Let height of cylinder be y inside the liquid (Initially in equilibrium).

Then, weight of the cylinder = upthrust due to liquid displaced

$$\therefore \; Ah\rho g = Ay\rho_l g$$

When cork of cylinder is slightly depressed by $\Delta y$ and released, a restoring force is equal to additional upthrust, acts on it.




Restoring force,

$F$ = – (volume × Density of liquid) × $g$

$$= -(Ay\rho_l g)$$

Acceleration of cylinder, $a = \frac{F}{m} = \frac{-Ay\rho_l g}{m}$

As mass of cylindrical cork $m = \text{volume} \times \text{Density}$

$$= Ah\rho$$

$$\therefore \; a = \frac{-Ay\rho_l g}{Ah\rho} = \frac{-\rho_l g}{\rho h} y$$

But $\frac{-\rho_l g}{\rho h}$ =constant

$\therefore$ Acceleration $(a) \propto$ –displacement $(y)$

So, Motion of cylindrical cork is SHM

Comparing eq$^n$. (I) with a = – $\omega^2 x$,

$$\omega = \sqrt{\frac{\rho_l g}{\rho h}}$$

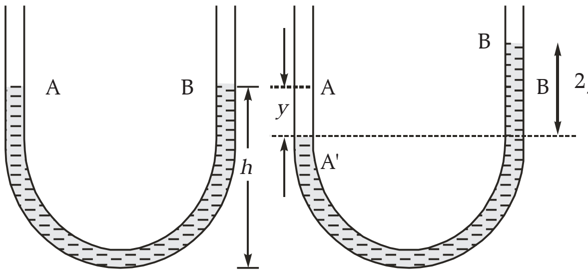
Time period, $T = \frac{2\pi}{\omega}$

$$T = 2\pi\sqrt{\frac{\rho h}{\rho_l g}}$$

**Q. 16. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the liquid column of mercury in the U-tube executes simple harmonic motion.**

**[NCERT Ex. Q. 14.19, Page 366]**

**Ans.** The suction pump creates the pressure difference. Hence mercury rises in one limb of the U-tube. When it is removed a net force acts on the liquid column due to the difference in levels of mercury in the two limbs and therefore the liquid column executes S.H.M. which can be explained as follows :




The mercury contained in a vertical U-tube upto the level A and B in its two limbs.

Suppose ρ = density of the mercury.

L = Total length of the mercury column in both the limbs.

A = internal cross-sectional area of U-tube.

$m$ = mass of mercury in U-tube = LAρ.

Suppose the mercury be depressed in left limb to A′ by a small distance $y$, then it rises by the same amount in the right limb to position B′

$\therefore$ Difference in levels in the two limbs

= A′B′ = 2$y$.

$\therefore$ Volume of mercury contained in the column of length 2$y$ = A × 2$y$

$\therefore$ $m$ = A × 2$y$ × ρ.

When $w$ = weight of liquid contained in the column of length 2y.

Then $w = mg$ = A × 2$y$ × ρ × $g$.

This weight produced the restoring force (F) which tends to bring back the mercury to its equilibrium position.

F= – 2 A$y$ρ$g$ = – (2Aρ$g$)$y$

When $a$ = acceleration produced in the liquid column, Then

$$a = \frac{F}{m} = -\frac{(2A\rho g)y}{LA\rho}$$

$$\text{or } a = -\frac{2\rho y g}{L\rho} = -\frac{2\rho g}{2h\rho} y \qquad ...(i)$$

$(\therefore$ L = 2$h)$

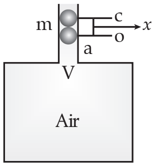
where $h$ = height of mercury in each limb. Now from (i). it is clear that $a \propto y$ and $-ve$ sign shows that it acts opposite to $y$, so the motion of mercury in U-tube is simple harmonic in nature having time period (T) given by

$$T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{2h\rho}{2\rho g}}$$

$$= 2\pi\sqrt{\frac{h\rho}{\rho g}}$$

$$T = 2\pi\sqrt{\frac{h}{g}}$$

**Q. 17. An air chamber of volume V has a neck area of cross-section a into which a ball of mass m just fits and can move up and down without any friction (fig. below). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal.**




**[NCERT Ex. Q. 14.20, Page 366]**

**Ans.** Before the ball is pressed,

Pressure inside chamber = pressure outside chamber = atmospheric pressure

Assume ball is depressed by x units. ,

As a result –

Volume decrease (in air), $\Delta V = ax$

[a=cross-sectional area by neck].

Volumetric strain $= \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$

$= \frac{ax}{V}$

Volumetric Stress=Bulk Stress=B

But $B = \frac{-P}{\left(\frac{ax}{V}\right)}$ [negative sign indicates decrease in volume].

Therefore, Increase in pressure, $P = -\frac{Bax}{V}$

Restoring force on ball, $F = P$a

$$= -\frac{Bax}{V}.a$$

$$= -\frac{Ba^2x}{V}$$

$$F \propto -x(\text{displacement}) \qquad \left[\because \frac{Ba^2}{V} = \text{constant}\right]$$

Hence, this motion is Simple Harmonic Motion (SHM).

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{mV}{Ba^2}}$$

**Q. 18. You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.**

**[NCERT Ex. Q. 14.21, Page 366]**

**Ans.** **(a)** Mass of the automobile m = 3000kg

Displacement in suspension System x = 15 cm = 0.15 m

There are 4 spring in parallel to the support of the mass of the automobile.

The equation for the restoring force for the system $f = -4\,kx = mg$

where, $k$ is the spring constant of the suspension system

Time period, $T = 2x\sqrt{\frac{m}{4k}}$

And $k = mg/4x = \frac{3000 \times 10}{4 \times 0.15}$

$= 5\times10^4$ Nm

Spring Constant, $k = 5\times10^4$ Nm

(b) Each wheel supports a mass, $m$ = 3000/4 = 750kg

For damping factor b, the equation for displacement is written as

$X = x_0e^{-bt/2m}$

The amplitude of oscillation decreases by 50%

$\therefore$ $x = x_0/2$

$x_0/2 = x_0e^{-bt/2m}$

log2 = bt/2$m$

$\therefore\ b = \frac{2m\log e^2}{t}$

where,

Time period,

$$t = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\sqrt{\frac{3000}{4\times5\times10^4}} = 0.7695\text{s} = 0.77s$$

$$\therefore\ b = \frac{2\times750\times0.693}{0.77} = 1350\,\text{kg / s}$$

Therefore, the damping constant of the spring is 1350 kg/s

**Q. 19. A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire.**

**(Torsional spring constant $\alpha$ is defined by the relation J = – $\alpha\theta$, where *J* is the restoring couple and $\theta$ the angle of twist).**

**[NCERT Ex. Q. 14.23, Page 366]**

**Ans.** Given: mass $m = 10$ kg,

Radius, $R = 15$ cm $= 0.15$ m.

Time, $T = 1.5$ sec.

$$\text{moment of inertia of disc } = \frac{1}{2}mR^2$$

$$= \frac{1}{2} \times 10 \times (0.15)^2$$

$$T = 2\pi\sqrt{\frac{I}{\alpha}}$$

$$\text{or } \alpha = \frac{4\pi^2 I}{T^2}$$

$$\text{or } \alpha = 4 \times \left(\frac{22}{7}\right)^2 \times \frac{1}{2} \times \frac{10 \times (0.15)^2}{(1.5)^2}$$

$$\alpha = 1.97 \text{ Nm/radian}$$

**Q. 20. A body describes SHM with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm (b) 3 cm, (c) 0 cm.**

**[NCERT Ex. Q. 14.24, Page 366]**

**Ans.** Given: $a = 5$ cm $= 0.05$ m

$T = 0.2$ s

$$\omega = \frac{2\pi}{T} = 10\pi \text{ rad/s}$$

**(a)** $y = 5$ cm $= 0.05$ m

$$a = -\omega^2 y = -(10\pi)^2 \times 0.05 = -5\pi^2 m/s^2$$

$$v = 10\pi\sqrt{(0.05)^2 - (0.05)^2} = 0 \quad \left[\because v = \omega\sqrt{a^2 - u^2}\right]$$

**(b)** $y = 3$ cm $= 0.03$ m

$$a = -(10\pi)^2 \times 0.03 = -3\pi^2 m/s^2;$$

$$v = +10\pi\sqrt{(0.05)^2 - (0.03)^2} = +0.4\pi m/s$$

(c) $y = 0$

$$a = -\omega^2 y = 0$$

$$v = 10\pi\sqrt{(0.05)^2 - 0} = 0.5\pi m/s$$

**Q. 21. A mass attached to a spring is free to oscillate, with angular velocity $w$, in a horizontal plane without friction or damping. It is pulled to a distance $x_0$ and pushed towards the centre with a velocity $v_0$ at time $t = 0$. Determine the amplitude of the resulting oscillations in terms of the parameters $w$, $x_0$ and $v_0$.[Hint: start with the equation $x = a \cos(wt + \theta)$ and note that the initial velocity is negative.]**

**[NCERT Ex. Q. 14.25, Page 366]**

**Ans.** $x = a \cos(\omega t + \theta)$

$$v = \frac{dx}{dt} = -a\omega \sin(\omega t + \theta)$$

At $t = 0$, $x = x_0$; $\frac{dx}{dt} = -v_0$

$$\therefore x_0 = a\cos\theta \qquad \text{(A)}$$

$$-v_0 = -a\omega\sin\theta$$

$$a\sin\theta = \frac{v_0}{\omega} \qquad \text{(B)}$$

Squaring (A) & (B), then adding

$$a^2(\cos^2\theta + \sin^2\theta) = x_0^2 + \frac{v_0^2}{\omega^2}$$

$$a = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$

# TOPIC-2
## Energy in S.H.M., Forced and Damped Oscillations

## Quick Review

- **Energy of S.H.M :** A particle executing S.H.M. possesses two types of energy :
  **(a) Potential Energy :** This energy is on account of the displacement of the particle from its mean position.
  **(b) Kinetic Energy :** This energy is on account of the velocity of the particle.
- **(a) Free oscillations :** A system capable of oscillating is said to be executing free oscillations if it vibrates with its own natural frequency without the help of any external periodic force.
  The natural frequency depends upon inertia factors, and elastic properties & dimension of system.
  **(b) Forced Oscillations :** When a body oscillates with the help of an external periodic force with a frequency different from the natural frequency of the body, its oscillations are called forced oscillations.
  **(c) Resonant Oscillations :** When a body oscillates with its own natural frequency $\nu_0$ with the help of an external periodic force whose frequency $\nu$ is equal to the natural frequency of the body $\nu_0$, the oscillations of the body are called resonant oscillations.
- **Resonance :** It is a phenomenon of setting a body into vibrations with the help of another body vibrating with the same frequency.

or

The phenomenon of increase in amplitude when the frequency of the driving force is close to natural frequency of the oscillator is called resonance.

## Know the Terms

- **Undamped Oscillations :** When a simple harmonic oscillator oscillates with a constant amplitude which does not change with time, its oscillations are undamped S.H.M.
- **Damped Oscillations :** When a simple harmonic oscillator oscillates with a decreasing amplitude with time, its oscillations are called damped S.H.M.
- **Coupled Oscillations :** Those oscillations in which two or more oscillating bodies connected to each other oscillate together & affect each other oscillations.

## Know the Formulae

- **Potential Energy, S.H.M.** $U = \frac{1}{2}mw^2y^2 = \frac{1}{2}ky^2$
- **Kinetic Energy, S.H.M.** $K = \frac{1}{2}mw^2(a^2 - y^2)$ $= \frac{1}{2}k(a^2 - y^2)$
- **Total Energy, S.H.M.** $E = -mw^2a^2 = \frac{1}{2}ka^2$

**Unit :** $\nu$ = Hz or $s^{-1}$

U, K, E = J.

## Know the Links

- *www.vedantu.com*
- *www.learncbse.in*
- *www.toppr.com*

# MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. Four pendulums A, B, C and D are suspended from the same elastic support as shown in Fig. 14. 1. A and C are of the same length, while B is smaller than A and D is larger than A. If A is given a transverse displacement,**

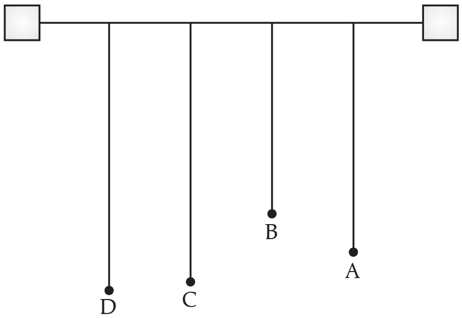



**Fig 14.1**

**(a) D will vibrate with maximum amplitude.**
**(b) C will vibrate with maximum amplitude.**
**(c) B will vibrate with maximum amplitude.**
**(d) All the four will oscillate with equal amplitude.**

**[NCERT Exemp. Q. 14.7, Page 98]**

**Ans. Correct option:** (b)

*Explanation:* As the length of A and C is same $\therefore$ they will have same time and frequency of vibration due to which resonance will occur and C will vibrate with maximum amplitude

**Q. 2. The displacement time graph of a particle executing S.H.M. is shown in Fig. 14.5. which of the following statement is/are true?**

**(a) The force is zero at $t = \frac{3T}{4}$.**

**(b) The acceleration is maximum at $t = \frac{4T}{4}$.**

**(c) The velocity is maximum at $t = \frac{T}{4}$.**

**(d) The P.E. is equal to K.E. of oscillation at $t = \frac{T}{2}$.**

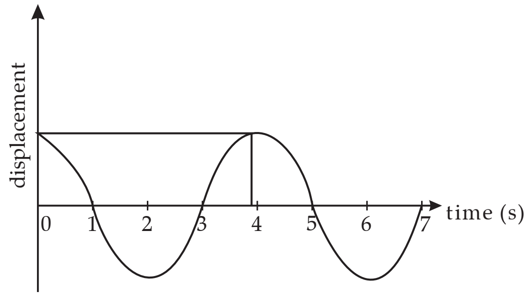



**Fig 14.5**

**[NCERT Exemp. Q. 14.16, Page 101]**

**Ans. Correct option:** (a), (b) and (c)

*Explanation:*

$$A_0 \cos wt = A_0 \cos \frac{2\pi}{T} t$$

(a) $F = mw^2 y, at\, t = \frac{3T}{4}, y = A_0 \cos\left(\frac{2\pi}{T} \times \frac{3T}{4}\right)$

$$= A_0 \cos\left(\frac{3\pi}{2}\right) = 0$$

$\therefore F = 0$

(b) $a = \omega^2 y$

$$y = A_0 \cos\left(\frac{2\pi}{T} \times T\right) = A_0 \cos 2\pi = +A_0$$

$\therefore a = \omega^2 A_0$

this is maximum value of a

(c) $v = w\sqrt{A_0^2 - y^2}$

$$y = A_0 \cos\left(\frac{2\pi}{T} \times \frac{T}{4}\right)$$

$$= A_0 \cos\left(\frac{\pi}{2}\right) \text{ or } y = 0$$

$\therefore v = \omega A_0$

which is maximum value of velocity

(d) $y = A_0 \cos\left(\frac{2\pi}{T} \times \frac{T}{2}\right) = -A_0$

displacement is maximum, it means PE is max and KE is zero

**Q. 3. A body is performing S.H.M. Then its**

**(a) average total energy per cycle is equal to its maximum kinetic energy.**

**(b) average kinetic energy per cycle is equal to half of its maximum kinetic energy.**

**(c) mean velocity over a complete cycle is equal to $\frac{2}{\pi}$ times of its maximum velocity.**

**(d) root mean square velocity is $\frac{1}{\sqrt{2}}$ times of its maximum velocity.**

**[NCERT Exemp. Q. 14.17, Page 101]**

**Ans. Correct option:** (a), (b) and (d)

*Explanation:*

Average KE per cycle

$$= \frac{0 + k_0}{2} = \frac{k_0}{2}$$

$$v_{mean} = \frac{\int_0^{2A} wA \cos\theta d\theta}{2\pi}$$

$$= \frac{wA}{2\pi}[\sin\theta]_0^{2\pi} = 0$$

$$\therefore v_{mean} \neq \frac{2}{\pi} v_{\max}$$

$$v_{rms} = \sqrt{\frac{v_{\min}^2 + v_{\max}^2}{2}}$$

$$= \sqrt{\frac{0 + v_{\max}^2}{2}}$$

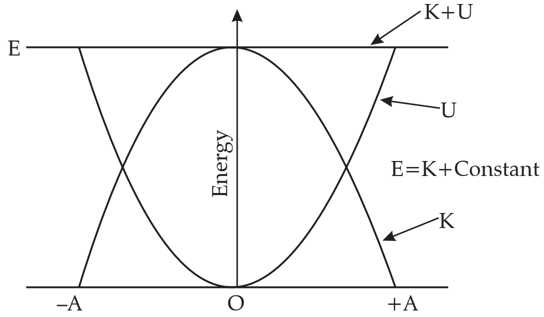
$$= \frac{1}{\sqrt{2}} v_{\max}$$

## Very Short Answer Type Questions **(1 mark each)**

**Q. 1. Draw a graph to show the variation of P.E., K.E. and total energy of a simple harmonic oscillator with displacement.**

**[NCERT Exemp. Q. 14.27, Page 103]**

**Ans.**




E = K + U = constant

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. Find the displacement of a simple harmonic oscillator at which its P.E. is half of the maximum energy of the oscillator.**
**[NCERT Exemp. Q. 14.31, Page 103]**

**Ans.** PE of oscillator, $U = \frac{1}{2}mw^2y^2$

{y=displacement}

Maximum-energy of oscillator, $E = \frac{1}{2}mw^2A^2$

$$U = \frac{1}{2}E$$

or $\frac{1}{2}mw^2y^2 = \frac{1}{4}mw^2A^2$

or $y^2 = \frac{A^2}{2}$

or $y = \pm\frac{A}{\sqrt{2}}$

**Q. 2. A body of mass $m$ is situated in a potential field $U(x) = U_\circ(1 - \cos \alpha x)$ when $U_\circ$ and $\alpha$ are constants. Find the time period of small oscillations.**
**[NCERT Exemp. Q. 14.32, Page 103]**

**Ans.** $U(x) = U_0(1 - \cos \alpha x)$

Differentiating both sides with respect to $x$

$$\frac{dU(x)}{dx} = U_0[0 + \alpha \sin \alpha x] = U_0\alpha \sin \alpha x$$

$$\therefore F = -\frac{dU(x)}{dx} = -U_0\alpha \sin \alpha x$$

When oscillations are small, $\sin \theta = \theta$

or $\sin ax = ax$

$$\therefore F = -U_0\alpha(\alpha x) = -U_0\alpha^2 x$$

$$\therefore F = -(U_0\alpha^2)x \quad \text{... (i)}$$

We know that $F = -kx$ ...(ii)

$K = \mu_0\alpha^2$ {from (i) & (ii)}

$$\therefore T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{U_0\alpha^2}}$$

## Long Answer Type Questions (5 marks each)

**Q. 1. Show that for a particle in linear SHM, the average kinetic energy over a period of oscillation equals the average potential energy over the same period.**
**[NCERT Ex. Q. 14.22, Page 366]**

**Ans.** In SHM, $K.E. = \frac{1}{2}mv^2$

Average, $KE = \frac{1}{T}\int_0^T \frac{1}{2}mv^2dt$

For SHM, Displacement, $x = A \sin wt$

$$v = \frac{dx}{dt} = w\,A \cos wt$$

Average $K.E. = \frac{1}{T}\int_0^T \frac{1}{2}mw^2A^2\cos^2(wt)dt$

$$= \frac{1}{2T}mw^2A^2\int_0^T\left(\frac{1+\cos 2wt}{2}\right)dt$$

$$= \frac{1}{4T}mw^2A^2\left[t + \frac{\sin 2wt}{2w}\right]_0^T$$

$$= \frac{1}{4T}mw^2A^2T$$

Average $KE = \frac{1}{4}mw^2A^2$

In SHM, $P.E. = \frac{1}{2}Kx^2$

Average $P.E. = \frac{1}{2T}\int_0^T Kx^2dt$

$$= \frac{1}{2T}\int_0^T mw^2A^2\sin^2 wt\,dt$$

$[\therefore K = mw^2]$

$$= \frac{1}{2T}\int_0^T mw^2A^2\left(\frac{1-\cos 2wt}{2}\right)dt \quad \left[As \cos 2wt = 1 - 2\sin^2 wt\right]$$

$$= \frac{1}{2T}mw^2A^2\int_0^T\left(\frac{1-\cos 2wt}{2}\right)dt$$

$$= \frac{1}{2T}mw^2A^2\left[\frac{1-\cos 2wt}{2}\right]_0^T$$

$$= \frac{1}{4}mw^2A^2$$

Hence,

Average P.E. = Average K.E.

## Some Commonly Made Errors

- Students skips the topic of oscillations due to spring at simple pendulum.
- Students got confused in differentiating between periodic and oscillatory motion.

## EXPERT ADVICE

- *The motion of simple pendulum is simple harmonic for small angular displacement.*
- *Good understanding of periodic motion and its characteristics are required for all the topics involved in this unit.*

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CHAPTER

# 15

# WAVES

## Chapter Objective

**This chapter will help you understand :**

- *Wave Motion : Transverse and longitudinal waves speed of wave motion.*
- *Principle of superposition of waves, reflection of waves, reflection of waves, standing waves in strings and organ pipes. Fundamental mode and harmonics. Beat, Doppler's effect.*

## TOPIC-1
## Waves & Wave Motion



### Extra Info

- **Wave motion** is a kind of disturbance which travels through a medium on account of repeated periodic vibrations of the particles of the medium about their mean position.
- **The medium** for wave propagation should have three properties :
  - **(a)** elasticity
  - **(b)** inertia
  - **(c)** minimum frictional resistance.
- **Kinds of waves :**

**(a) On the basis of necessity of material medium :**

- **(i) Mechanical waves** or elastic waves like sound waves, waves on the surface of water, waves on strings. All these waves require a material medium for propagation.
- **(ii) Electromagnetic waves** like light waves, radio waves which require no medium for propagation.

**(b) On the basis of vibrations of particles :**

- **(i) Longitudinal waves :** in which particles vibrate in the direction of propagation of waves.
- **(ii) Transverse waves :** in which particles vibrate in a direction perpendicular to the direction of propagation of waves.

**(c) On the basis of energy propagation :**

- **(i) Progressive waves—** in which energy is propagated.
- **(ii) Stationary waves—**in which energy is confined in a particular region.

- **Waves :**
  - **(a) Longitudinal waves** travel through a medium in the form of compressions and rarefactions involving changes in pressure and volume and can travel in all modes and cannot be polarised. The medium required must possess elasticity of volume. Sound waves in air are longitudinal.
  - **(b) Transverse waves** travel through a medium in the form of crests and troughs involving changes in shape can travel in solid and liquid and can be polarised. The medium required must possess elasticity of shape. Vibrations in strings are transverse.
- **Laplace correction :** According to laplace, the changes in pressure & volume of a gas, when sound waves propagate through it are not isothermal but it is adiabatic. Because of :

**(a)** Velocity of sound in gas is quite large.

**(b)** A gas is bad conductor of heat.

$\therefore$ Velocity of sound, $v = \sqrt{\dfrac{B_a}{\rho}}$

$B_a$ = Bulk modulus = $\gamma P$, $\rho$ = density of gas

## Know the Terms

- A crest is a portion of the medium which is raised temporarily above the normal position of rest of the particles of the medium, when a transverse wave passes through it.
- A trough is a portion of the medium which is depressed temporarily below the normal position of rest of the particles of the medium, when a transverse wave passes through it.
- A compression is a region of the medium in which particles are compressed *i.e.*, particles come closer or distance between particles become less than the normal distance between them. Thus, there is a temporary decrease in volume and a consequent increase in density of the medium in the region of compression.
- A rarefaction is a region of the medium in which *i.e.*, particles get farther apart than what they normally are. Thus there is a temporary increase in volume and a consequent decrease in density of the medium in the region of rarefaction.
- **Some parameters related to wave motion**

**(a)** **Displacement** of a particle is the distance covered by the particle from the mean position.

**(b)** **Amplitude** is the maximum displacement of the particle from the equilibrium position.

**(c)** **One wavelength** is the distance travelled by the wave, during the time the particle completes one vibration about its mean position. We may also define, one wavelength = smallest distance between two particles vibrating in the same phase = distance between the centres of two consecutive crests/troughs/compressions/rarefactions.

**(d)** **Angular wave number or propagation constant : It is $2\pi$ times the no. of waves that can be accomodated per unit length. It is denoted by K.**

**(e)** **Frequency : It is the no. of complete wavelengths traversed by the wave in one second.**

**(f)** **Time period : It is equal to time taken by wave to travel a distance equal to wavelength.**

**(g)** **Particle velocity** = velocity of particle executing SHM = $\dfrac{dx}{dt}$. Its value changes with time.

**(h)** **Wave velocity** is the velocity with which disturbance travels in the medium.

$$v = n\lambda = \frac{\lambda}{T} = \text{constant for a wave motion.}$$

**(i)** **Group velocity** ($v_g$) is the velocity with which the group of waves travel.

$$v_g = \frac{d\omega}{dK}$$

- **Some parameters related to sound waves :**

**(i)** **Audible range : This frequency range lies between 20 Hz to 20,000 Hz. It is sensible to human ear.**

**(ii)** **Ultrasonic range : Any vibration whose frequency is greater than 20,000 Hz.**

**(iii)** **Infrasonic range : Sound waves which have frequencies less than the audible range are called infrasonic waves.**

- **Phase or phase angle** is the physical quantity which tells us by what amount the two waves or the two particles lag or lead in terms of angle or time or distance.
- **Energy density** is defined as the energy associated with unit volume of the medium, *i.e.*,

  Energy density ($u$) = Energy/Volume.

  It is measured in $\dfrac{\text{Joule}}{\text{m}^3}$.

## Know the Formulae

- **A longitudinal wave** can be represented by

$$x = a\sin(\omega t \pm kx)$$

**A transverse wave** can be represented by

$$y = a\sin(\omega t \pm kz)\ ;\ y = a\sin(\omega t \pm kx)$$
$$z = a\sin(\omega t \pm kx)\ ;\ z = a\sin(\omega t \pm ky)$$
$$x = a\sin(\omega t \pm ky)\ ;\ x = a\sin(\omega t \pm kz)$$

- **Harmonic wave function :** $y(x, t) = r\sin\left[\frac{2\pi}{\lambda}(vt - x) + \phi_o\right]$

**or** $y(x, t) = r\cos\left[\frac{2\pi}{\lambda}(vt - x) + \phi_o\right]$

**where** $\phi_0$ = **Initial phase.**

- **Relation between phase difference, path difference and time difference**

A phase difference of $2\pi$ radian is equivalent to a path difference of $\lambda$ and a time difference of time period T, *i.e.*, $2\pi = \lambda = T$

So, Phase difference, $\phi = \frac{2\pi}{\lambda} \times$ Path difference

Also, $\frac{x}{\lambda} = \frac{t}{T} = \frac{\phi}{2\pi}$

where T is time period and it is time for a path $x$ or phase $\phi$.

- **Relation between particle velocity & wave velocity.**

$$u(x, t) = -v\frac{d}{dx}[y(x, t)]$$

Where $u$ = particle velocity, $v$ = wave velocity.

- **Particle Acceleration :** $a(x, t) = -(2\pi\nu)^2 y = -\omega^2 y$.
- **Plane progressive wave :**

**(a) Standard Equations :** $y = r\sin\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$

or $y = r\cos\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$

where $y$ = displacement, $r$ = amplitude, T= time period, $\lambda$ = wavelength, $x$ = starting distance of wave from origin.

**(b) Angular frequency :** $\omega = \frac{2\pi}{T}$

**(c) Propagation constant :** $x = \frac{2\pi}{\lambda}$

**(d) Velocity of wave :** $v = \nu\lambda = \frac{\lambda}{T}$

**(e) Velocity of particle :** $u = \frac{dy}{dt}, u_{max} = r\omega$

**(f) Acceleration of particle :** $a = \frac{d^2y}{dt^2}, a_{max} = -\omega^2 r$

**(g) Phase/path difference two waves :**

**(a) For two waves :** $y_1 = a\sin\omega t, y_2 = b\cos\omega t$

**Phase Difference :** $(\omega t + \frac{\pi}{2} - \omega t) = \frac{\pi}{2}$

**(b) For two waves :** $y_1 = 10^{-6}\sin\left(100t + \frac{x}{50} + 0.5\right)$

$$y_2 = 10^{-6}\cos\left(100t + \frac{x}{50}\right)$$

**Path Difference** = 0.5 radian.

- **Newton's corrected formula for velocity of sound :**

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

## Know the Links

- *www.toppr.com*
- *www.vedantu.com*
- *www.learncbse.in*

## MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. Water waves produced by a motor boat sailing in water are**

**(a) neither longitudinal nor transverse.**
**(b) both longitudinal and transverse.**
**(c) only longitudinal.**
**(d) only transverse.**

**[NCERT Exemp. Q. 15.1, Page 105]**

**Ans. Correct option:** (b)

*Explanation:* The waves, produce transverse as well as lateral vibrations in the particles of the medium. The water molecules at the surface move up and down and back and forth simultaneously describing nearly circular paths. As the wave passes, water molecules at the crests move in the direction of the wave will those at the through move in the opposite direction.

**Q. 2. Sound waves of wavelength λ travelling in a medium with a speed of $v$ m/s enter into another medium where its speed is 2 $v$ m/s. Wavelength of sound waves in the second medium is**

**(a)** λ
**(b)** $\frac{\lambda}{2}$
**(c)** 2λ
**(d)** 4λ

**[NCERT Exemp. Q. 15.2, Page 105]**

**Ans. Correct option:** (c)

*Explanation:* Wavelength in I medium,

$$\lambda = \frac{v}{\upsilon} \quad \text{(i)}$$

Wavelength in II medium,

$$\lambda' = \frac{2v}{\upsilon} \quad \text{(ii)}$$

$\therefore \nu = \upsilon'$ $\{\because$ frequency remains unchanged$\}$

$(ii) \div (i)$, we get

$$\frac{\lambda'}{\lambda} = 2$$

*or* $\lambda' = 2\lambda$

**Q. 3. Speed of sound wave in air**

**(a) is independent of temperature.**
**(b) increases with pressure.**
**(c) increases with increase in humidity.**
**(d) decreases with increase in humidity.**

**[NCERT Exemp. Q. 15.3, Page 106]**

**Ans. Correct option:** (c)

*Explanation:* Speed of sound $v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v \propto \frac{1}{\sqrt{\rho}}$

due to presence of humidity density of air decreases so, sound speed increases.

**Q. 4. Change in temperature of the medium changes**

**(a) frequency of sound waves.**
**(b) amplitude of sound waves.**
**(c) wavelength of sound waves.**
**(d) loudness of sound waves.**

**[NCERT Exemp. Q. 15.4, Page 106]**

**Ans. Correct option:** (c)

*Explanation:* Speed of sound in a medium

$$v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \sqrt{T}$$

If temperature *i.e.*, change then speed is change

As, $v = n\lambda \Rightarrow v \propto \lambda$ (here $n$ frequency constant)

hence wavelength changes.

**Q. 5. With propagation of longitudinal waves through a medium, the quantity transmitted is**

**(a) matter.**
**(b) energy.**
**(c) energy and matter.**
**(d) energy, matter and momentum.**

**[NCERT Exemp. Q. 15.5, Page 106]**

**Ans. Correct option:** (b)

*Explanation:* A wave is a disturbance which propagates energy and momentum from one place to the other without the transport of matter. In Propagation of longitudinal waves through a medium without matter being transmitted there is no movement of matter (mass) and momentum.

**Q. 6. Which of the following statements are true for wave motion?**

**(a) Mechanical transverse waves can propagate through all mediums.**
**(b) Longitudinal waves can propagate through solids only.**
**(c) Mechanical transverse waves can propagate through solids only.**
**(d) Longitudinal waves can propagate through vacuum.**

**[NCERT Exemp. Q. 15.6, Page 106]**

**Ans. Correct option:** (c)

*Explanation:* When mechanical transverse wave propagates through a medium element of the medium is subjected to shearing stress. Solids and strings have shear modulus. That is why they sustain shearing stress.

**Q. 7. A transverse harmonic wave on a string is described by $y(x, t)=3.0 \sin (36t+0.018x+\pi/4)$ where x and y are in cm and t is in s. The positive direction of x is from left to right.**

**(a) The wave is travelling from right to left.**

**(b) The speed of the wave is 20m/s.**

**(c) Frequency of the wave is 5.7 Hz**

**(d) The least distance between two successive crests in the wave is 2.5 cm.**

**[NCERT Exemp. Q. 15.11, Page 108]**

**Ans. Correct option:** (a), (b) and (c)

*Explanation:* Given that :

$$y = 3.0 \sin \left(36 t + 0.018 x + \frac{\pi}{4}\right) \quad ...(i)$$

and $y = a \sin (\omega t + kx + \phi)$ ...(ii)

**(a)** Since there is negative sign between ωt and kx, the wave travels from right to left.

**(b)** Compare equation (i) and (ii) we get

$\omega = 36$ and $k = 0.018$

$$\therefore\ v = \frac{\omega}{k} = \frac{36}{0.018} = 2000 \text{ cm/sec} = 20 \text{ m/sec}$$

**(c)** frequency $v = \frac{\omega}{2\pi} = \frac{36}{2\times 3.14} = 5.7$ Hz.

**Q. 8. Speed of sound waves in a fluid depends upon**

**(a) directly on density of the medium.**

**(b) square of Bulk modulus of the medium.**

**(c) inversely on the square root of density.**

**(d) directly on the square root of density.**

**[NCERT Exemp. Q. 15.13, Page 108]**

**Ans. Correct option:** (c) and (d)

*Explanation:* Speed of sound waves in a fluid

$$v = \sqrt{\frac{B}{\rho}}$$

here B = Bulk modulus of the medium

$\rho$ = density of the medium

**Q. 9. During propagation of a plane progressive mechanical wave**

**(a) all the particles are vibrating in the same phase.**

**(b) amplitude of all the particles is equal.**

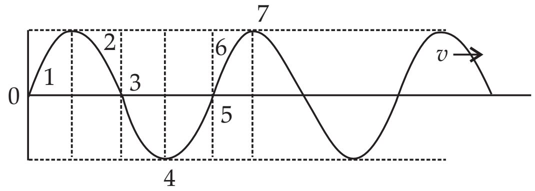
**(c) particles of the medium executes S.H.M.**

**(d) wave velocity depends upon the nature of the medium.**

**[NCERT Exemp. Q. 15.14, Page 108]**

**Ans. Correct option:** (b), (c) and (d)

*Explanation:*




**(b)** Energy particle in progressive wave has same amplitude.

**(c)** Every particle of the medium in S.H.M.

**(d)** Speed $v = \sqrt{\frac{B}{\rho}} \Rightarrow v \propto \frac{1}{\sqrt{\rho}}$

As $\rho$ depends upon nature of the medium.

## Very Short Answer Type Questions (1 mark each)

**Q. 1. The displacement of an elastic wave is given by the function**

$$y = 3 \sin \omega t + 4 \cos\omega t.$$

**Where $y$ is in cm and $t$ is in second. Calculate the resultant amplitude.**

**[NCERT Exemp. Q. 15.21, Page 109]**

**Ans.**

$y = 3\sin \omega t + 4\cos \omega t$ (i)

*Let*, $3 = a\cos\phi$ (ii)

$4 = a\sin\phi$ (iii)

$\therefore\ y = a\cos\phi \sin \omega t + a\sin\phi \cos \omega t$ (iv)

$y = a\sin(\omega t + \phi)$

from eq. (s) (i) &(iii)-

$$\tan\phi = \frac{3}{4} \text{ or } \phi = \tan^{-1}\frac{3}{4}$$

Squaring (ii) & (iii), then adding-

$$a^2\cos^2\phi + a^2\sin^2\phi = 3^2 + 4^2$$

$$a^2\left(\cos^2\phi + \sin^2\phi\right) = 9 + 16$$

$$a^2 = 25$$

or $a = 5$ or amplitude, $a = 5$cm

$$\therefore\ y_1 = 5\sin(\omega t + \phi), \quad \because \phi = \tan^{-1}\frac{4}{3}$$

Hence, new amplitude, $a$ = 5cm.

**Q. 2. Explain why (or how):**

**(a) in a sound wave, a displacement node is a pressure anti-node and vice versa.**

**(b) bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes",**

**(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes.**

**(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and**

**(e) the shape of a pulse gets distorted during propagation in a dispersive medium.**

**[NCERT Ex. Q. 15.19, Page 393]**

**Ans.** **(a)** Displacement node is a point of zero displacement, so pressure is maximum at this point where as on anti-nodes the reverse happens, hence it is a pressure anti-node.

**(b)** Bats produce and detect ultrasonic waves and emit sound of frequencies between 20-100 kHz for this purpose, hence they developed 'Echo-location' system.

**(c)** Because nature of waveforms and presence of overtones are different in the given two cases.

**(d)** Gases cannot sustain shearing stress as in case of transverse wave propagation, the medium must have property of sustaining shearing. So gases cannot support transverse wave propagation.

**(e)** This is the property of dispersive medium that waves of different wave length travel with different speeds in different directions or with different velocities. Thus, when sound pulse (a combination of waves of different wavelengths) travels through it, gets distorted.

**Q. 3. You have learnt, that a travelling wave in one dimension is represented by a function $y = f(x, t)$, where $x$ and $t$ must appear in the combination $(x - vt)$ or $(x + vt)$, *i.e.*, $y = f(x \pm vt)$. Is the converse true ? That is, does every function of $(x - vt)$ or $(x + vt)$ represent a travelling wave ? Examine, if the following function for $y$ can possibly represent a travelling wave ?**

**(i)** $(x - vt)^2$ **(ii)** $\log\left[\frac{x+vt}{x_0}\right]$

**(iii)** $e^{-(x-vt)^2}$ **(iv)** $\frac{1}{x+vt}$

**[NCERT Ex. Q. 15.5, Page 391]**

**Ans.** No, the converse is not true. The basic requirement for a wave function to represent a travelling wave is that for all values of $x$ and $t$, wave function must have a finite value.

Out of the given functions for y only (iii) satisfies this condition. The other three functions (i), (ii) and (iv) cannot represent a travelling wave as the necessary condition is not satisfied by these functions.

**Q. 4. Given below are some functions of $x$ and $t$ to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent**

**(i) a travelling wave,**

**(ii) a stationary wave, or**

**(iii) none of all ?**

**(a)** $y = 2\cos(3x)\sin(10t)$

**(b)** $y = 2\sqrt{x - vt}$

**(c)** $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

**(d)** $y = \cos x \sin t + \cos 2x \sin 2t$

**[NCERT Ex. Q. 15.13, Page 392]**

**Ans.** **(a)** It represents a stationary wave in harmonic functions of $x$ and $t$ are contained separately in the equation.

**(b)** It cannot represent any type of wave.

**(c)** It represents a progressive travelling harmonic wave.

**(d)** This equation is sum of two functions each representing a stationary wave. Therefore, it represents superposition of two stationary waves.

**Q. 5. At what temperatures (in °C) will the speed of sound in air be 3 times its value at 0° C?**

**[NCERT Exemp. Q. 15.23, Page 110]**

**Ans.**

$$v \propto \sqrt{T}$$

or $\frac{v_T}{v_0} = \sqrt{\frac{T}{T_0}}$

From question, $v_T = 3v_0$

$\therefore \frac{3v_0}{v_0} = \sqrt{\frac{T}{273+0}}$

or $\sqrt{T} = 3\sqrt{273}$

$T = 9 \times 273 = 2457\,K$

$T = 2457 - 273$

$T = 2184\ °C$

## Short Answer Type Questions (2 or 3 marks each)

**Q. 1. A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 $ms^{-1}$ ? ($g$ = 9.8 $ms^{-2}$)**

**[NCERT Ex. Q. 15.2, Page 391]**

**Ans.** Let total time $t = t_1 + t_2$, where $t_1$ is the time taken from top to surface of water and $t_2$ is the time taken by sound to reach the top.

**Calculation of $t_1$ :**

Using $s = ut_1 + \frac{1}{2}at_1^2$

Putting, $u = 0, a = g = 9.8\ ms^{-1}$

and $s = 300$ m, we get

$$300 = \frac{1}{2} \times 9.8 \times t_1^2$$

or $t_1 = 7.8255$ s

**Calculation of $t_2$ :**

Using $v = \frac{s}{t_2}$,

$$t_2 = \frac{s}{v} = \frac{300}{340} = 0.88 \text{ s}$$

$$t = t_1 + t_2$$

$$= 7.82 + 0.88 = 8.70 \text{ s}$$

**Q. 2. A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound ? Speed of sound in air is 340 ms$^{-1}$ and in water 1,486 ms$^{-1}$.**

**[NCERT Ex. Q. 15.6, Page 391]**

**Ans.** **(a) The reflected sound :**

Using $v = \frac{\lambda}{T} = \lambda\nu$

$$\lambda = \frac{v}{\nu} = \frac{340}{10^6}$$

$$= 3.4 \times 10^{-4} \text{ m}$$

**(b) The transmitted sound :**

Here, $\lambda' = \frac{1486}{10^6}$

$$= 1.486 \times 10^{-3} \text{ m}$$

**Q. 3. A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wave length of sound in the tissue in which the speed of sound is 1.7 kms$^{-1}$ ? The operating frequency of the scanner is 4.2 MHz.** **[NCERT Ex. Q. 15.7, Page 391]**

**Ans.** Using $\lambda = \frac{v}{\nu}$,

or $\lambda = \frac{1.7 \times 10^3}{4.2 \times 10^6}$

$$= 0.405 \times 10^{-3}\, m.$$

## Long Answer Type Questions (5 marks each)

**Q. 1. The earth has a radius of 6400 km. The inner core of 1000 km radius is solid. Outside it, there is a region from 1000 km to a radius of 3500 km which is in molten state. Then again from 3500 km to 6400 km the earth is solid. Only longitudinal (P) waves can travel inside a liquid. Assume that the P wave has a speed of 8 km s$^{-1}$ in solid parts and of 5 km s$^{-1}$ in liquid parts of the earth. An earthquake occurs at some place close to the surface of the earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if wave travels along diameter?**

**[NCERT Exemp. Q. 15.32, Page 111]**

**Ans.** In liquid speed of wave = 5 km/s

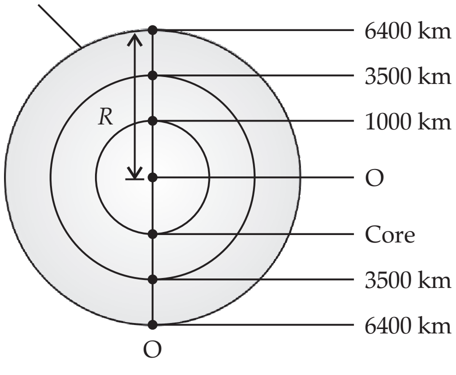
$$\text{Time required} = \frac{\text{Distance}}{\text{speed}}$$

$$\text{Time} = \left[\frac{1000-0}{8} + \frac{3500-1000}{5} + \frac{6400-3500}{8}\right] \times 2$$

$$\text{Time} = \left[\frac{1000}{8} + \frac{2500}{5} + \frac{2900}{8}\right] \times 2$$

Time required = 1975 sec

Time required = 32 min 55 sec.




**Q. 2. If $c$ is r.m.s speed of molecules in a gas and $v$ is the speed of sound waves in the gas, show that $c/v$ is constant and independent of temperature for all diatomic gases.**

**[NCERT Exemp. Q. 15.33, Page 112]**

**Ans.** RMS speed of molecules of a gas

$$c = \sqrt{\frac{3P}{\rho}}$$

$$c = \sqrt{\frac{3RT}{M}} \quad [M = \text{Molar mass}] \quad ...(i)$$

$\because$ $PV = nRT$

$n = 1$

or $P = \frac{RT}{V}$

$$\therefore \frac{P}{\rho} = \frac{RT}{M} \quad \left[\because \frac{P}{\rho} = \frac{\frac{RT}{V}}{\frac{M}{V}} = \frac{RT}{M}\right]$$

$n = 1$

or $P = \frac{RT}{V}$

$$\therefore \frac{P}{\rho} = \frac{RT}{M} \quad \left[\because \frac{P}{\delta} = \frac{\frac{RT}{V}}{\frac{M}{V}} = \frac{RT}{M}\right]$$

speed of sound wave in gas, $v = \sqrt{\frac{\gamma P}{\rho}}$

$$v = \sqrt{\frac{\gamma RT}{M}} \quad (II)$$

Dividing eq. n (II) by eq. n (I),

$$\frac{c}{v}=\frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{\gamma RT}{M}}}$$

$$\frac{c}{v}=\sqrt{\frac{3}{\gamma}} \quad [\gamma = \text{adiabatic constant for diatomic gas}]$$

$$\gamma=\frac{7}{5}$$

Thus, $\frac{c}{v}=$ constant

**Q. 3. Given below are some functions of $x$ and $t$ to represent the displacement of an elastic wave.**

**(a) $y = 5\cos(4x)\sin(20t)$**

**(b) $y = 4\sin(5x - t/2) + 3\cos(5x - t/2)$**

**(c) $y = 10\cos[(252 - 250)\pi t]\cos[(252 + 250)\pi t]$**

**(d) $y = 100\cos(100\pi t + 0.5x)$**

**State which of these represent**

**(a) a travelling wave along $-x$ direction**

**(b) a stationary wave**

**(c) beats**

**(d) a travelling wave along 4 + $x$ direction.**

**Given reasons for your answers.**

**[NCERT Exemp. Q. 15.34, Page 112]**

**Ans.** **(a)** Equation $y = 100\cos(100\pi t + 0.5x)$ represents a travelling wave along x-direction.

**(b)** Equation $y = 5\cos(4x)\sin(20t)$ represents stationary wave, as this is combination of progressive waves ( contains sin, cos, terms)

**(c)** Equation

$y = 100\cos[(252 - 250)\pi t]\cos[(252 + 250)\pi t]$

represents beats formation because this equation involve sum & difference of two nearby frequencies i.e., 252, 250.

**(d)** Equation

$$y=4\sin\left(5x-\frac{t}{2}\right)+3\cos\left(5x-\frac{t}{2}\right)$$

represents a travelling wave along +x direction because it involves negative sign with x.

**Q. 4. In the given progressive wave**

**$y = 5\sin(100\pi t - 0.4\pi x)$**

**where $y$ and $x$ are in m, $t$ is in s. What is the**

**(a) amplitude**

**(b) wavelength**

**(c) frequency**

**(d) wave velocity**

**(e) particle velocity amplitude.**

**[NCERT Exemp. Q. 15.36, Page 112]**

**Ans.** Standard progressive wave-

$$y=a\sin(\omega t-kx+\phi)$$

$$y=5\sin(100\pi t-0.4\pi x+0)$$

**(a)** Amplitude, $a = 5$ m

**(b)** Wavelength, $\lambda$

$$k=\frac{2\pi}{\lambda} \text{ or } \lambda=\frac{2\pi}{k}$$

$$k=0.4\pi$$

$$\lambda=\frac{2\pi}{0.4\pi}=5 \text{ m}$$

**(c)** Frequency $\nu$,

$$\omega=2\pi\nu$$

$$\text{or } \nu=\frac{\omega}{2\pi}$$

$$\because \ \omega=100\pi$$

$$\nu=\frac{100\pi}{2\pi}=50 \text{ Hz}$$

**(d)** Wave velocity, $v = \nu\lambda = 50 \times 5$
$= 250$ m/s

**(e)** $y=5\sin(100\pi t-0.4\pi x)$

$$\frac{dy}{dt}=5\times100\pi\cos(100\pi t-0.4\pi x)$$

For maximum velocity at mean position-

$$\cos(100\pi t-0.4\pi x)=1$$

$$\text{or } 100\pi t-0.4\pi x=0$$

$$\therefore \left(\frac{dy}{dt}\right)_{\text{maximum}}=5\times100\pi\times1$$

$$v_{\max}=500\pi \text{ m/s.}$$

**Q. 5. Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air :**

**(a) is independent of pressure.**

**(b) increases with temperature.**

**(c) increases with humidity.**

**[NCERT Ex. Q. 15.4, Page 391]**

**Ans.** **(a)** The formula for velocity of sound in air is given by.

$$v=\sqrt{\frac{\gamma P}{\rho}} \quad ..(1)$$

where $\gamma$ = constant for a given gas, *i.e.*, air

$\rho$ = density of the gas

From gas equation,

$$PV = RT$$

$$\text{or} \quad P=\frac{RT}{V} \quad ...(2)$$

$\therefore$ From equation (1) and (2),

$$v=\sqrt{\frac{\gamma RT}{\rho V}}=\sqrt{\frac{\gamma RT}{M}} \quad ...(3)$$

where $\rho V = M$ = molecular weight of air or gas.

For a given gas, M = constant

R is also constant.

When T = constant, then from equation (3), we conclude that $v$ independent of the pressure of air (gas) if temperature remains constant.

**(b) Effect of temperature :**

From eqn. (3) $v = \sqrt{\frac{\gamma RT}{M}}$

Since $\gamma$, R and M are constants, so

$$v \propto \sqrt{T}$$

It means velocity of sound in a gas is directly proportional to the square root of its temperature, hence we conclude that the velocity of sound in air increases with increase in temperature.

**(c) Effect of humidity :** The presence of water vapours in air changes the density of air, thus the velocity of sound changes with humidity of air.

Let $\rho_m$ = density of moist air

$\rho_d$ = density of dry air

$v_m$ = velocity of sound in moist air

$v_d$ = velocity of sound in dry air

From eqn. (1) $v = \sqrt{\frac{\gamma P}{\rho}}$,

or, $$v_m = \sqrt{\frac{\gamma P}{\rho_m}} \quad ..(i)$$

and $$v_d = \sqrt{\frac{\gamma \rho}{\rho_d}} \quad ..(ii)$$

Dividing equation (i) by equation (ii),

$$\frac{v_m}{v_d} = \sqrt{\frac{\rho_d}{\rho_m}}$$

Also we know that density of water vapours is less than the density of dry air. It means dry air is heavier than water vapours as the molecular mass of water is less than that of $N_2$ (28) and $O_2$ (32), so

$$\rho_m < \rho_d$$

or $$\frac{\rho_d}{\rho_m} > 1 \quad ..(iv)$$

$\therefore$ from equations (iii) and (iv),

$$\frac{v_m}{v_d} > 1$$

or $$v_m > v_d$$

It means velocity of sound in air increases with humidity, *i.e.*, velocity of sound in moist air is greater than the velocity of sound in dry air. That is why sound travels faster on rainy day than on a dry day.

# TOPIC-2
## Superposition Principle and Doppler Effect

## Quick Review

➢ **Principle of Superposition of Waves :**

**(a)** According to this principle, overlapping waves add algebraically to produce a resultant wave or a net wave.

**or**

When any number of waves meet simultaneously at a point in a medium, the net displacement at a given time is the algebraic sum of displacements due to each wave at that time.

*i.e.*, $$y = y_1 + y_2 + \ldots\ldots + y_n$$

Applications of Superposition Principle

**(i)** Stationary waves

**(ii)** Beats

**(iii)** Interference of waves.

➢ **Laws of Vibrations of Stretched Strings.**

Fundamental Frequency of vibration of stretched string

$$v = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{lD}\sqrt{\frac{T}{\pi\rho}}$$

**(a) Law of Length :** $$v \propto \frac{1}{L}$$

Fundamental frequency is inversely proportional to length.

**(b) Law of Tension :** Fundamental frequency is directly proportional to square root of tension. *i.e.*

$$v \propto \sqrt{T}$$

**(c) Law of Mass :** Fundamental frequency is inversely proportional to the square root of mass :

$$\nu \propto \frac{1}{\sqrt{m}}$$

**(d) Law of Diameter :** $\nu \propto \frac{1}{D}$

**(e) Law of Density :** $\nu \propto \frac{1}{\sqrt{\rho}}$

- **Doppler Effect :** According to Doppler effect, wherever there is a relation between a source of sound & listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by source.
  Doppler effect is the motion related to change in frequency of sound.

## Know the Terms

- **Stationary Waves :** If two waves of same type having same amplitude, same frequency and same wavelength, travelling with same speed in opposite directions along a straight line superimpose each other, they give rise to a new kind of waves known as stationary wave.
- **Longitudinal stationary waves** are formed as a result of super-imposition of two identical longitudinal waves travelling in opposite directions.
- **Transverse stationary waves** are formed as a result of super-imposition of two identical transverse waves travelling in opposite direction.
- **Anti-nodes** are certain points in the medium in a standing wave, the amplitude of vibration of which is maximum.
  Distance between two anti-nodes is $\lambda/2$.
- **Nodes** are certain points in the medium in a standing wave which are permanently at rest.
  Distance between two consecutive nodes is $\lambda/2$.
- **Beats** is the phenomenon of regular variation in the intensity of sound when two sources of nearly equal frequencies are sounded together.
  One minima of sound followed by one maxima is said to constitute one beat.
  The essential conditions for production of beats are :

**(i)** The amplitudes of two waves should be nearly equal.
**(ii)** The difference in frequencies of two sources must be less than 10 per second.

- **Beat period** is the time interval between two successive beats.
- **Beat frequency** is the number of beats produced per second.

## Know the Formulae

- **Equation of stationary wave :** $y = 2r \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda}$.
- **Normal modes of vibration of strings :**

**(a) Fundamental frequency :** $\nu_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{m}}$

$\nu$ = frequency, L = resonance length, T = Tension.

**(b) I overtone** $\nu_2 = 2\nu_1$
**(c) II overtone** $\nu_3 = 3\nu_1$ & so on.

- **Organ Pipes :**

**(a) Closed organ pipe :**

**(i)** Fundamental note $\nu_1 = \frac{v}{4L}$

**(ii)** I$^{st}$ overtone or III$^{rd}$ harmonic $\nu_2 = 3\nu_1$
**(iii)** II$^{nd}$ overtone or V$^{th}$ harmonic $\nu_3 = 5\nu_1$ & so on

**(b) Open organ pipe :**

(i) Fundamental note $\nu_1 = \frac{v}{2L}$

(ii) $I^{st}$ overtone or $II^{nd}$ harmonic $\nu_2 = 2\nu_1$

(iii) $II^{nd}$ overtone or $III^{rd}$ harmonic $\nu_3 = 3\nu_1$ & so on

➢ **Beats :** Beat frequency $m = n_1 - n_2$ or $n_2 - n_1$

$n_2 = n_1 \pm m.$

➢ **Doppler's Effect**

$$\nu' = \frac{\{(v + v_m) - v_L\}\nu}{(v + v_m) - v_s}$$

$\nu'$ = Apparent frequency of sound heard

$\nu$ = Actual frequency of sound

$v_m$ = Velocity of medium (air)

$v_s$ = Velocity of source

$v_L$ = Velocity of listener.

## Know the Links

☞ *www.vedantu.com*

☞ *www.learncbse.in*

☞ *www.toppr.com*

# MCQ/Fillups/True or False

**(1 mark each)**

### (A) Multiple Choice Questions

**Q. 1. A sound wave is passing through air column in the form of compression and rarefactions, In consecutive compressions and rarefactions,**

**(a) density remains constant.**

**(b) Boyle's law is obeyed.**

**(c) bulk modulus of air oscillates.**

**(d) there is no transfer of heat.**

**[NCERT Exemp. Q. 15.7, Page 106]**

**Ans. Correct option:** (d)

***Explanation:*** If sound wave is passing through air column. The air particle compressed and barefaced, so the time of compression and rarefaction is too small, i.e., we can assume adiabatic process and hence no transfer of heat.

**Q. 2. Equation of a plane progressive wave is given by $y = 0.6\sin 2\pi\left(t - \frac{x}{2}\right)$. On reflection from a denser medium its amplitude becomes 2/3 of the amplitude of the incident wave. The equation of the reflected wave is**

**(a)** $y = 0.6\sin 2\pi\left(t + \frac{x}{2}\right)$

**(b)** $y = -0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

**(c)** $y = 0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

**(d)** $y = -0.4\sin 2\pi\left(t - \frac{x}{2}\right)$.

**[NCERT Exemp. Q. 15.8, Page 106]**

**Ans. Correct option:** (b)

***Explanation:***

Amplitude of reflected wave

$$= \frac{2}{3} \times 0.6$$

$$= 0.4$$

There will be phase change of $\pi$ because of reflection from denser to rarer medium

$$y = 0.4\sin 2\pi\left(t + \frac{x}{2} + \pi\right) = -0.4\sin 2\pi\left(t + \frac{x}{2}\right)$$

**Q. 3. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in**

**(a) one second**

**(b) 0.5 second**

**(c) 2 seconds**

**(d) data given is insufficient.**

**[NCERT Exemp. Q. 15.9, Page 107]**

**Ans. Correct option:** (b)

*Explanation:*

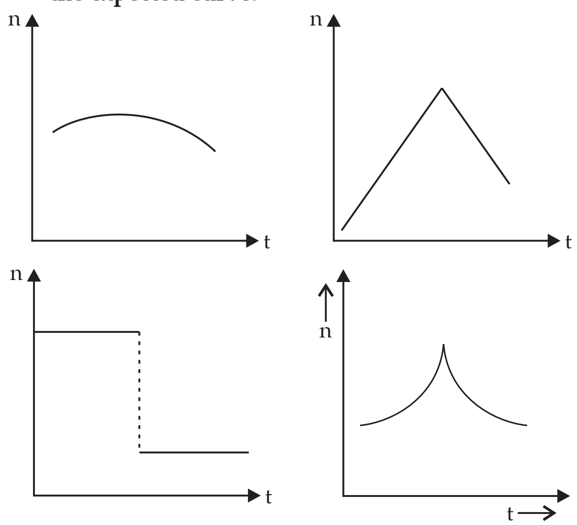
$$\mu = \frac{M}{L} = \frac{2.5}{20\text{ m}} kg = 0.125 \text{kg/m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200\text{ N}}{0.125\text{kg/m}}}$$

$$= 40 \text{m/s}$$

$$t = \frac{L}{v} = \frac{20\text{ m}}{40\text{ m/s}} = 0.5\text{ s}$$

**Q. 4. A train whistling at constant frequency is moving towards a station at a constant speed $v$. The train goes past a stationary observer on the station. The frequency $n'$ of the sound as heard by the observer is plotted as a function of time $t$(Fig 15.1). Identify the expected curve.**




**[NCERT Exemp. Q. 15.10, Page 107]**

**Ans. Correct option:** (c)

*Explanation:* When train is approaching towards the observance

Apparent frequency $n_a = n_o\left(\frac{v}{v - v_s}\right)$ here $n_a > n_o$

now when the train is going away from the observance

Apparent frequency $n_a = n_o\left(\frac{v}{v + v_s}\right)$ $(v_s = -v_e)$

so it is clear that $n_a > n_o$

hence, the expected curve is (c).

**Q. 5. The displacement of a string is given by**

$$y(x, t) = 0.06\, sin(2\pi x / 3)\cos(120\pi t)$$

**Where $x$ and $y$ are in m and $t$ in s. The length of the string is 1.5m and its mass is 3.0 × $10^{-2}$kg.**

**(a) It represents a progressive wave of frequency 60Hz.**

**(b) It represents a stationary wave of frequency 60Hz.**

**(c) It is the result of superposition of two waves of wavelength 3 m, frequency 60Hz each travelling with a speed of 180 m/s in opposite direction.**

**(d) Amplitude of this wave is constant.**

**[NCERT Exemp. Q. 15.12, Page 108]**

**Ans. Correct option:** (b) and (c)

*Explanation:* Given that

$$y = 0.06 \sin \frac{2\pi x}{3} \cos 120\,\pi t \qquad ...(i)$$

and $y = 2a \sin kx \cos \omega t$ ...(ii)

Compare equation (i) and (ii)

$k = \frac{2\pi}{3}$ and $\omega = 120\,\pi$

**(b)** $\omega = 2\pi n = 120\pi \Rightarrow n = 60$ Hz

**(c)** $k = \frac{2\pi}{3} = \frac{2\pi}{\lambda} \Rightarrow \lambda = 3$ m

now $v = n\lambda = 60 \times 30 = 180$ m/sec

**Q. 6. The transverse displacement of a string (clamped at its both ends) is given by $y(x,t) = 0.06 \sin(2\pi x / 3)\cos(120\pi t)$.**

**All the points on the string between two consecutive nodes vibrate with**

**(a) same frequency**

**(b) same phase**

**(c) same energy**

**(d) different amplitude.**

**[NCERT Exemp. Q. 15.15, Page 108]**

**Ans. Correct option:** (a), (b) and (d)

*Explanation:* Given that $y = 0.06 \sin \frac{2\pi x}{3} \cos 120\,\pi t$

This represents stationary wave. All the point on the string between two consecutive nodes vibrate with same frequency. But are having different amplitude of $0.06 \sin \frac{2\pi x}{3}$ and because of different amplitudes they are having different energies.

**Q. 7. A train, standing in a station yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with a speed of 10 m/s. Given that the speed of sound in still air is 340 m/s,**

**(a) the frequency of sound as heard by an observer standing on the platform is 400Hz.**

**(b) the speed of sound for the observer standing on the platform is 350 m/s.**

**(c) the frequency of sound as heard by the observer standing on the platform will increase.**

**(d) the frequency of sound as heard by the observer standing on the platform will decrease.**

**[NCERT Exemp. Q. 15.16, Page 109]**

**Ans. Correct option:** (a) and (b)

*Explanation:* **(a)** As there is no relative motion between the source and observer, hence frequency observed will be the same as natural frequency so = 400 Hz.

**(b)** When the wind is blowing in the same direction as that of sound wave, then not speed of the wave is sum of speed of sound wave and speed of the wind. The speed of sound v = 340 + 10 = 350 m/sec.

**Q. 8. Which of the following statements are true for a stationary wave?**

**(a) Every particle has a fixed amplitude which**

**is different from the amplitude of its nearest particle.**

**(b) All the particles cross their mean position at the same time.**

**(c) All the particles are oscillating with same amplitude.**

**(d) There is no net transfer of energy across any plane.**

**(e) There are some particles which are always at rest.**

**[NCERT Exemp. Q. 15.17, Page 109]**

**Ans. Correct option:** (a), (b), (d) and (e)

*Explanation:* **(a)** In stationary wave any particle at a given Position have amplitude 2a sin kt

**(b)** The time period of oscillation of all the particles is same, hence all the particles cross their mean position as the same time.

**(d) and (e)** Nodes are the paints which is always at rest hence no transfer of energy across the nodes. It means the energy is a stationary wave is confined between two nodes.

## Very Short Answer Type Questions (1 mark each)

**Q. 1. A sonometer wire is vibrating in resonance with a tuning fork. Keeping the tension applied same, the length of the wire is doubled. Under what conditions would the tuning fork still be in resonance with the wire?**

**[NCERT Exemp. Q. 15.18, Page 109]**

**Ans.** When a wire of length L vibrates its resonant frequency in nth mode after stretching it by tension T, then frequency, $\nu = \frac{n}{2L}\sqrt{\frac{T}{m}}$

[Here, m= mass per unit length of stretched wire]

Let us consider two given cases:

$$\nu_1 = \frac{n_1}{2L_1}\sqrt{\frac{T_1}{m_1}} \ \& \ \nu_2 = \frac{n_2}{2L_2} = \sqrt{\frac{T_2}{m_2}}$$

from question,

$T_1 = T_2 = T,\ m_1 = m_2 = m$ (as wires are same)

$L_2 = 2L_1$

$$\frac{\nu_1}{\nu_2} = \left(\frac{n_1}{2L_1}\sqrt{\frac{T}{m}}\right) \div \left(\frac{n_2}{2(2L_1)}\sqrt{\frac{T}{m}}\right) = \frac{2n_1}{n_2}$$

As turning fork is same, $\therefore\ \nu_1 = \nu_2$

$$\frac{2n_1}{n_2} = 1 \quad \text{or } n_2 = 2n_1$$

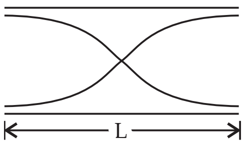
Hence, when length of wire double the number of harmonics double for same resonant frequency.

**Q. 2. An organ pipe of length $L$ open at both ends is found to vibrate in its first harmonic when sounded with a tuning fork of 480 Hz. What should be the length of a pipe closed at one end, so that it also vibrates in its first harmonic with the same tuning fork?**

**[NCERT Exemp. Q. 15.19, Page 109]**

**Ans.** Let us consider the following diagram organ is open at both ends,

for first harmonic-




$$L = \frac{\lambda}{2}$$

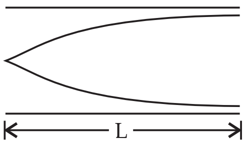
or $\lambda = 2L$

$$\left[\begin{array}{l} As, \lambda = \frac{c}{v} \\ c = \text{speed of sound air} \end{array}\right],$$

$$\frac{c}{v} = 2L$$

$$\text{or } v = \frac{c}{2L}$$

For pipe closed at one end-




$$v_1 = \frac{c}{4L_1}$$

But $\nu = \nu_1$ [for resonance same tuning fork]

$$\frac{c}{2L} = \frac{c}{4L_1} \qquad [\therefore \text{speed remains constant}]$$

$$\frac{L_1}{L} = \frac{2}{4} = \frac{1}{2}$$

$$\text{or } L_1 = \frac{L}{2}$$

**Q. 3. A tuning fork A, marked 512 Hz, produces 5 beats per second, when sounded with another unmarked tuning fork B. If B is loaded with wax the number of beats is again 5 per second. What is the frequency of the tuning fork B when not loaded?**

**[NCERT Exemp. Q. 15.20, Page 109]**

**Ans.** Frequency of tuning fork A,

$$\nu_A = 512 \text{ Hz}$$

Now, frequency of tuning fork B may be-

$$\nu_B = \nu_A \pm 5$$
$$= 512 \pm 5$$
$$= 517 \text{ or } 507 \text{ Hz}$$

As on loading B, frequency decreases

If $\nu_B$= 517 Hz, if might reduced to 507 Hz, given again a beat of 5 Hz.

If $\nu_B$= 507 Hz, reduction will always increase the frequency of beat, so, $\nu_B$= 517 Hz.

**Q. 4. A sitar wire is replaced by another wire of same length and material but of three times the earlier radius. If the tension in the wire remains the same, by what factor will the frequency change?**

**[NCERT Exemp. Q. 15.22, Page 109]**

**Ans.** Wire is stretched, so frequency of stretched wire is,

$$\nu = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

Mass per unit length,

$$m = \frac{\text{Mass}}{\text{Length}} = \frac{\pi r^2 l \rho}{l}$$

$$\text{or } m = \pi r^2 \rho$$

$$\nu = \frac{n}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}$$

$$\text{or } \nu \propto \sqrt{\frac{1}{r^2}} \propto n$$

$$\nu \propto \frac{1}{r} \propto n$$

So, frequency of sitar reduced by $\frac{1}{3}$rd of it's previous value.

**Q. 5.** When two waves of almost equal frequencies $\boldsymbol{n_1}$ and $\boldsymbol{n_2}$ reach at a point simultaneously, what is the time interval between successive maxima?

**[NCERT Exemp. Q. 15.24, Page 110]**

**Ans.** Assume, $n_1 > n_2$

Frequency of Beat, $\nu_b = n_1 - n_2$

$$\therefore \text{ Time Period, } T_b = \frac{1}{\nu_b}$$

$$= \frac{1}{n_1 - n_2} s$$

## Short Answer Type Questions

**(2 or 3 marks each)**

**Q. 1. A steel wire has a length of 12 m and a mass of 2.10 kg. What will be the speed of a transverse wave on this wire when a tension of 2.06 × 10$^4$ N is applied?**

**[NCERT Exemp. Q. 15.25, Page 110]**

**Ans.** Given: $l = 12$ m, mass = 2.10 kg

Tension, $T = 2.06 \times 10^4$ N,

$$m = \frac{M}{l} = \frac{2.10}{12}$$

$$\therefore\ v = \sqrt{\frac{T}{m}} = \sqrt{\frac{2.06 \times 10^4 \times 12}{2.10}}$$

$$= \sqrt{11.77 \times 10^4}$$

$$v = 3.43 \times 10^2 \text{ m/s}$$

$$v = 343 \text{ m/s}$$

**Q. 2. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a source of 1237.5 Hz? (sound velocity in air = 330 m s$^{-1}$)**

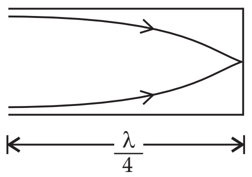
**[NCERT Exemp. Q. 15.26, Page 110]**

**Ans.** Given: $l = 20$ cm = 0.2 m

$v_{given}$= 1237.5 Hz

$v$ = 330 m/s

$$l = \frac{\lambda}{4} \text{ or } \lambda = 4l$$




$$\nu_{\text{fundamental}} = \frac{v}{4l}$$

$$\nu_{\text{fundamental}} = \frac{330}{4 \times 0.2} = 412.5\,\text{Hz}$$

$$\nu_{\text{given}} = 1237.5 \text{ Hz}$$

$$\frac{\nu_{\text{given}}}{\nu_{\text{fundamental}}} = \frac{1237.5}{412.5} = \frac{3}{1}$$

So, 3$^{rd}$ harmonic node of pipe is excited by 1237.5 Hz frequency.

**Q. 3. A train standing at the outer signal of a railway station blows a whistle of frequency 400 Hz still air. The train begins to move with a speed of 10 ms$^{-1}$ towards the platform. What is the frequency of the sound for an observer standing on the platform? (sound velocity in air = 330 m s$^{-1}$)**

**[NCERT Exemp. Q. 15.27, Page 110]**

**Ans.** Given: $n_0$ = 400 Hz

velocity of train, $v_t$ = 10 m/s

Velocity of sound in air, $v_a$ = 330 m/s

Apparent frequency when source is moving

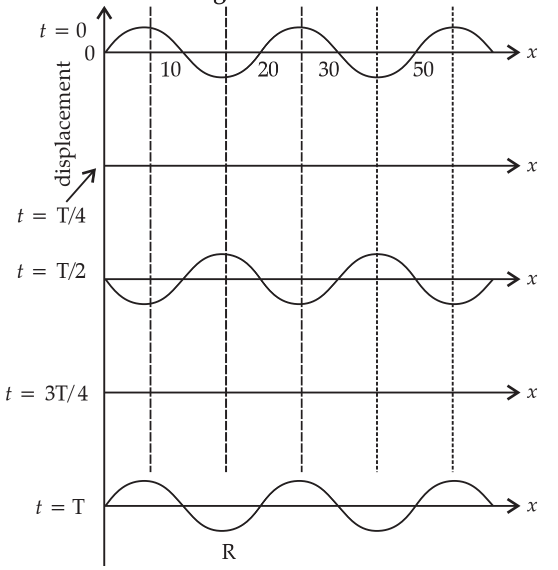
$$v_{app} = \frac{v_a}{(v_a - v_t)} v_0$$

$$= \frac{330 \times 400}{(330 - 10)}$$

$$v_{app} = \frac{825}{2} = 412.5\,\text{Hz}$$

$$v_{app} = 412.5\,\text{Hz}$$

**Q. 4. The wave pattern on a stretched string is shown in Fig. 14.2. Interpret what kind of wave this is and find its wavelength.**




**[NCERT Exemp. Q. 15.28, Page 110]**

**Ans.** Nature of two waves is decided by observing the displacement and position of different points. The graph shows stationary wave. Points on positions $x$ = 10, 20, 30, 40, there are nodes.

They are stationary waves :

Distance between successive nodes = $\frac{\lambda}{2}$

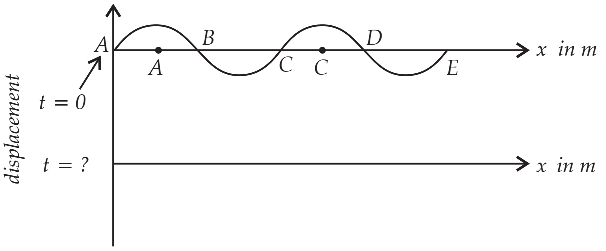
$\lambda$ = 2 × (node- to- node distance)

= 2 × (20 − 10)

= 2 × 10

$\lambda$ = 20 cm

**Q. 5. The pattern of standing waves formed on a stretched string at two instants of time are shown in Fig. The velocity of two waves superimposing to form stationary waves is 360 ms$^{-1}$ and their frequencies are 256 Hz.**




**(a) Calculate the time at which the second curve is plotted.**

**(b) Mark nodes and anti-nodes on the curve.**

**(c) Calculate the distance between $A'$ and $C'$.**

**[NCERT Exemp. Q. 15.29, Page 111]**

**Ans.** Given: frequency of wave, □ = 256 Hz

$$\text{Time period, } T = \frac{1}{\nu} = \frac{1}{256}$$

$$= 3.9 \times 10^{-3}\,\text{s}$$

**(a)** Time taken to pass through mean position

$$t = \frac{T}{4}$$

$$= \frac{3.9 \times 10^{-3}}{4}\,s$$

$$= 9.8 \times 10^{-4}\,s$$

**(b)** Nodes- A, B, C, D, E ( zero displacement)

Anti-nodes-$A'$, $C'$ (maximum displacement)

**(c)** At A′, C′, there are consecutive anti-nodes, so distance between A′ and C′,

$$\lambda = \frac{v}{\nu} = \frac{360}{256}$$

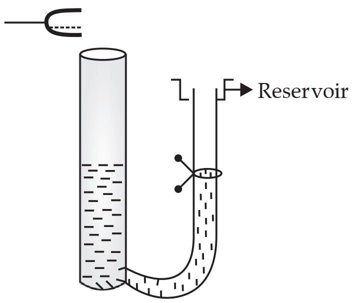
$$\lambda = 1.41\,\text{m}$$

**Q. 6. A tuning fork vibrating with a frequency of 512 Hz is kept close to the open end of a tube filled with water (Fig. 15.4). The water level in the tube is gradually lowered. When the water level is 17 cm below the open end, maximum intensity of sound is heard. If the room temperature is 20°C, calculate**

**(a) speed of sound in air at room temperature**

**(b) speed of sound in air at 0°C**

**(c) if the water in the tube is replaced with mercury, will there be any difference in your observations?**




**[NCERT Exemp. Q. 15.30, Page 111]**

**Ans.** Let us consider the following diagram

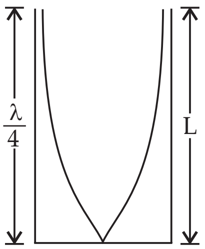



Fig 14.5

For maximum intensity-

(a) $L = \frac{\lambda}{4}$

or $\lambda = 4L$

$v = \nu\lambda = \nu \times 4L$

$= 512 \times 4 \times 17 \times 10^{-2}$

$= 348.16$ m/s

(b) As, $v \propto \sqrt{T}$ (T = Temperature)

$$\frac{v_{20}}{v_0} = \sqrt{\frac{273+20}{273+0}} = \sqrt{\frac{293}{273}}$$

$$\frac{v_{20}}{v_0} = 1.04$$

or $v_0 = \frac{v_{20}}{1.04} = \frac{348.16}{1.04} = 334.8$ m/s

**(c)** Water and mercury in tube reflects the sound into air column to form stationary wave and reflection is more in mercury than water as mercury is more denser than water. So, intensity of sound heard will be longer but reading does not change as medium in tube (air) and tuning fork are same.

**Q. 7. Show that when a string fixed at its two ends vibrates in 1 loop, 2 loops, 3 loops and 4 loops, the frequencies are in the ratio 1:2:3:4.**

**[NCERT Exemp. Q. 15.31, Page 111]**

**Ans.** Length for each loop $= \frac{\lambda}{2}$

Now,

$$L = \frac{n\lambda}{2}$$

$$\lambda = \frac{2L}{n} \qquad (1)$$

But $v = \upsilon\lambda$ or $\lambda = \frac{v}{\upsilon}$

Putting in eqn. (1)

$$\frac{v}{\upsilon} = \frac{2L}{n}$$

$$\nu = \frac{n}{2L}v$$

$$\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \qquad \left[\because v = \sqrt{\frac{T}{\mu}}\right]$$

for n = 1, $\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu_0$

for n = 2, $\nu_2 = \frac{1}{2L}\sqrt{\frac{T}{m}} = 2\nu_0$

Therefore, $\upsilon_1 : \upsilon_2 : \upsilon_3 : \upsilon_4 = n_1 : n_2 : n_3 : n_4$

$\upsilon_1 : \upsilon_2 : \upsilon_3 : \upsilon_4 = 1:2:3:4$

**Q. 8. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end ?**

**[NCERT Ex. Q. 15.1, Page 391]**

**Ans.** Using $v = \sqrt{\frac{T}{m}}$

Putting, $m = \frac{2.5}{20} = 0.125\ kgm^{-1}$, T = 200 N,

$$v = \sqrt{\frac{200}{0.125}}$$

$= 40\ ms^{-1}$

Time, $t = \frac{l}{v} = \frac{20}{40} = 0.5$ s

**Q. 9. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C = 343 $ms^{-1}$**

**[NCERT Ex. Q. 15.3, Page 391]**

**Ans.** Using $v = \sqrt{\frac{T}{m}}$

we have

$T = v^2 m$

where, $m$ is mass per unit length

Now $m = \frac{M}{l} = \frac{2{\cdot}1}{12}$

$= 0.175\ kg\ m^{-1}$

and $v = 343\ ms^{-1}$

we get $\mathbf{T} = (343)^2 \times 0.175 = 20588.6$

$= 2.06 \times 10^4$ N

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. A transverse harmonic wave on a string is described by $y(x, t) = 3.0 \sin (36t + 0.018x + \pi/4)$ where $x$ and $y$ are in cm and $t$ in sec. The positive direction of $x$ is from left to right.**

**(a) Is this a travelling wave or a stationary wave ?**

**If it is travelling, what are the speed and direction of its propagation ?**

**(b) What are its amplitude and frequency ?**

**(c) What is the initial phase at the origin ?**

**(d) What is the least distance between two successive crests in the wave ? [NCERT Ex. Q. 15.8, Page 391]**

**Ans.** Here $y(x, t) = 3\sin (36t + 0.018x + \pi/4)$

$= 3\sin [0.018 (2000t + x) + \pi/4]$

Comparing with standard equation

$$y(x, t) = r \sin\left[\frac{2\pi}{\lambda}(vt + x) + \phi\right]$$

It is observed that the given equation represents a travelling waveform right to left.

**(a)** Here $v = 2000\ cms^{-1} = 20\ ms^{-1}$

**(b)** $r = 3$ cm

Also, $\frac{2\pi}{\lambda} = 0.018$

or $\lambda = \frac{2\pi}{0.018}$ cm

Using $v = \lambda\nu$,

$$\nu = \frac{v}{\lambda} = \frac{2000}{2\pi} \times 0.018$$

$$= 5.73\ s^{-1}$$

**(c)** Initial phase at origin, $\phi = \frac{\pi}{4}$ radian

**(d)** Distance between successive crests

$$= \lambda = \frac{2\pi}{0.018}$$

$$= 349 \text{ cm} = 3.49 \text{ m}$$

**Q. 2. For the wave described in Exercise 15.8, plot the displacement ($y$) versus ($t$) graphs for $x$ = 0.2 and 4 cm. What are the shapes of these graphs ? In which aspects does the oscillatory motion in travelling waves differ from one point to another : amplitude, frequency or phase?**

**[NCERT Ex. Q. 15.9, Page 391]**

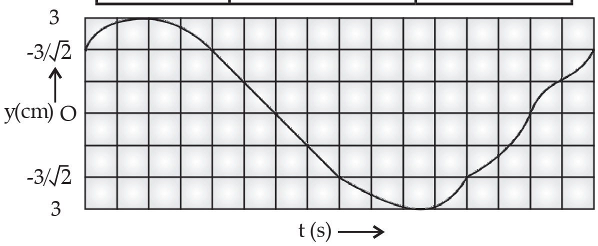
**Ans.** At $x = 0$, The equation becomes-

$$y(0,t) = 3.0\sin\left(36t + \frac{\pi}{4}\right)$$

But $\omega = \frac{2\pi}{T} = 36$ rad/s

or $T = \frac{2\pi}{\omega} = \frac{2\pi}{36} = \frac{\pi}{18}$ sec

| S.No. | Time, $T(s)$ | $y$(cm) |
|---|---|---|
| 1 | 0 | $\frac{3}{\sqrt{2}}$ |
| 2 | $\frac{T}{8}$ | 3 |
| 3 | $\frac{2T}{8}$ | $\frac{3}{\sqrt{2}}$ |
| 4 | $\frac{3T}{8}$ | 0 |
| 5 | $\frac{4T}{8}$ | $-\frac{3}{\sqrt{2}}$ |
| 6 | $\frac{5T}{8}$ | −3 |
| 7 | $\frac{6T}{8}$ | $-\frac{3}{\sqrt{2}}$ |
| 8 | $\frac{7T}{8}$ | 0 |




Similarly, graphs for $x = 2$ and $x = 4$ can be plotted. Amplitude and frequency of these waves are same *i.e.* They differ in phase.

**Q. 3. For the travelling harmonic wave**

**$y(x, t) = 2.0 \cos 2\pi [10t – 0.0080x + 0.35]$**

**where $x$ and $y$ are in cm and $t$ in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of (a) 4 cm, (b) 0.5 m, (c) $\lambda/2$ (d) $3\lambda/4$.**

**[NCERT Ex. Q. 15.10, Page 391]**

**Ans.** Here

$$y = 2\cos 2\pi(10t – 0.0080x + 0.35)$$

$$= 2\cos [2\pi(10\,t – 0.0080x) + 2\pi(0.35)]$$

$$= 2\cos\left[2\pi \times 0.0080\left(\frac{10}{0.0080}t - x\right) + 2\pi \times 0.35\right]$$

Standard equation for a travelling wave is

$$y = r\cos\left[\frac{2\pi}{\lambda}(vt - x) + \phi\right]$$

Here $\phi = \frac{2\pi}{\lambda}x = 2\pi \times 0.008x$

$$\left[\because \frac{1}{\lambda} = 0.008\text{cm} = 125 \text{ cm}\right]$$

**(a)** when $x = 4$ m $= 400$ cm,

$\phi = 2\pi \times 0.008 \times 400$

$= 6.4\pi$ rad

**(b)** when $x = 0.5$ m $= 50$ cm,

$\phi = 2\pi \times 0.008 \times 50$

$= 0.8\pi$ rad

**(c)** when $x = \frac{\lambda}{2}$, $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$ rad

**(d)** when $x = \frac{3}{4}\lambda$, $\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2}$ rad

**Q. 4. The transverse displacement of a string (clamped at its both ends) is given by**

$$y(x,t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\ \pi t)$$

**Where $x$ and $y$ are in $m$ and $t$ in $s$. The length of the string is 1.5 m and its mass is $3.0 \times 10^{-2}$ kg.**

**Answer the following:**

**(a) Does the function represent a travelling wave or a stationary wave?**

**(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave?**

**(c) Determine the tension in the string.**

**[NCERT Ex. Q. 15.11, Page 392]**

**Ans.** **(a)** Given: $y(x,t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$

It is same as the equation for stationary wave-

$y(x, t) = 2a \sin kx \cos \omega t$

Therefore, this represents stationary wave.

**(b)** Standing wave produced by two waves-

$y(x, t) = 2a \sin kx \cos \omega t$

Due to superposition of these waves-

$y_1(x, t) = a \sin (\omega t - kx)$

$y_2(x, t) = a \sin (\omega t + kx)$

Equation given-

$$y(x,t) = 0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$$

$$\text{or } k = \frac{2\pi}{\lambda} = \frac{2\pi}{3} \text{ or } \lambda = 3 \text{ m}$$

$$\omega = 120\pi \text{ rad/s}$$

$$\omega = 2\pi\nu \text{ or } \nu = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi}$$

As, $\nu = 60 \text{ Hz}$

**(c)** Speed of wave, $v = \nu\lambda$

$= 60 \times 3$

$= 180$ m/s

$2a = 0.06$ or $a = 0.03$

$$\therefore\ y_1 = 0.03\sin\left(120\pi t - \frac{2\pi}{3}x\right)$$

$$y_2 = 0.03\sin\left(120\pi t + \frac{2\pi}{3}x\right)$$

Wave's speed = 180 m/s

Mass per unit length

$$\mu = \frac{m}{l}$$

$$= \frac{3.0}{1.5}\times 10^{-2} \text{ kg } m^{-1}$$

$$= 2\times 10^{-2} \text{ kg } m^{-1}$$

Therefore, $T = v^2\mu$

$T = (180)^2 \times 2 \times 10^{-2}$

Tension, T = 648 N

**Q. 5. For the wave on a string described in Q. 4 do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers. (ii) What is the amplitude of a point 0.375 m away from one end?**

**[NCERT Ex. Q. 15.13, Page 392]**

**Ans.** **(i)** Transverse Displacement is-

$$y(x,t) = 0.06\sin\frac{2\pi}{3}x\cos 120\pi t$$

**(a)** Yes, cos 120 πt represents its frequency, as this function does not depend on x, so oscillation's frequency of all points on the string is same.

**(b)** Yes, phase of all points on string is same. [same reason in (a)]

**(c)** Now, amplitude at a point-0.375 m away from one end-

$$a = 0.06\sin\frac{2\pi}{3}\times 0.375$$

$$= 0.06\sin 0.7854$$

$$= 0.06\times 0.707$$

$$a = 0.042 \text{ m}$$

**Q. 6. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency 45 Hz. The mass of the wire is $3.5 \times 10^{-2}$ kg and its linear density is $4.0 \times 10^{-2}$ kg m$^{-1}$. What is**

**(a) the speed of a transverse wave on the string, and**

**(b) the tension in the string ?**

**[NCERT Ex. Q. 15.14, Page 392]**

**Ans.** Using $\mu = \frac{M}{l}$,

$$= \frac{3.5\times 10^{-2}}{4\times 10^{-2}}$$

$= 0.875$ m

**(a)** In fundamental mode,

$$l = \frac{\lambda}{2}$$

or $\lambda = 2l = 2 \times 0.875 = 1.75$ m

Velocity $= n\lambda = 45 \times 1.75$

$= 78.75 \text{ ms}^{-1}$

**(b)** $v = \sqrt{T/m}$

or $T = v^2 m$

$= (78.75)^2 \times 4 \times 10^{-2}$

$\approx 248$ N.

**Q. 7. A 1 metre long tube open at one end, with a movable piston at other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.**

**[NCERT Ex. Q. 15.15, Page 392]**

**Ans.** Using frequency of $n^{th}$ mode vibration of the closed organ pipe

$$\nu_n = \frac{(2n-1)v}{4l}, \quad \text{(i)}$$

$$340 = \frac{(2n_1 - 1)}{4\times 25.5}$$

and frequency of $(n + 1)^{th}$ mode of vibration of closed pipe of length

$$\nu_{A+1} = [2(n+1)-1]\frac{\nu}{4l_2} = (2n+1)\frac{\nu}{4l_2} \quad \text{(ii)}$$

From the above equation

$$\frac{2n_1-1}{25.5} = \frac{2n_1+1}{79.3}$$

or $n_1 \approx 1$

then from eqn (i), $v = \dfrac{\nu_n \times 4l}{(2n_1-1)}$

$$= \frac{340 \times 4 \times 25.5}{(2\times1-1)}$$

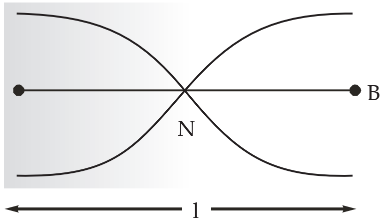
$= 34680 \text{ cms}^{-1}$

$= 346.8 \text{ ms}^{-1}$

**Q. 8. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel ?**

**[NCERT Ex. Q. 15.16, Page 392]**

**Ans.** When a rod clamped in the middle it has anti-nodes (A) at its ends and node (N) at the point of clamping. In fundamental mode, so the length of the rod is




$$l = \frac{\lambda}{2} \text{ or } \lambda = 2l,$$

where $l$ = length of rod

and $\lambda$ = wavelength of the wave

**Given :** $l = 100$ cm, $v = 2.53 \text{ kHz} = 2.53 \times 10^3$ Hz

$\therefore$ $\lambda = 2 \times 100 = 200$ cm

When $v$ be the speed of sound in steel, then

$v = \nu\lambda = 2.53 \times 10^3 \times 200$

$= 506 \times 10^3 \text{ cms}^{-1}$

$= 5.06 \times 10^3 \text{ ms}^{-1}$

$v = 5.06 \times 10^3 \text{ ms}^{-1}$

**Q. 9. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 ms$^{-1}$).**

**[NCERT Ex. Q. 15.17, Page 392]**

**Ans.** Given: Pipe's length, $l = 20$ cm

$= 0.2$ m

frequency, $v = 430$ Hz

speed of sound = 340 m/s

For closed pipe, fundamental frequency,

$$\nu_1 = \frac{v}{4l} = \frac{340}{4\times0.2} = 425 \text{ Hz}$$

Hence, fundamental frequency will resonate with the source.

For open pipe, fundamental frequency,

$$\nu_2 = \frac{v}{2l} = \frac{340}{2\times0.2} = 850 \text{ Hz}$$

Then, there will be no resonance.

**Q. 10. Two sitar strings A and B playing the note '*Ga*' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?**

**[NCERT Ex. Q. 15.18, Page 392]**

**Ans.** Let $\nu_A$, $\nu_B$ be frequency of A and B respectively

$\nu_A - \nu_B = \pm 6$

or $\nu_B = \nu_A \pm 6 = 324 \pm 6$

$\nu_B = 330 \text{ or } 318$ Hz

When tension is reduced, frequency also decreases-

$$\nu \propto \sqrt{T}$$

$\therefore$ No. of Beats also decreases

Hence, $\nu_B = 324 - 6 = 318$ Hz

**Q. 11. A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air.**

**(i) What is the frequency of the whistle for a platform observer when the train**

**(a) approaches the platform with a speed of 10 ms$^{-1}$,**

**(b) recedes from the platform with a speed of 10 ms$^{-1}$?**

**(ii) What is the speed of sound in each case? The speed of sound in still air can be taken as 340 ms$^{-1}$.**

**[NCERT Ex. Q. 15.20, Page 393]**

**Ans. (i)** Given: $\nu = 400$ Hz, $v = 340$ m/s

**(a)** Velocity of train approaching platform $v_s = 10$ m/s

$$\nu' = \frac{v}{v - v_s} \times \nu = \frac{340 \times 400}{340 - 10}$$

$\nu' = 412.12$ Hz

**(b)** frequency, when the train recedes-

$$\nu'' = \frac{v \times \nu}{v + v_s} = \frac{340 \times 400}{340 + 10}$$

$\nu'' = 388.6$ Hz

(c) Speed of sound remains same in both cases, ***i.e.*** 340 m/s

**Q. 12. A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the**

**station with a speed of 10 $ms^{-1}$. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 $ms^{-1}$? The speed of sound in still air can be taken as 340 $ms^{-1}$.**

**[NCERT Ex. Q. 15.21, Page 393]**

**Ans.** Given: $\nu$ = 400Hz, $v_w$ = 10m/s, $v$ = 340m/s.

When wind is blowing in sound's direction –

$\therefore$ Effective speed of sound = $v + v_w$

= (340 + 10) m/s

= 350 m/s

When the source and listener both are at rest therefore, frequency remains unchanged *i.e.*, 400Hz

For stationary observer,

$$\text{Sound's wavelength}, \lambda' = \frac{v+v_m}{v}$$

$$= \frac{350}{400} = 0.875 \text{ m.}$$

When observer is running towards the yard.

Velocity of observer, $v$ = −10 m/s, velocity of source, $v_o$ = 0.

$$\nu' = \frac{v - v_o}{v - v_s} \times \nu$$

$$= \frac{340+10}{340-0} \times 400$$

$$= \frac{350}{340} \times 400$$

$$= 411.8 \text{ Hz}$$

As sound wave's wavelength is not affected by observer's motion, it remains unchanged. Speed of sound relative to the observer –

$\nu''$ = 340 + 10 = 350 m/s.

Hence, the situations in both the cases are not exactly identical.

**Q. 13. A travelling harmonic wave on a string is described by**

**$y(x, t)$ = 7.5 sin (0.0050$x$ + 12$t$ + $\pi$/4)**

**(a) what are the displacement and velocity of oscillation of a point at $x$ = 1 cm, and $t$ = 1 s? Is this velocity equal to the velocity of wave propagation?**

**(b) Locate the points of the string which have the same transverse displacements and velocity as the $x$ = 1 cm point at $t$ = 2 s, 5 s and 11 s.**

**[NCERT Ex. Q. 15.22, Page 393]**

**Ans.** (a) Given:

$$y\ (x,\ t) = 7.5\sin(0.0050x + 12t + \frac{\pi}{4}) \qquad (I)$$

Comparing with standard equation

$$y\ (x,t) = a\sin(kx + \omega t + \phi)$$

Velocity of particle –

$$\frac{dy}{dt} = a\omega\cos(kx + \omega t + \phi)$$

$$v = 7.5 \times 12\cos\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

At $x$ = 1cm, $t$ = 1s.

From $eq^n$ (I) –

$$y(1,1) = 7.5\sin\left(0.005\times1 + 12\times1 + \frac{\pi}{4}\right).$$

$$= 7.5\sin\left(12.005 + \frac{\pi}{4}\right)$$

$$= 7.5\sin(12.79\,\text{radian})$$

$$= 7.5\sin(735.81°)$$

$$= 7.5 \times 0.2217$$

$$= 1.6629\,\text{cm}$$

$$\approx 1.663\,\text{cm}.$$

from $eq^n$ (II), at $x$ = 1 cm, $t$ = 1 s.

$$\text{particle s velocity}, v = 7.5\times12\cos\left(0.005\times1 + 12\times1 + \frac{\pi}{4}\right)$$

$$= 90\cos(12.005 + 0.785)$$

$$= 90\cos(12.79)\text{rad}$$

$$= 90\cos(732.81°)$$

$$= 90 \times 0.975$$

$$= 87.75 \text{ cm/s.}$$

$$\text{velocity of wave}, v = \frac{\omega}{k}$$

$$= \frac{12}{0.005}$$

$$= 24 \text{ m/s}$$

Sign between $kx$ and $\omega t$ is same, therefore the wave is travelling along negative $x$ axis, so, velocity of wave $v_\omega$ = −24 m/s.

This is not point or particles velocity.

**(b)** As, $$k = \frac{2\pi}{\lambda} \text{ or } \frac{2\pi}{k}$$

$$= \frac{2\times3.14}{0.005}$$

$$= 1256\,\text{cm}$$

$$= 12.56\text{m}$$

All points located at distance n$\lambda$ (where n = integer)

From the point $x$ = 1 cm have the same transverse displacement and velocity.

**Q. 14. A narrow sound pulse (for example, a short Pip by a whistle) is sent across a medium.**

**(a) Does the pulse have a definite (i) wavelength, (ii) frequency, (iii) speed of propagation ?**

**(b) If the pulse rate is 1 after energy 20 s, *i.e.*, the whistle is blown for a split second after every 20 second, is the frequency of the note produced by the whistle equal to 1/20 = 0.05 Hz ?**

**[NCERT Ex. Q. 15.23, Page 393]**

**Ans. (a)** A short pip by a whistle has neither a definite wavelength nor a definite frequency. However, its speed of propagation is fixed, being equal to speed of sound in air.

**(b)** No, frequency of the note produced by whistle is not 1/20 = 0.05 Hz. Rather 0.05 Hz is the frequency of repetition of the short pip of the whistle.

**Q. 15. One end of a long string of linear mass density 8.0 × $10^{-3}$ kg $m^{-1}$ is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At $t = 0$, the left end (fork end) of the string $x = 0$ has zero transverse displacement ($y = 0$) and is moving along positive $y$-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement $y$ as function of $x$ and $t$ that describes the wave on the string.**

**[NCERT Ex. Q. 15.24, Page 393]**

**Ans.** Wave is travelling along + $x$ – axis direction. So,

$$y(x, t) = a \sin(wt - kx)$$

Amplitude, $a$ = 5 cm = 0.05 m, m = 90kg

Tension in the string, T = mg

$$= 90 \times 9.8 = 882 \text{ N}$$

Velocity of Transverse wave, $v = \sqrt{\dfrac{T}{\mu}}$

$$v = \sqrt{\frac{882}{8.0 \times 10^{-3}}}$$

$$= 332 \text{ m/s}$$

Angular frequency, $\omega = 2\pi\nu$

$$= 2 \times 3.14 \times 256$$

$$= 1607.6$$

$$= 1.6 \times 10^3 \text{ rad/s.}$$

Wavelength, $\lambda = \dfrac{v}{\upsilon} = \dfrac{332}{256}$ m.

Propagation constant, $k = \dfrac{2\pi}{\lambda}$

$$k = \frac{2 \times 3.14}{(332/256)}$$

$$= \frac{2 \times 3.14 \times 256}{332}$$

$$= 4.84 \text{ m}$$

Thus, required equation of wave is –

$y(x, t)$ **= 0.05 sin (1.6 × $10^3 t$ – 4.84$x$) m.**

**Q. 16. A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km $h^{-1}$. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 $ms^{-1}$.**

**[NCERT Ex. Q. 15.25, Page 393]**

**Ans.** Given: Frequency of SONAR, $v$ = 40 kHz = 40 × $10^3$ Hz

Speed of observer, $v_\circ$ = 360 km/h = 100m/s

Speed of sound, $v$ = 1450 m/s

The observer is moving towards the source (at rest),

∴ Apparent frequency received by submarine,

$$\nu' = \frac{(v + \nu_\circ)\nu}{v}$$

$$\nu' = \frac{(1450 + 100) \times 40 \times 10^3}{1450}$$

$$\nu' = 4.276 \times 10^4 \text{ Hz.}$$

This is reflection frequency by energy submarine and is observed by SONAR –

∴ $v_s$ = 360 km/s = 100 m/s, $\nu_\circ$ = 0.

so, Apparent frequency, $\nu'' = \left(\dfrac{v\nu'}{v - v_s}\right)$

$$= \left(\frac{1450 \times 4.276 \times 10^4}{1450 - 100}\right)$$

$$\nu'' = 4.59 \times 10^4 \text{ Hz.}$$

**Q. 17. A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?**

**[NCERT Ex. Q. 15.27, Page 394]**

**Ans.** Given: Frequency of sound emitted by bat, $v$ = 40 kHz.

Velocity of bat, $v = 0.03v_s$ [$v_s$ = speed of sound]

Apparent frequency (after striking the wall),

$$\nu^1 = \left(\frac{v}{v - v_s}\right) \times \nu$$

$$= \left(\frac{v}{v - 0.03v}\right) 40 \times 10^3$$

$$= \frac{40 \times 10^3}{0.97} \text{ Hz.}$$

This frequency is reflected by wall, and bat receives.

∴ $v_s = 0$, $v_b = 0.03\,v$

Now, Apparent frequency which bat can hear,

$$\nu'' = \frac{v + v_b}{v}\nu'$$

$$= \left(\frac{v + 0.03v}{v}\right) \times \frac{40 \times 10^3}{0.97}$$

$$= 42.47 \times 10^3 \text{ Hz}$$

$$\nu'' = 42.47 \text{ KHz.}$$

**TIPS... & TRICKS...**

- *Study and understands about periodic function and S.H.M.*
- *Learn about Time period in S.H.M. and Various conditions.*
- *Understand about velocity and acceleration in S.H.M.*
- *Study Energy in S.H.M. and its types.*
- *Study various type of oscillations with examples.*
- *Understand about Resonance with examples.*
- *Study and understands about waves and kinds of waves.*
- *Learn speed of sound waves in various Medium.*
- *Study Progressive waves and its various displacement formulas.*
- *Study Principle of super position.*
- *Understand about stationery waves, Beats and Interference of waves.*
- *Study and understands about tone and overtone about organ pipes.*
- *Study about Doppler Effect.*

## Some Commonly Made Errors

- Students make mistakes while converting the units of pressure. Always remember / atom = 101325 Pa.

### EXPERT ADVICE

- *Make a short summary for specific heat capacity for monoatomic, diatomic and polyatomic gases.*
- *Remember pressure of fluid is not only exerted on the wall, it is exerted everywhere in a fluid.*
- *Kinetic theory explains the behaviour of gases. Brief understanding of behaviour of gases in required. Study them using graph for different processes.*

## OSWAAL LEARNING TOOLS

***For* Suggested Online Videos**

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# WRITING NOTES

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